Math 1540 Final - Extra Study Guide

1. Be able to use the product rule: If y = f(x)g(x), then y' = f(x)g'(x) + g(x)f'(x)

$$z = 8x^2e^x$$

$$\frac{dz}{dx} = 16xe^x + 8x^2e^x$$

$$f(x) = (2x^5 - 2x^3 - 6)(5x^2 - 5\sqrt{x})$$

$$f'(x) = (2x^5 - 2x^3 - 6)\left(10x - \frac{5}{2\sqrt{x}}\right) + (5x^2 - 5\sqrt{x})(10x^4 - 6x^2)$$

2. Know how to use the quotient rule: If y = $\frac{f(x)}{g(x)}$ Then y' = $\frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$

$$y = \frac{5x^2 + x - 1}{x^3 - 9x^2}$$

$$\frac{dy}{dx} = \frac{-5x^4 - 2x^3 + 12x^2 - 18x}{(x^3 - 9x^2)^2}$$

$$y = \frac{x^2 + 2x - 2}{x^2 - 2x + 2}$$

$$\frac{dy}{dx} = \frac{-4x^2 + 8x}{(x^2 - 2x + 2)^2}$$

3. Know how to use the Chain Rule (see Chain Rule video on web site)

If $y = f(u)$, $and\ u = g(x)$, $then\ y' = \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$$g(x) = \left(5x^4 + 4x + \frac{4}{x^2}\right)^{8/5}$$

$$g'(x) = \frac{8}{5}\left(5x^4 + 4x + \frac{4}{x^2}\right)^{3/5}\left(20x^3 + 4 - \frac{8}{x^3}\right)$$

$$f(x) = (2x^5 - 4x^4 + 3)^{308}$$

$$f'(x) = 308(2x^5 - 4x^4 + 3)^{307}(10x^4 - 16x^3)$$

4. Know how to calculate integrals using the substitution rule (see video on web site)

$$\int_0^1 \frac{4x^3}{(1+x^4)^4}\,dx = \frac{7}{24}$$

$$\int \frac{t^3}{\sqrt[4]{3+t^4}}\,dt = \frac{1}{3}(3+t^4)^{3/4} + C$$

5. Understand implicit differentiation

Suppose that x and y are related by the equation $x^3 + (2y + 1)^2 = y^2$. Use implicit differentiation to determine $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{-3x^2}{2(3y + 2)}$$

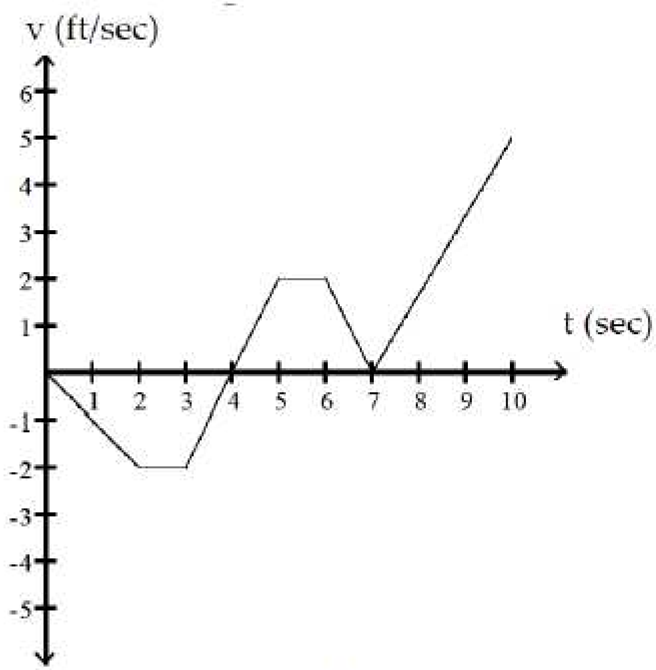
Use implicit differentiation to find dy/dx.

