

Math 2013 Final Study Practice Quiz

Name: Last _____, First _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question. You must show your work to get credit.

Find the angle between u and v in radians.

1) $u = 2i - 3j - 3k$, $v = 10i + 4j - 4k$

☒ A) 1.19

B) 1.56

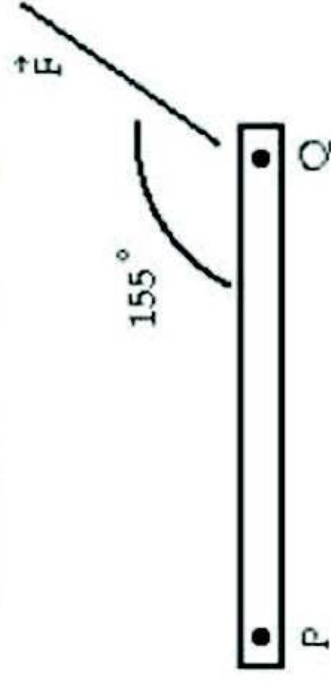
C) 0.38

D) 1.44

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right) = \cos^{-1}\left(\frac{(2i - 3j - 3k) \cdot (10i - 4j - 4k)}{\sqrt{4+9+9} \sqrt{100+16+16}}\right) = \cos^{-1}\left(\frac{20-12+12}{\sqrt{22}\sqrt{132}}\right) = 1.19 A$$

Solve the problem.

- 2) Find the magnitude of the torque in foot-pounds at point P for the following lever:



$$|\vec{PQ}| = 4 \text{ in. and } |\vec{F}| = 30 \text{ lb}$$

A) 4.23 ft-lb

B) 120 ft-lb

C) 14.37 ft-lb

D) 9.06 ft-lb

$$\text{Torque} = |r||F|\sin\theta$$

$$|\vec{PQ}| = 4/12 \text{ ft}$$

$$\text{Torque} = \left(\frac{4}{12}\right)(30)\sin 155^\circ$$

$$= \text{A) } 4.23 \text{ ft-lb}$$

Find parametric equations for the line described below.

3) The line through the point $P(3, -4, 5)$ parallel to the vector $-8\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$

A) $x = 8t + 3, y = 4t - 4, z = -4t + 5$

C) $x = 8t - 3, y = 4t + 4, z = -4t - 5$

B) $x = -8t + 3, y = 4t - 4, z = -4t + 5$

D) $x = -8t - 3, y = 4t + 4, z = -4t - 5$

$P(3, -4, 5)$ parallel to $-8\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$
start dir stat

$x = 3 - 8t, y = -4 + 4t, z = 5 - 4t$ B

Calculate the requested distance.

- 4) The distance from the point $S(-6, -6, 9)$ to the plane $2x + 2y + z = 6$

A) $\frac{7}{3}$

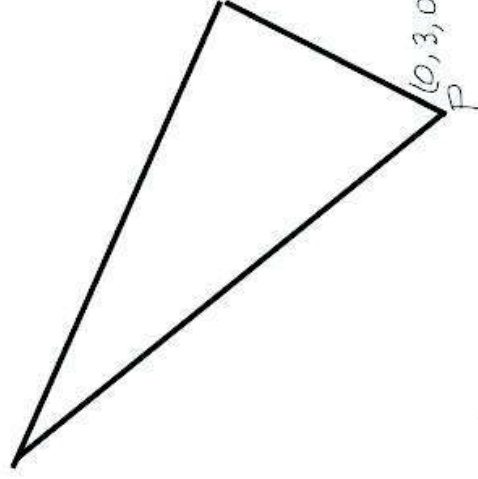
B) 7

C) 9

D) 3

$S(-6, -6, 9)$ to plane $2x + 2y + z = 6 \Rightarrow$ normal vector to plane is $2i + 2j + k = n$

$$|n| = \sqrt{4+4+1} = 3$$



$$\vec{PS} = -6i - 9j + 9k$$

Pick an easy pt. on plane

$$d = \left| \vec{PS} \cdot \frac{n}{|n|} \right| \quad d = \left| (-6i - 9j + 9k) \cdot \frac{(2i + 2j + k)}{3} \right| = \left| \frac{-12 - 18 + 9}{3} \right| = \frac{21}{3} = 7 \quad \underline{\underline{B}}$$

Write the equation for the plane.

5) The plane through the point $P(-6, -6, -5)$ and normal to $\mathbf{n} = -8\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}$.

A) $8x + 7y - 4z = 26$

B) $6x + 6y - 5z = 26$

C) $-6x - 6y + 5z = 26$

D) $-8x - 7y + 4z = 70$

Let (x, y, z) be point on the plane, $(x+6)\mathbf{i} + (y+6)\mathbf{j} + (z+5)\mathbf{k}$ is a vector on the plane which must be \perp to $(-8\mathbf{i} - 7\mathbf{j} + 4\mathbf{k})$, so $[(x+6)\mathbf{i} + (y+6)\mathbf{j} + (z+5)\mathbf{k}] \cdot [-8\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}] = 0$

$$-8(x+6) - 7(y+6) + 4(z+5) = 0, \quad -8x - 48 - 7y - 42 + 4z + 20 = 0$$

$$-8x - 7y + 4z = 48 + 42 - 20 = 70$$

$$\boxed{-8x - 7y + 4z = 70} \quad \text{D}$$

Find the intersection of the line and the plane.

6) $x = 7 + 8t, y = -9 + 2t, z = 3 + 8t; 2x + 8y + 7z = 7$

A) $(15, -7, 11)$

B) $(-1, -11, -5)$

☒ C) $(11, -8, 7)$

D) $\left(3, -\frac{20}{11}, -3\right)$

Need to find t that causes line to intersect plane

$$2(7+8t) + 8(-9+2t) + 7(3+8t) = 7$$

$$14 + 16t - 72 + 16t + 21 + 56t = 7$$

$$88t = 7 - 14 + 72 - 21 = 44$$

$$t = \frac{1}{2}$$

So, pt. of intersection is

$$x = 7 + 8\left(\frac{1}{2}\right) = 11, y = -9 + 2\left(\frac{1}{2}\right) = -8, z = 3 + 8\left(\frac{1}{2}\right) = 7$$

the pt. is $(11, -8, 7)$ C

The position vector of a particle is $\mathbf{r}(t)$. Find the requested vector.

7) The acceleration at $t = 3$ for $\mathbf{r}(t) = (5t - 2t^4)\mathbf{i} + (10 - t)\mathbf{j} + (8t^2 - 2t)\mathbf{k}$

A) $\mathbf{a}(3) = -216\mathbf{i} + 16\mathbf{k}$

B) $\mathbf{a}(3) = 216\mathbf{i} + 16\mathbf{k}$

C) $\mathbf{a}(3) = -216\mathbf{i} - \mathbf{j} + 16\mathbf{k}$

D) $\mathbf{a}(3) = -54\mathbf{i} + 16\mathbf{k}$

Need 2nd derivative

$$\mathbf{r}(t) = (5 - 8t^3)\mathbf{i} - \mathbf{j} + (16t - 2)\mathbf{k}$$

$$\mathbf{a}(t) = -24t^2\mathbf{i} + 16\mathbf{k}$$

$$\mathbf{a}(3) = -24(3)^2\mathbf{i} + 16\mathbf{k}$$

$$= -216\mathbf{i} + 16\mathbf{k} \quad A$$

Write the equation for the plane.