$$\cos xy + x^7 = y^7$$

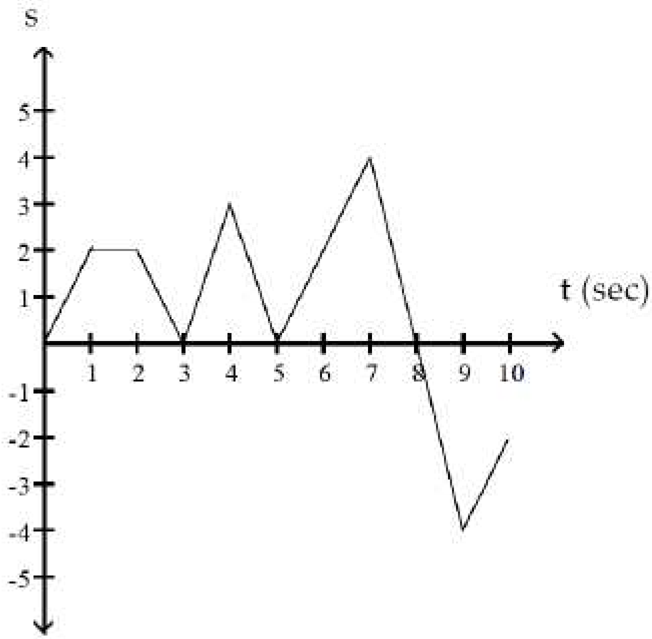
$$\mathbf{dy/dx} = \frac{7x^6 - y \sin xy}{7y^6 + x \sin xy}$$

6. Position s, velocity v, and acceleration.

$v = s'$, $a = v' = s''$ To get velocity, you need the derivative of the position. To get acceleration, you need to take the derivative of the velocity or the second derivative of the position.

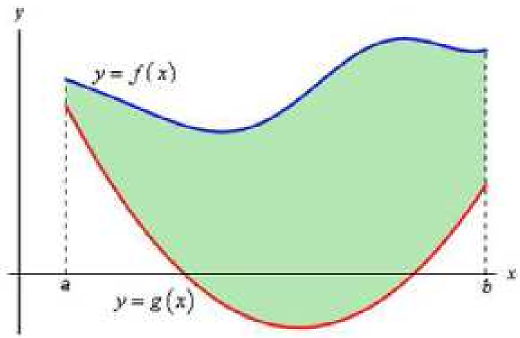


What is the body's acceleration when t = 1 sec?



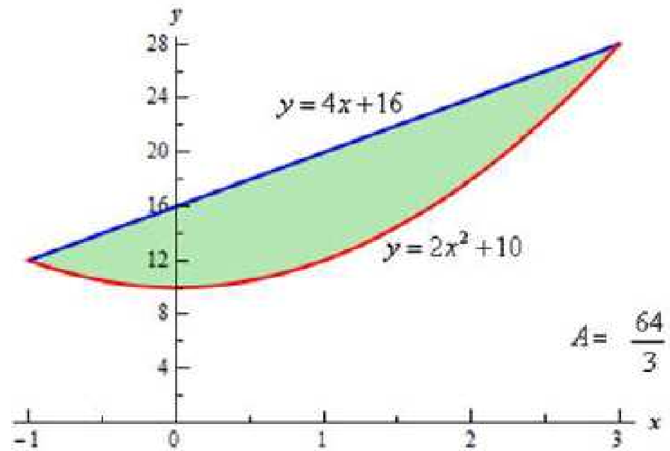
What is the body's velocity when t = 8 sec?

7. Area between curves

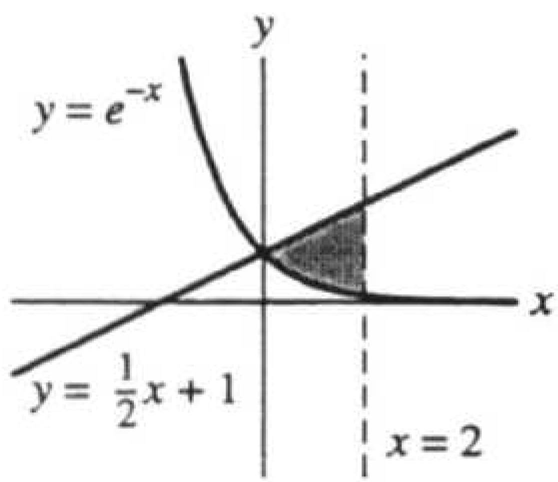


$$A=\int_a^b f(x)-g(x)dx$$

$$A=\int_a^b \begin{pmatrix}\text{upper}\\ \text{function}\end{pmatrix}-\begin{pmatrix}\text{lower}\\ \text{function}\end{pmatrix}dx,$$

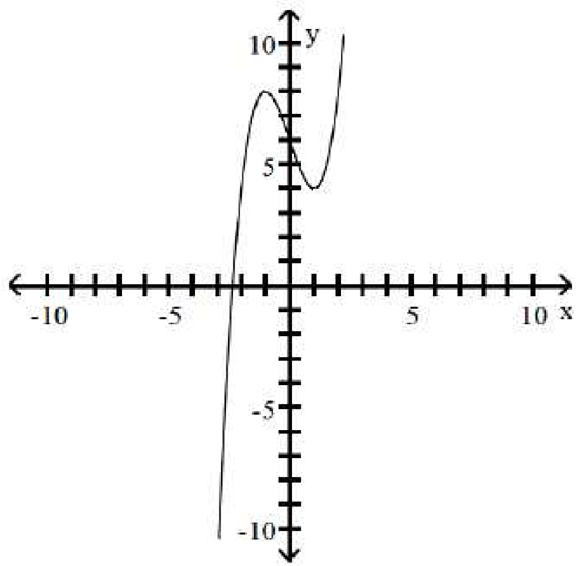


Use the graph below to determine the area of the shaded region.



answer $2 + e^{-2}$

9. A function $y = f(x)$ is increasing if $y' > 0$ and is decreasing if $y' < 0$. A graph is concave up if $y'' > 0$ and concave down if $y'' < 0$ On the graph below, state where $y' > 0$ and < 0. Label local min and max. Label intervals where function is concave up or down and where it is concave up.



10. To find the equation of the tangent line to a graph at a point (x_1, y_1), you must find

$y' = f'(x_1) = m$ which will be the slope of the tangent line at the point.

Then write $y_1 = mx_1 + b$ and solve $b = y_1 - mx_1$ Then the equation is y $= mx + b$

Find an equation of the tangent line at the indicated point on the graph of the function.

1) $y = f(x) = x - x^2$, $(x, y) = (4, -12)$

$y = -7x + 16$

2) $y = f(x) = 10\sqrt{x} - x + 1$, $(x, y) = (100, 1)$

$y = -\frac{1}{2}x + 51$

11. Finding limits. a) Plug in b) Use calculator, c) factor, d)use "trick" or try L'Hopital's Rule

$$\lim_{x \to -1} \frac{x^2 - 1}{\sqrt{x^2 + 3} - 2} = 4$$

$$\lim_{x \to 0} \frac{\sqrt{36 + x} - \sqrt{36 - x}}{x} = \frac{1}{6}$$

12. Finding maxima and minima: a) Calculate critical points (y' = 0 or undefined) b) Determine concavity at critical points. y'' > 0 (concave up ∪ relative min), y'' < 0 (concave down ∩ relative max) c) Check y value at any given end points if y is defined on an interval [a, b].

$$f(x) = \frac{7x}{x^2 + 1}$$

Relative minimum: $\left(-1, -\frac{7}{2}\right)$, relative maximum: $\left(1, \frac{7}{2}\right)$

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$f(x) = -x^3 + 3x^2 - 2$

Relative minimum: (0, −2); relative maximum: (2, 2)

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$f(x) = x^3 - 3x + 5; [-1, 3]$

Absolute maximum: 23, absolute minimum: 3

13. Fundamental Theorem of Calculus Part 1. If $F(x) = \int_a^x f(t)\,dt$. $Then\ F'(x) = f(x)$

However, if $F(x) = \int_a^{g(x)} f(t)\,dt$. $Then\ F'(x) = f(g(x))g'(x)$ **Remember the Chain Rule**