8) The plane through the point $A(5, 7, 4)$ perpendicular to the vector from the origin to A .

A) $5x + 7y + 4z = \sqrt{90}$

B) $5x + 7y + 4z = -90$

C) $5x + 7y + 4z = 90$

D) $5x + 7y + 4z = 16$

Vector from origin to A is $(5\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$, vector from pt. on plane to A is $(x-5)\mathbf{i} + (y-7)\mathbf{j} + (z-4)\mathbf{k}$

To be perpendicular the dot products must be zero, so

$$(5\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}) \cdot [(x-5)\mathbf{i} + (y-7)\mathbf{j} + (z-4)\mathbf{k}] = 0$$

$$5(x-5) + 7(y-7) + 4(z-4) = 0$$

$$5x - 25 + 7y - 49 + 4z - 16 = 0$$

$$5x + 7y + 4z = 25 + 49 + 16 = 90 \quad \text{so}$$

$$\boxed{5x + 7y + 4z = 90} \quad \text{C}$$

Solve the problem.

9) Find an equation for the level surface of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ that passes through the point $(4, 3, 12)$.

A) $x^2 + y^2 + z^2 = 13$

B) $x + y + z = \pm 13$

☒ C) $x^2 + y^2 + z^2 = 169$

D) $x + y + z = 13$

$\sqrt{x^2 + y^2 + z^2} = C$ level curve, $\sqrt{16 + 9 + 144} = \sqrt{169} = C = 13$
 $\uparrow 4 \uparrow 3 \uparrow 12$ $\sqrt{x^2 + y^2 + z^2} = 13$

$x^2 + y^2 + z^2 = 169$
 C

Find f_x , f_y , and f_z .

10) $f(x, y, z) = \frac{\cos y}{xz^2}$

A) $f_x = \frac{\cos y}{z^2}$; $f_y = \frac{\sin y}{xz^2}$; $f_z = \frac{2 \cos y}{xz}$

B) $f_x = -\frac{\cos y}{z^2}$; $f_y = -\frac{\sin y}{xz^2}$; $f_z = -\frac{2 \cos y}{xz}$

C) $f_x = \frac{\cos y}{x^2 z^2}$; $f_y = \frac{\sin y}{xz^2}$; $f_z = \frac{2 \cos y}{xz^3}$

~~D) $f_x = -\frac{\cos y}{x^2 z^2}$; $f_y = -\frac{\sin y}{xz^2}$; $f_z = -\frac{2 \cos y}{xz^3}$~~

$$f_x = \frac{\cos y}{z^2} \frac{d}{dx} x^{-1} = -\frac{\cos y}{x^2 z^2}, \quad f_y = -\frac{\sin y}{x^2 z^2}, \quad f_z = -\frac{2 \cos y}{x^2 z^3} \quad \textcircled{D}$$

Solve the problem.

11) Evaluate $\frac{dw}{dt}$ at $t = 3\pi$ for the function $w = x^2 - y^2 - 5x$; $x = \cos t$, $y = \sin t$.

A) -9

B) -7

☒ C) 0

D) -3

Chain Rule $\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt}$

$$= (2x - 5)(-\sin t) - 2y \cos t = (2 \cos t - 5)(-\sin t) - 2 \sin t \cos t \Big|_{t=3\pi}$$

$$= (2 \cos 3\pi - 5)(-\sin 3\pi) - 2 \sin 3\pi \cos 3\pi = 0 \quad C$$

Compute the gradient of the function at the given point.

12) $f(x, y, z) = 2xy^3z^2$, $(2, 8, 4)$

A) $\nabla f = 16,384\mathbf{i} + 8192\mathbf{j} + 24,576\mathbf{k}$

(C) $\nabla f = 16,384\mathbf{i} + 12,288\mathbf{j} + 16,384\mathbf{k}$

B) $\nabla f = 8192\mathbf{i} + 8192\mathbf{j} + 6144\mathbf{k}$

D) $\nabla f = 8192\mathbf{i} + 12,288\mathbf{j} + 4096\mathbf{k}$

$$\nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$$

$$\nabla f = 2y^3z^2 \mathbf{i} + 6xy^2z^2 \mathbf{j} + 4xy^3z \mathbf{k},$$

$$\nabla f|_{2,8,4}$$

$$= 2(8)^3(4)^2 \mathbf{i} + 6(2)(8)^2(4)^2 \mathbf{j} + 4(2)(8)^3(4) \mathbf{k}$$

$$= 16,384 \mathbf{i} + 12,288 \mathbf{j} + 16,384 \mathbf{k} \quad \text{(C)}$$

Provide an appropriate response.

13) Find the direction in which the function is increasing most rapidly at the point P_0 .

$$f(x, y, z) = xy - \ln(z), P_0(2, -2, 2)$$

A) $\frac{1}{\sqrt{33}}(-4i - 4j + k)$

C) $\frac{33}{\sqrt{33}}(-4i + 4j - k)$

B) $\frac{1}{\sqrt{33}}(-4i + 4j - k)$

D) $\frac{1}{33}(-4i + 4j - k)$

$$\nabla f = y\mathbf{i} + x\mathbf{j} - \frac{1}{z}\mathbf{k}$$

$$\nabla f|_{2,-2,2} = -2\mathbf{i} + 2\mathbf{j} - \frac{1}{2}\mathbf{k}$$

$$|V| = \sqrt{4+4+1/4} = \sqrt{\frac{33}{4}} = \frac{\sqrt{33}}{2}$$

$$\text{So direction} = \frac{-2\mathbf{i} + 2\mathbf{j} - \frac{1}{2}\mathbf{k}}{\frac{\sqrt{33}}{2}}$$

$$= \frac{-4\mathbf{i} + 4\mathbf{j} - \mathbf{k}}{\sqrt{33}} \quad \text{B}$$

Find the derivative of the function at P_0 in the direction of u .

14) $f(x, y, z) = 4x + 3y + 9z$, $P_0(2, -8, 6)$, $u = 3i - 6j - 2k$

A) $-\frac{24}{7}$

B) $-\frac{33}{7}$

C) $-\frac{12}{7}$

D) $-\frac{15}{7}$

Derivative in direction of u is $\nabla f \cdot \frac{u}{|u|}$, $\nabla f = 4i + 3j + 9k$, $|u| = \sqrt{9+36+4} = 7$

$$\text{So } \nabla f \cdot \frac{u}{|u|} = \frac{(4i + 3j + 9k) \cdot (3i - 6j - 2k)}{7} = \frac{12 - 18 - 18}{7} = -\frac{24}{7} \quad \text{A}$$

Find the linearization of the function at the given point.

15) $f(x, y, z) = 6xy + 2yz - 9zx$ at $(1, 1, 1)$

A) $L(x, y, z) = -3x + 8y - 7z + 2$

C) $L(x, y, z) = 6x + 2y - 9z + 2$

B) $L(x, y, z) = 6x + 2y - 9z + 1$

D) $L(x, y, z) = -3x + 8y - 7z + 1$

$$L(x, y, z)|_{P_0} = f(x_0, y_0, z_0) + \frac{\partial f}{\partial x}|_{P_0}(x - x_0) + \frac{\partial f}{\partial y}|_{P_0}(y - y_0) + \frac{\partial f}{\partial z}|_{P_0}(z - z_0)$$

$$= (6 \times 1 + 2 \times 1 - 9 \times 1)|_{1,1,1} + (6y - 9z)(x - 1)|_{1,1,1} + (2y - 9x)(y - 1)|_{1,1,1} + (2z - 9x)(z - 1)|_{1,1,1}$$

$$= (6 + 2 - 9) - 3(x - 1) + 8(y - 1) - 7(z - 1) = -1 - 3x + 3 + 8y - 8 - 7z + 7 = -3x + 8y - 7z + 1$$

D

Find all the local maxima, local minima, and saddle points of the function.

16) $f(x, y) = x^3 + y^3 - 300x - 75y - 3$

A) $f(10, -5) = -1753$, saddle point; $f(-10, 5) = 1747$, saddle point

B) $f(-10, -5) = 2247$, local maximum; $f(10, 5) = -2253$, local minimum

C) $f(10, 5) = -2253$, local minimum; $f(10, -5) = -1753$, saddle point; $f(-10, 5) = 1747$, saddle point;

$f(-10, -5) = 2247$, local maximum

D) $f(-10, -5) = 2247$, local maximum

$\frac{\partial f}{\partial x} = 3x^2 - 300 = 0 \quad \frac{\partial f}{\partial y} = 3y^2 - 75 = 0$
 $x^2 = 100 \quad y^2 = 25$
 $x = \pm 10 \quad y = \pm 5$

Need to check

$(10, 5), (10, -5), (-10, 5), (-10, -5)$

$(10, 5)$
 $f_{xx} = 6x \Big|_0 = 60 > 0$
 $f_{xy} = 0, f_{yy} = 6y \Big|_5 = 30$

$f_{xx}f_{yy} - f_{xy}^2 = (60)(30) - 0 > 0$
 \Rightarrow No Saddle Point
 $f_{xx} > 0 \Rightarrow$ Concave up
 Local Min $f(10, 5) = -2253$

$(10, -5)$
 $f_{xx} = 6x \Big|_0 = 60 > 0$
 $f_{xy} = 6y \Big|_{-5} = -30$
 $f_{xy} = 0$
 $f_{xx}f_{yy} - f_{xy}^2 = 60(-30) < 0$
 \Rightarrow Saddle Point
 $f(10, -5) = -1753$

$(-10, 5)$
 $f_{xx} = 6x \Big|_{-10} = -60 < 0$
 $f_{xy} = 0, f_{yy} = 6y \Big|_5 = 30$
 $f_{xx}f_{yy} - f_{xy}^2 = -60(30) < 0$
 \Rightarrow Saddle Point
 $f(-10, 5) = 1747$

$(-10, -5)$
 $f_{xx} = 6x \Big|_{-10} = -60 < 0$
 $f_{xy} = 0, f_{yy} = 6y \Big|_{-5} = -30$
 $f_{xx}f_{yy} - f_{xy}^2 = -60(-30) > 0$
 \Rightarrow No Saddle pt.
 $f_{xx} < 0$ Concave down
 Local MAX $f(-10, -5) = 2247$

C

17) Find the extreme values of the function subject to the given constraint.

$$f(x, y) = xy, \quad x^2 + y^2 = 800$$

- (A) Maximum: 400 at (20, 20) and (-20, -20); minimum: -400 at (20, -20) and (-20, 20)
 B) Maximum: 400 at (20, 20); minimum: -400 at (-20, -20)
 C) Maximum: 400 at (20, 20); minimum: 0 at (0, 0)
 D) Maximum: 400 at (20, -20) and (-20, 20); minimum: -400 at (20, 20) and (-20, -20)

$$\nabla f = \lambda \nabla g \quad f = xy, \quad g = x^2 + y^2 - 800, \quad \nabla f = y\mathbf{i} + x\mathbf{j} = \lambda(2x\mathbf{i} + 2y\mathbf{j}), \text{ so } y = 2\lambda x, \quad x = 2\lambda y \quad (2)$$

$x=0, y=0$ is a solution to last two equations but doesn't satisfy the constraint $x^2 + y^2 = 800$

solving (1) & (2) for λ we get $\lambda = \frac{y}{2x} = \frac{x}{2y}$, so $2y^2 = 2x^2$, this leads to $x = \pm y$, substitute in $x^2 + y^2 = 800$

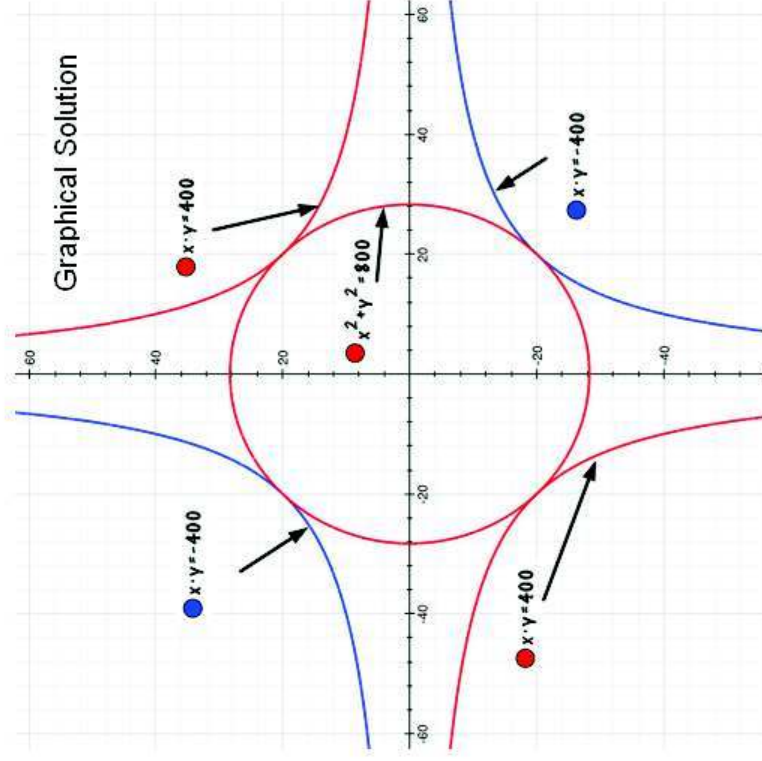
$$\text{we get } y^2 + y^2 = 800, y^2 = 400 \Rightarrow y = \pm 20$$

solutions

$$(20, 20), (-20, -20), (20, -20), (-20, 20)$$

$$f = 400 \text{ at } (20, 20), (20, -20), \text{ MAX}$$

$$f = -400 \text{ at } (-20, -20), (-20, 20) \text{ MIN}$$



18)

$$f(x, y) = x^2 + y^2, \quad xy^2 = 686$$

- A) Maximum: none; minimum: 147 at $(7, \pm 7\sqrt{2})$
 B) Maximum: none; minimum: 0 at $(0, 0)$
 C) Maximum: 147 at $(7, 7\sqrt{2})$; minimum: -147 at $(7, -7\sqrt{2})$
 D) Maximum: 147 at $(7, \pm 7\sqrt{2})$; minimum: 0 at $(0, 0)$