$$\frac{d}{dx}\int_0^x 7t\cos(t^6)\,dt = 7x\cos(x^6)$$

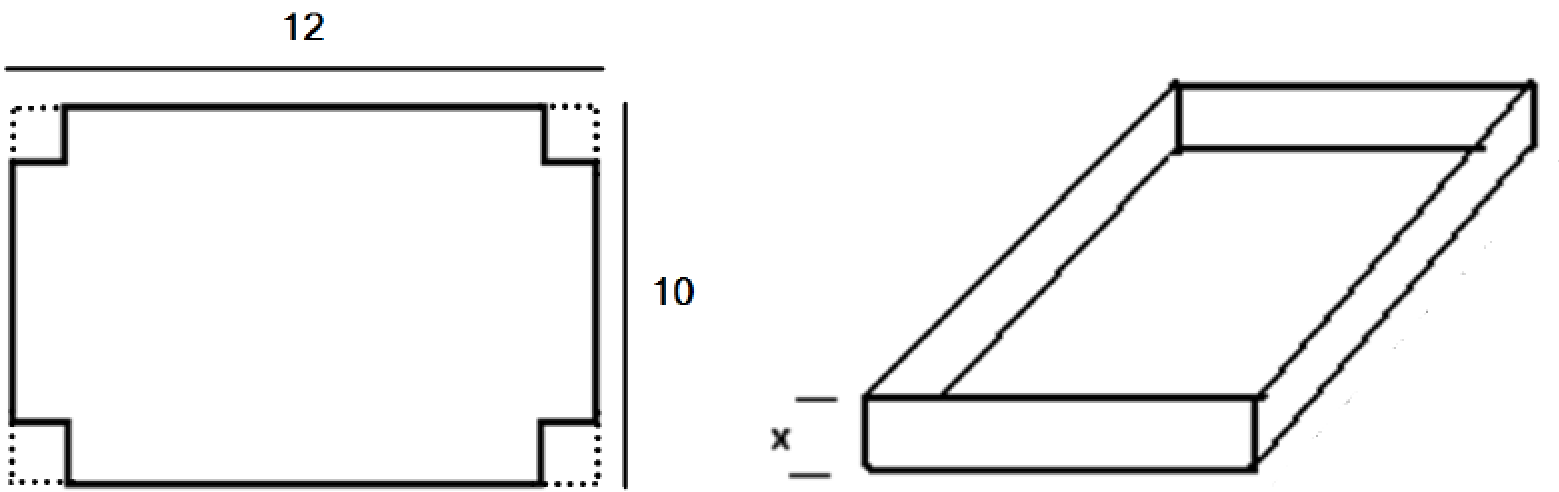
$$\frac{d}{dx}\int_0^{\sin x} \frac{1}{1 - t^2}\,dt = \frac{\cos x}{1 - \sin^2 x}$$

14. Fundamental Theorem of Calculus Part 2. If $F(x) is\ an\ antiderivative\ of\ f(x) on\ [a, b]. then$

$$\int_a^b f(x)\,dx = F(b) - F(b)$$

15. Optimization

A sheet of metal 12 inches by 10 inches is to be used to make a open box. Squares of equal sides x are cut out of each corner then the sides are folded to make the box. Find the value of x that makes the volume maximum. What are the dimensions and the volume?



V(1.81) = 96.77 sq. inches. Dimensions 8.38" x 6.38" x 1.81"

This problem can be solved with Calculus or by graphing the correct function.

16. Related rates.

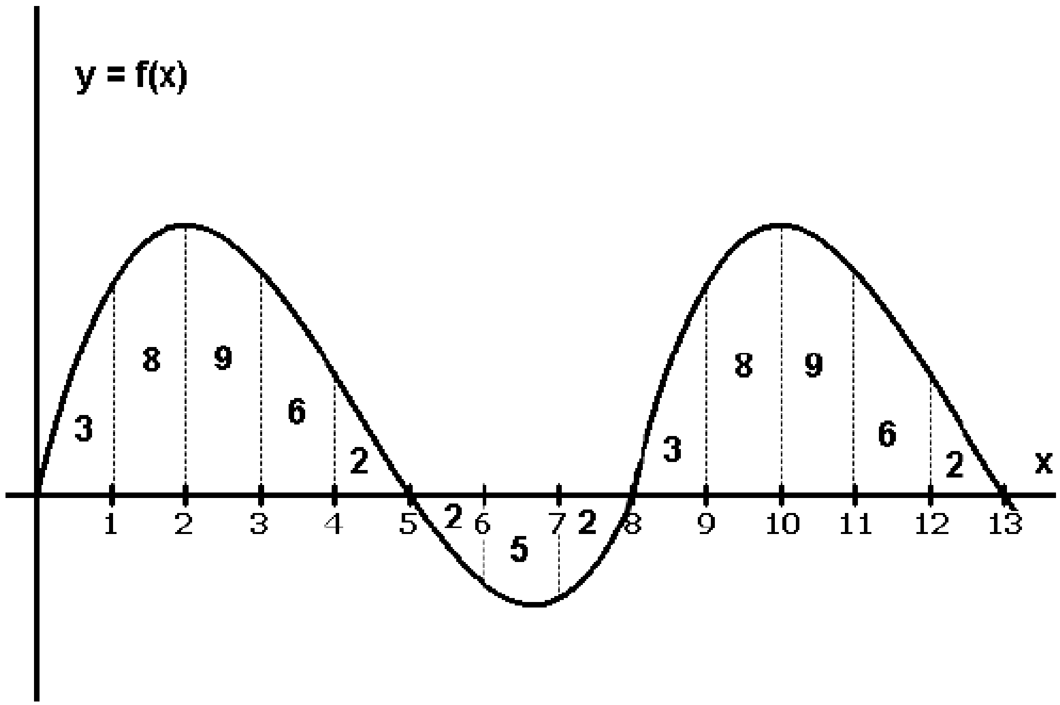
1) Water is falling on a surface, wetting a circular area that is expanding at a rate of 9 mm^2/s. How fast is the radius of the wetted area expanding when the radius is 166 mm? (Round your answer to four decimal places.)