$$\nabla f = \lambda \nabla g, \quad g = xy^2 - 686$$

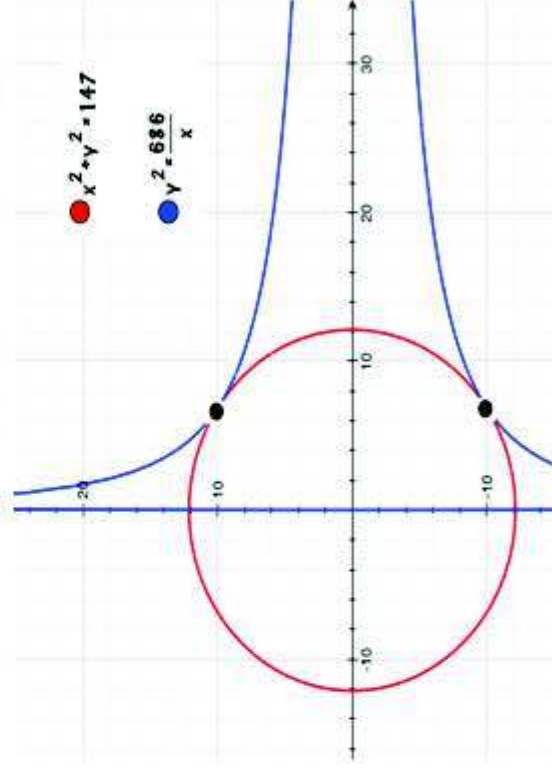
$$\nabla f = 2x\mathbf{i} + 2y\mathbf{j} = \lambda y^2\mathbf{i} + 2\lambda xy\mathbf{j}, \quad \text{so } 2x = y^2\lambda, \quad 2y = 2\lambda xy, \quad \text{Note } x=y=0 \text{ is not a solution (doesn't satisfy } xy^2=686)$$

$$\text{Solving (1) for } \lambda \text{ we get } \lambda = \frac{2x}{y^2}, \text{ Subs. in (2) } 2y = 2xy\left(\frac{2x}{y^2}\right) = \frac{4x^2}{y} \Rightarrow 2y^2 = 4x^2 \text{ or } y^2 = 2x^2, \text{ sub into } xy^2 = 686$$

$$\text{we get } x(2x^2) = 686, \quad x^3 = 343, \quad x = 7, \quad y^2 = \frac{686}{x} = \frac{686}{7} = 98 = 2(49) \text{ so } y = \pm 7\sqrt{2}$$

So $f(7, \pm 7\sqrt{2}) = 49 + 49(2) = 147$ is a minimum, There is no max. because y can be very small and y will approach ∞ as $x \rightarrow 0$

A



Evaluate the integral.

$$19) \int_1^3 \int_0^y x^2 y^2 \, dx \, dy$$

A) $\frac{350}{3}$

☒ B) $\frac{364}{9}$

C) $\frac{350}{9}$

D) $\frac{364}{3}$

$$\int_1^3 \int_0^y x^2 y^2 \, dx \, dy = \int_0^3 \left[\frac{x^3 y^2}{3} \right]_0^y \, dy = \int_1^3 \frac{y^5}{3} \, dy = \left[\frac{y^6}{18} \right]_1^3 = \frac{(27)(27)}{18} - \frac{1}{18} = \frac{728}{18} = \frac{364}{9} \quad \textcircled{B}$$

Write an equivalent double integral with the order of integration reversed.

$$20) \int_0^9 \int_0^x dy \, dx$$

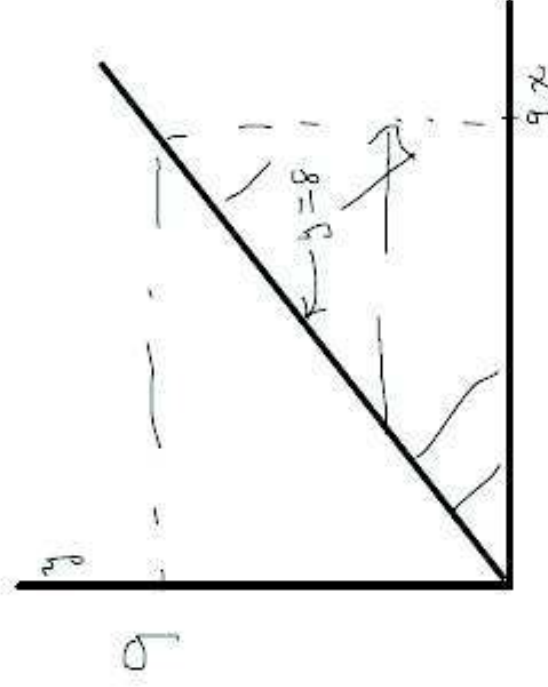
$$A) \int_0^9 \int_9^y dx \, dy$$

$$C) \int_0^9 \int_{-9}^y dx \, dy$$

$$B) \int_0^x \int_0^9 dx \, dy$$

$$D) \int_0^9 \int_y^9 dx \, dy$$

$$\int_0^9 \int_0^x dy \, dx = \int_0^9 \int_y^9 dx \, dy \quad \textcircled{D}$$



Find the volume of the indicated region.

21) the region under the surface $z = x^2 + y^4$, and bounded by the planes $x = 0$ and $y = 25$ and the

cylinder $y = x^2$

A) $\frac{292,982,500}{33}$

B) $\frac{11,732,500}{33}$

C) $\frac{2,357,500}{33}$

D) $\frac{58,607,500}{33}$

$$z = x^2 + y^2$$

$$V = \int_0^5 \int_{x^2}^{25} \int_0^{x^2+y^4} dz dy dx = \int_0^5 \int_{x^2}^{25} (x^2+y^4) dy dx$$

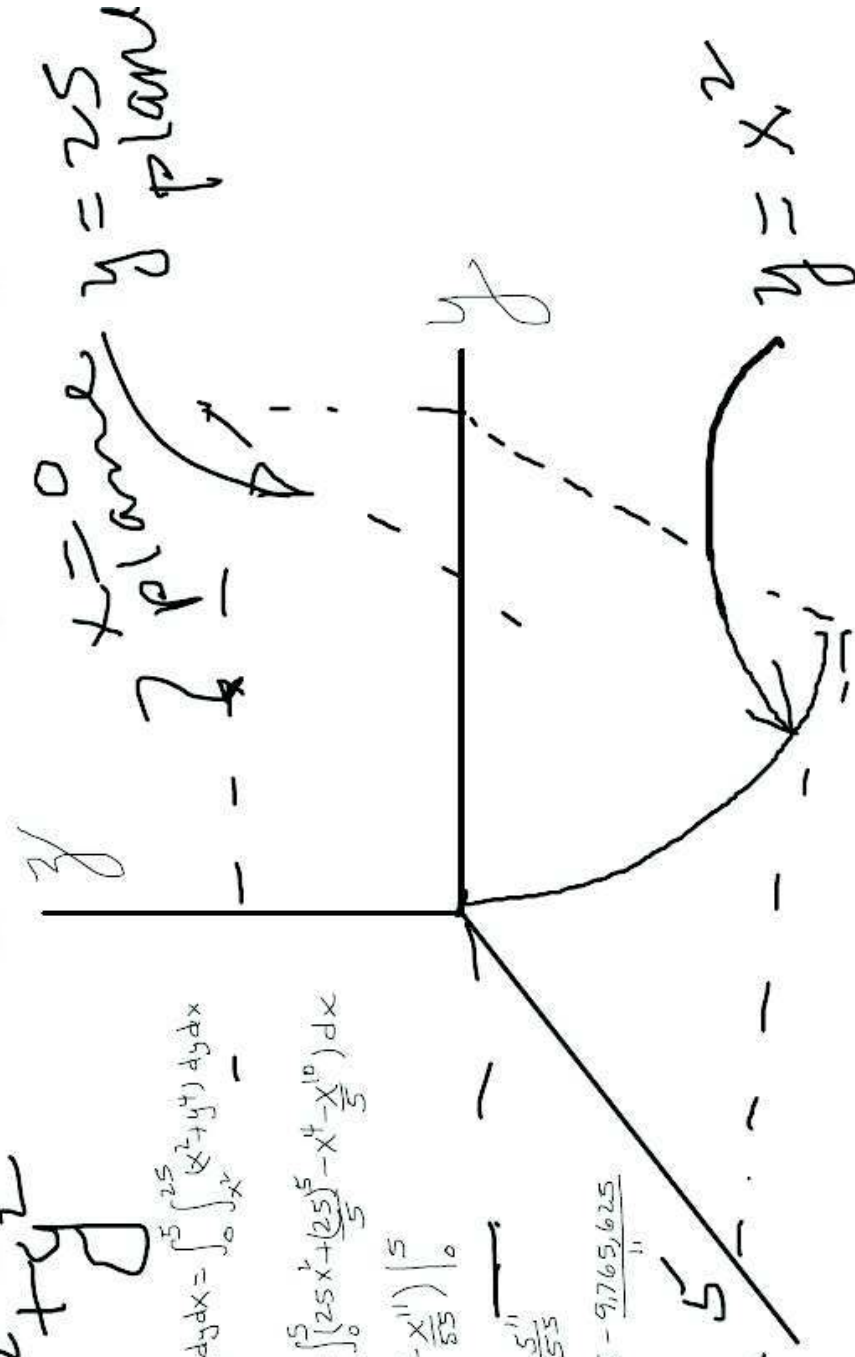
$$= \int_0^5 \left(x^2 y + \frac{y^5}{5} \right) \Big|_{x^2}^{25} dx = \int_0^5 \left(25x^2 + \frac{(25)^5}{5} - x^4 - \frac{x^{10}}{5} \right) dx$$

$$= \left(\frac{25x^3}{3} + \frac{(25)^5}{5} x - \frac{x^5}{5} - \frac{x^{11}}{\frac{5}{11}} \right) \Big|_0^5$$

$$= \frac{25(125)}{3} + \frac{(25)^5}{5} \cdot 5 - \frac{5^5}{5} - \frac{5^{11}}{\frac{5}{11}}$$

$$= \frac{3125}{3} + 9,765,625 - 625 - 9,765,625$$

$$= 292,982,500 \text{ (A)}$$



Find the volume of the indicated region.

- 22) the region bounded by the paraboloid $z = 100 - x^2 - y^2$ and the xy -plane

A) $\frac{10000}{3}\pi$

B) $\frac{5000}{3}\pi$

☒ C) 5000π

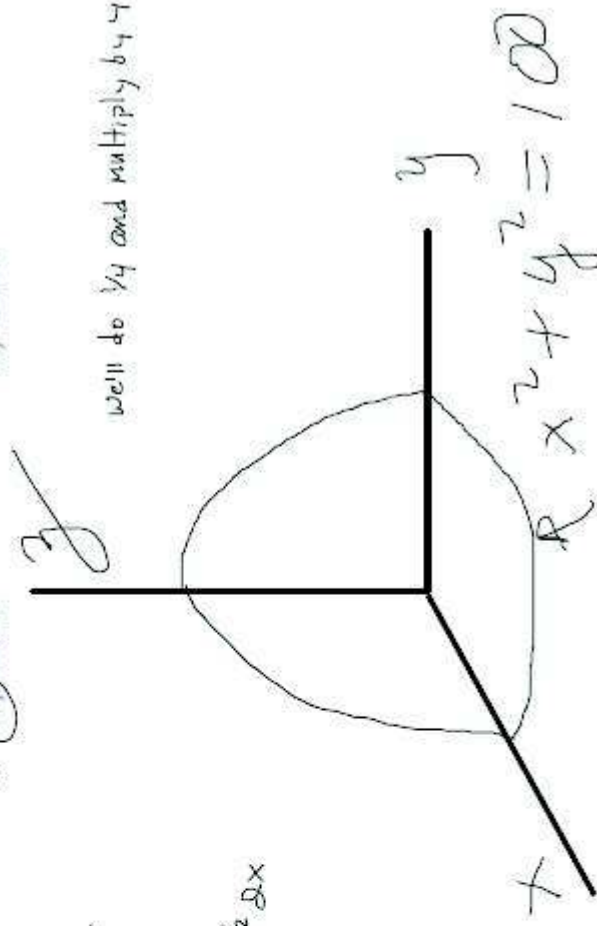
D) 2500π

$$V = 4 \int_0^{10} \int_0^{\sqrt{100-x^2}} (100-x^2-y^2) dy dx = 4 \int_0^{10} \int_0^{\sqrt{100-x^2}} (100-x^2-y^2) dy dx$$

$$= 4 \int_0^{10} \left[(100-x^2)y - \frac{y^3}{3} \right] \Big|_0^{\sqrt{100-x^2}} dx = 4 \int_0^{10} (100-x^2)^{3/2} - \frac{(100-x^2)^{3/2}}{3} dx$$

$$= 4 \int_0^{10} \frac{2}{3} (100-x^2)^{3/2} dx \quad \text{do the cal}$$

$$= 15,107.963 = 5000\pi \text{ C}$$



Solve the problem.

23) Write an iterated triple integral in the order $dx \, dy \, dz$ for the volume of the tetrahedron cut from the

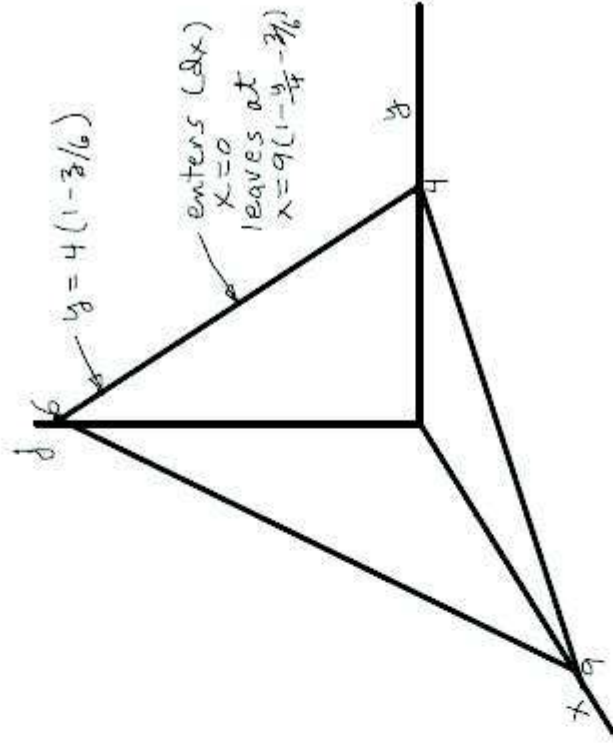
first octant by the plane $\frac{x}{9} + \frac{y}{4} + \frac{z}{6} = 1$.