0.0086 mm/s

2) Water is discharged from a pipeline at a velocity v (in ft/sec) given by $v = 1064p^{(1/2)}$, where p is the pressure (in psi). If the water pressure is changing at a rate of 0.266 psi/sec, find the acceleration (dv/dt) of the water when $p = 32.0$ psi.

25.0 ft/sec^2

Additional Note on Fundamental Theorem of Calculus Part 1.



t	S(t)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

Given $f(x)$ *in the graph above we define* $s(t) = \int_0^t f(x)dx$.

The numbers under the curve represent the true area in each section. On the chart below sketch s(t) and then answer the questions below.

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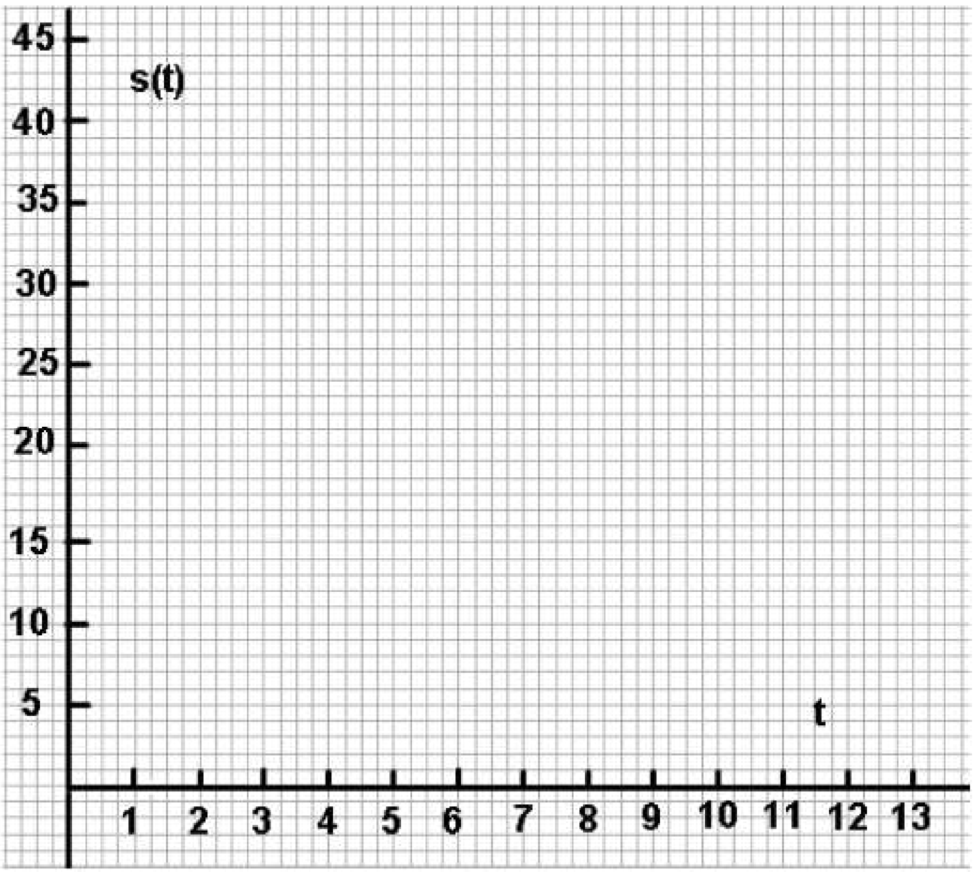

What is s'(t) ?__________________________

What is s''(t) ?_________________________

At what value of t does s(t) reach its first relative max? ______ . What is the sign of s''(t) at that point?

At what value of t does s(t) reach its first relative min? _______. What is the sign of s''(t) at that point?