A) $\int_0^6 \int_0^{1-y/4} \int_0^{1-y/4-z/6} dx \, dy \, dz$

B) $\int_0^6 \int_0^{9(1-y/4)} \int_0^{9(1-y/4-z/6)} dx \, dy \, dz$

C) $\int_0^6 \int_0^{1-z/6} \int_0^{1-y/4-z/6} dx \, dy \, dz$

D) $\int_0^6 \int_0^{4(1-z/6)} \int_0^{9(1-y/4-z/6)} dx \, dy \, dz$



$$V = \int_0^6 \int_0^{4(1-z/6)} \int_0^{9(1-y/4-z/6)} 1 \, dx \, dy \, dz$$

Evaluate the integral.

$$24) \int_{-1}^1 \int_0^5 \int_0^1 (x^2 + y^2 + z^2) dx dy dz$$

A) 126

☒ B) 90

C) 23.2

D) 124

$$\begin{aligned} &= \int_{-1}^1 \int_0^5 \left(\frac{x^3}{3} + (y^2 + z^2)x \right) \Big|_0^1 dy dz = \int_{-1}^1 \int_0^5 \left(\frac{1}{3} + y^2 + z^2 \right) dy dz = \int_{-1}^1 \left(\frac{5}{3} + \frac{125}{3} + 5z^2 \right) dz \\ &= \left(\frac{5z}{3} + \frac{125z}{3} + \frac{5z^3}{3} \right) \Big|_{-1}^1 = \frac{5}{3} + \frac{125}{3} + \frac{5}{3} = \frac{270}{3} = 90 \quad B \end{aligned}$$

Find the volume of the indicated region.

25) the region bounded by the coordinate planes, the parabolic cylinder $z = 4 - x^2$, and the plane $y = 5$

(B) $\frac{80}{3}$

A) 80

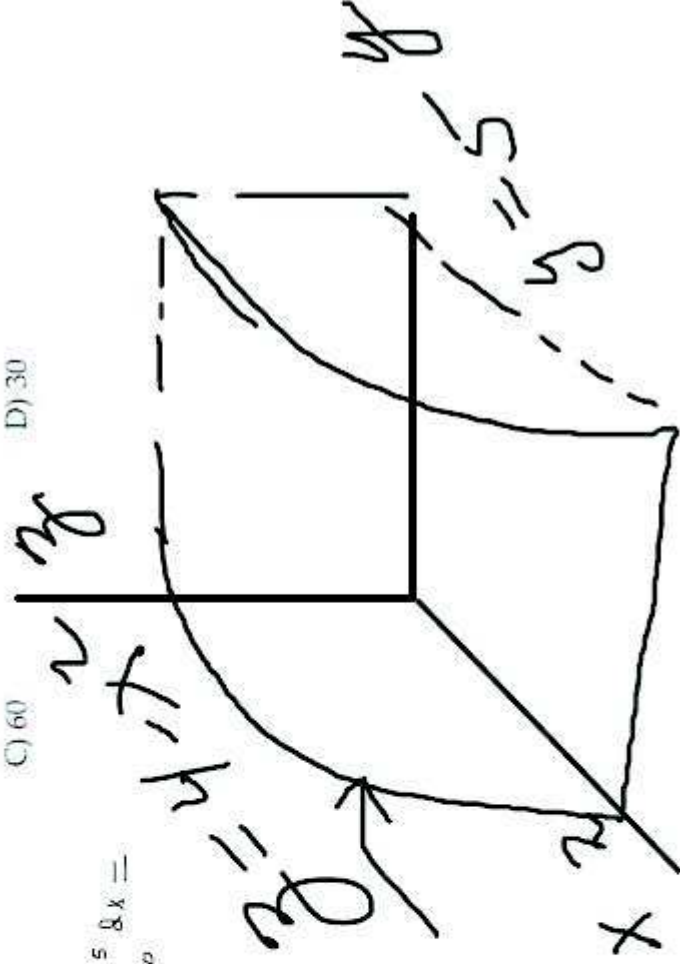
C) 60

D) 30

$$V = \int_0^2 \int_0^5 \int_0^{4-x^2} dz dy dx = \int_0^2 \int_0^5 (4-x^2) dy dx = \int_0^2 (4y - x^2 y) \Big|_0^5 dx =$$

$$\int_0^2 (20 - 5x^2) dx = \left(20x - \frac{5x^3}{3} \right) \Big|_0^2$$

$$= 40 - \frac{40}{3} = \frac{80}{3} \quad \text{(B)}$$



Find the volume of the indicated region.

- 26) the region bounded by the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 16$

A) $\frac{256}{3}\pi$

B) 128π

C) $\frac{1024}{3}\pi$

D) 384π

we'll do 1/4 and quadruple it

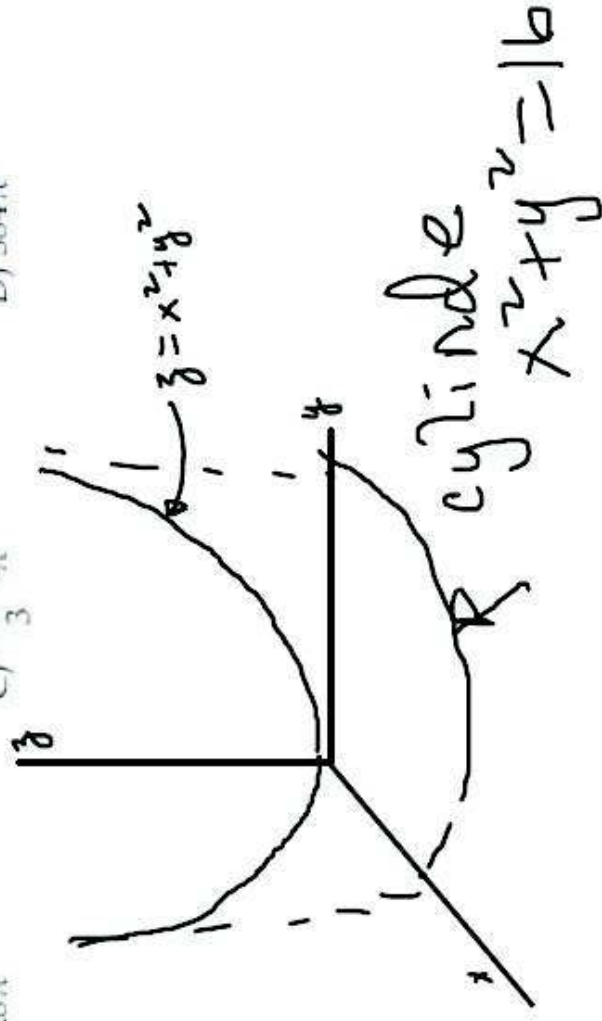
$$V = 4 \int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{x^2+y^2} dz dy dx$$

$$= 4 \int_0^4 \int_0^{\sqrt{16-x^2}} (x^2+y^2) dy dx = 4 \int_0^4 xy + \frac{y^3}{3} \Big|_0^{\sqrt{16-x^2}} dx$$

$$= 4 \int_0^4 x^2 \sqrt{16-x^2} + \frac{(16-x^2)^{3/2}}{3} dx$$

$$= 402.12396 = 128\pi$$

B



Evaluate the line integral along the curve C.

$$27) \int_C \frac{x+y+z}{5} ds, \quad C \text{ is the curve } \mathbf{r}(t) = 4t\mathbf{i} + (8 \cos \frac{3}{8}t)\mathbf{j} + (8 \sin \frac{3}{8}t)\mathbf{k}, \quad 0 \leq t \leq \frac{8}{3}\pi$$

A) $\frac{128}{9}\pi$

B) $\frac{128}{9}\pi^2 + \frac{256}{3}$

C) $\frac{128}{9} + \frac{128}{3}$

D) $\frac{128}{9}\pi^2 + \frac{128}{3}$

$$\int_C f(x, y, z) ds = \int_0^{\frac{8\pi}{3}} f(x(t), y(t), z(t)) |v(t)| dt, \quad v(t) = \frac{dr}{dt} = 4 - 3 \sin \frac{3}{8}t \mathbf{j} + 3 \cos \frac{3}{8}t \mathbf{k}, \quad |v(t)| = \sqrt{16 + 9} = 5$$

$$\oint_C f ds = \int_0^{\frac{8\pi}{3}} \underbrace{4t + 8 \cos \frac{3}{8}t + 8 \sin \frac{3}{8}t}_{\cancel{f}} dt = \left(2t^2 + \frac{64}{3} \sin \frac{3}{8}t - \frac{64}{3} \cos \frac{3}{8}t \right) \bigg|_0^{\frac{8\pi}{3}} = 2 \left(\frac{64}{9} \right) \pi^2 + 0 + \frac{64}{3} + \frac{64}{3}$$

$$= \frac{128\pi^2}{9} + \frac{128}{3} \quad D$$

Evaluate the line integral along the curve C.

28) $\int_C (y+z) ds$, C is the path from (0, 0, 0) to (-3, 3, 1) given by:

$$C_1: \mathbf{r}(t) = -3t^2\mathbf{i} + 3t\mathbf{j}, 0 \leq t \leq 1$$

$$C_2: \mathbf{r}(t) = -3\mathbf{i} + 3\mathbf{j} + (t-1)\mathbf{k}, 1 \leq t \leq 2$$

A) $\frac{25}{2}$

B) $\frac{13}{12}$

C) $\frac{15}{4}\sqrt{5} - \frac{11}{4}$

D) $\frac{15}{4}\sqrt{5} + \frac{11}{4}$

$C_1: \mathbf{r}(t) = -3t^2\mathbf{i} + 3t\mathbf{j}$ $0 \leq t \leq 1$
 $\mathbf{v}(t) = -6t\mathbf{i} + 3\mathbf{j}$ $|\mathbf{v}| = \sqrt{36t^2 + 9}$, $x = -3t^2, y = 3t, z = 0$, $C_2: \mathbf{r}(t) = -3\mathbf{i} + 3\mathbf{j} + (t-1)\mathbf{k}$
 $\mathbf{v} = \mathbf{j}$, $|\mathbf{v}| = 1$
 $x = -3, y = 3, z = t-1$

$$\int_C y+z ds = \int_0^1 3t \sqrt{36t^2+9} dt + \int_1^2 (3+t-1)(1) dt = \int_0^1 9t \sqrt{4t^2+1} dt + \int_1^2 (2+t) dt$$

$$\int_1^2 2+t dt = \left(2t + \frac{t^2}{2}\right) \Big|_1^2 = 4+2 - \frac{1}{2} = \frac{7}{2}$$

For 1st integral $u = 4t^2+1, du = 8t dt$, $\int_0^1 9t \sqrt{4t^2+1} dt = \int_1^5 \frac{9}{8} u^{1/2} du$

$$= \frac{9}{8} \left(\frac{2}{3}\right) u^{3/2} \Big|_1^5 = \frac{3}{4} u^{3/2} \Big|_1^5 = \frac{3}{4} \sqrt{125} - \frac{3}{4} \sqrt{1} = \frac{15\sqrt{5}}{4} - \frac{3}{4}$$

So, total = $\frac{15\sqrt{5}}{4} - \frac{3}{4} + \frac{7}{2} = \frac{15\sqrt{5}}{4} - \frac{3}{4} + \frac{14}{4} = \frac{15\sqrt{5} + 11}{4}$

Evaluate the line integral along the curve C.

29) $\int_C \frac{1}{x^2 + y^2 + z^2} ds$, C is the path given by:

$C_1: \mathbf{r}(t) = (5 \cos t)\mathbf{i} + (5 \sin t)\mathbf{j}$ from $(5, 0, 0)$ to $(0, 5, 0)$

$C_2: \mathbf{r}(t) = (5 \sin t)\mathbf{j} + (5 \cos t)\mathbf{k}$ from $(0, 5, 0)$ to $(0, 0, 5)$

$C_3: \mathbf{r}(t) = (5 \sin t)\mathbf{i} + (5 \cos t)\mathbf{k}$ from $(0, 0, 5)$ to $(5, 0, 0)$

A) $\frac{3}{10}\pi$

B) $\frac{\pi}{10}$

C) $-\frac{3}{10}\pi$

D) 0

$C_1: \mathbf{r}(t) = 5 \cos t \mathbf{i} + 5 \sin t \mathbf{j}$ $|v| = 5$
 $v(t) = -5 \sin t \mathbf{i} + 5 \cos t \mathbf{j}$
 $(5, 0, 0) \rightarrow (0, 5, 0) \Rightarrow 0 \leq t \leq \frac{\pi}{2}$

$C_2: \mathbf{r}(t) = 5 \sin t \mathbf{j} + 5 \cos t \mathbf{k}$ $|v| = 5$
 $v(t) = 5 \cos t \mathbf{j} - 5 \sin t \mathbf{k}$
 $(0, 5, 0) \rightarrow (0, 0, 5) \Rightarrow \frac{\pi}{2} \leq t \leq \pi$

$C_3: \mathbf{r}(t) = 5 \sin t \mathbf{i} + 5 \cos t \mathbf{k}$
 $v(t) = 5 \cos t \mathbf{i} - 5 \sin t \mathbf{k}$
 $(0, 0, 5) \rightarrow (5, 0, 0) \Rightarrow 0 \leq t \leq \frac{\pi}{2}$

Notice that in all cases $\frac{1}{x^2 + y^2 + z^2} = \frac{1}{25}$

So, $\int_C \frac{1}{x^2 + y^2 + z^2} ds = \int_0^{\frac{\pi}{2}} \frac{1}{25} (5) dt + \int_{\frac{\pi}{2}}^{\pi} \frac{1}{25} (5) dt + \int_0^{\frac{\pi}{2}} \frac{1}{25} (5) dt = \frac{\pi}{10} - \frac{\pi}{10} + \frac{\pi}{10} = \frac{\pi}{10}$ **B**

Find the work done by F over the curve in the direction of increasing t .

30) $F = -6yi + 6xj + 9z^3k$; $C: r(t) = \cos t i + \sin t j, 0 \leq t \leq 7$

(A) $W = 42$

B) $W = 0$

C) $W = 147$

D) $W = 84$

$$\begin{aligned}
 W &= \int_C F \cdot \frac{dr}{dt} dt & \begin{matrix} x = \cos t \\ y = \sin t \end{matrix} & \frac{dr}{dt} = -\sin t i + \cos t j, W = \int_0^7 (-6 \sin t i + 6 \cos t j) \cdot (-\sin t i + \cos t j) dt + \\
 &= \int_0^7 (6 \sin^2 t + 6 \cos^2 t) dt = \int_0^7 6 dt = 42 \quad A
 \end{aligned}$$

Calculate the flux of the field F across the closed plane curve C .

31) $F = y^3 \mathbf{i} + x^2 \mathbf{j}$; the curve C is the closed counterclockwise path formed from the semicircle

$r(t) = 5 \cos t \mathbf{i} + 5 \sin t \mathbf{j}$, $0 \leq t \leq \pi$, and the straight line segment from $(-5, 0)$ to $(5, 0)$

A) $-\frac{50}{3}$

B) $\frac{50}{3}$

C) 0

D) $\frac{100}{3}$

Flux = $\oint_C M dy - N dx$ for C_1 : $x = 5 \cos t$, $dx = -5 \sin t dt$, $y = 5 \sin t$, $dy = 5 \cos t dt$, $0 \leq t \leq \pi$

Flux = $\oint_C M dy - N dx = \oint y^3 dy - x^2 dx$

For C_2 $x = 5$, $dx = 0$, $-1 \leq t \leq 1$

$= \int_0^8 125 \sin^3 t \cos t dt - 25 \cos^2 t (-5 \sin t) dt + \int_{-1}^1 0 dy - 25 t^2 ds dt$

$= \int_0^\pi 625 \sin^3 t \cos t dt + \int_0^\pi 125 \cos^3 t \sin t dt - 125 \int_{-1}^1 t^2 dt = 83.333 - 125 \frac{t^3}{3} \Big|_{-1}^1$

83.333

$= 83.333 - 83.333 = 0$

Calculate the circulation of the field F around the closed curve C .

32) $F = xy\mathbf{i} + 3\mathbf{j}$, curve C is $\mathbf{r}(t) = 2 \cos t\mathbf{i} + 2 \sin t\mathbf{j}$, $0 \leq t \leq 2\pi$

A) $\frac{10}{3}$

B) 6

C) $\frac{26}{3}$

~~D) 0~~

Circulation = Flow around closed loop = $\int_C \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} dt$

$x = 2 \cos t$

$y = 2 \sin t$

$\mathbf{r} = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j}$

$\frac{d\mathbf{r}}{dt} = -2 \sin t \mathbf{i} + 2 \cos t \mathbf{j}$

$$C.F = \int_0^{2\pi} (2 \sin t \cos t \mathbf{i} + 3\mathbf{j}) \cdot (-2 \sin t \mathbf{i} + 2 \cos t \mathbf{j}) dt = \int_0^{2\pi} (-8 \cos t \sin^2 t + 6 \cos t) dt \stackrel{\text{>0}}{=} 0 \quad D$$

Calculate the flow in the field F along the path C .

33) $F = y^2\mathbf{i} + z\mathbf{j} + x\mathbf{k}$; C is the curve $\mathbf{r}(t) = (2 + 2t)\mathbf{i} + 3t\mathbf{j} - 3t\mathbf{k}$, $0 \leq t \leq 1$

A) 39

B) $\frac{9}{2}$

☒ C) $-\frac{15}{2}$

D) -3

$$\text{Flow} = \int_C \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} dt, \quad \mathbf{r} = (2+2t)\mathbf{i} + 3t\mathbf{j} - 3t\mathbf{k}, \quad \frac{d\mathbf{r}}{dt} = 2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}, \quad x = 2+2t, \quad y = 3t, \quad z = -3t$$

$$\begin{aligned} \text{Flow} &= \int_0^1 (9t^2 - 3t\mathbf{j} + 2+2t\mathbf{k}) \cdot (2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}) dt = \int_0^1 (18t^2 - 9t - 6 - 6t) dt = \int_0^1 (18t^2 - 15t - 6) dt \\ &= \left(6t^3 - \frac{15}{2}t^2 - 6t \right) \Big|_0^1 \\ &= 6 - \frac{15}{2} - 6 = -\frac{15}{2} \quad \text{C} \end{aligned}$$

Find the gradient field of the function.

34) $f(x, y, z) = x^7 y^8 + \frac{x^3}{z^4}$

A) $\nabla f = (7x^6 + 3x^2)\mathbf{i} + 8y^7\mathbf{j} - \frac{4}{z^5}\mathbf{k}$

C) $\nabla f = \left[7x^6 y^8 + \frac{3x^2}{z^4} \right] \mathbf{i} + 8x^7 y^7 \mathbf{j} - \frac{4x^3}{z^5} \mathbf{k}$

B) $\nabla f = 7x^6 y^8 \mathbf{i} + 8x^7 y^7 \mathbf{j} - \frac{4x^7}{z^5} \mathbf{k}$

D) $\nabla f = (7x^6 + 3x^2)\mathbf{i} + 8y^7 \mathbf{j} + \frac{4}{z^5} \mathbf{k}$

$$\begin{aligned} \nabla f &= \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k} \\ &= (7x^6 y^8 + \frac{3x^2}{z^4}) \mathbf{i} + 8x^7 y^7 \mathbf{j} - \frac{4x^3}{z^5} \mathbf{k} \quad \text{C} \end{aligned}$$

Find the potential function f for the field F .

35) $F = \frac{1}{z}i - 6j - \frac{x}{z^2}k$

☒ A) $f(x, y, z) = \frac{x}{z} - 6y + C$

B) $f(x, y, z) = \frac{x}{z} + C$

C) $f(x, y, z) = \frac{2x}{z} - 6y + C$

D) $f(x, y, z) = \frac{x}{z} - 6 + C$

$$\frac{\partial f}{\partial x} = \frac{1}{z}, \quad \frac{\partial f}{\partial y} = -6, \quad \frac{\partial f}{\partial z} = -\frac{x}{z^2}, \quad f = \frac{x}{z} + g(y, z), \quad \frac{\partial f}{\partial y} = 0 + \frac{\partial g}{\partial y} = -6 \Rightarrow g = -6y + h(z)$$

so far $f = \frac{x}{z} - 6y + h(z)$

$$\frac{\partial f}{\partial z} = -\frac{x}{z^2} + \frac{dh}{dz} = -\frac{x}{z^2} \Rightarrow h = C, \text{ so } f = \frac{x}{z} - 6y + C \quad \checkmark$$

Evaluate the work done between point 1 and point 2 for the conservative field F.

36) $\mathbf{F} = 6 \sin 6x \cos 4y \cos 6z \mathbf{i} + 4 \cos 6x \sin 4y \cos 6z \mathbf{j} + 6 \cos 6x \cos 4y \sin 6z \mathbf{k}$; $P_1(0, 0, 0)$, P_2

$$\left(\frac{1}{3}\pi, \frac{1}{2}\pi, \frac{\pi}{6} \right)$$

A) $W = 1$

B) $W = 0$

C) $W = -2$

D) $W = 2$

Need to find potential function $\frac{\partial f}{\partial x} = 6 \sin 6x \cos 4y \cos 6z \Rightarrow f = -\cos 6x \cos 4y \cos 6z + g(y, z)$

$$\frac{\partial f}{\partial y} = 4 \cos 6x \sin 4y \cos 6z + \frac{\partial g}{\partial y} = 4 \cos 6x \sin 4y \cos 6z \Rightarrow \frac{\partial g}{\partial y} = 0 \quad g = h(z)$$

$$\text{so far } f = -\cos 6x \cos 4y \cos 6z + h(z), \frac{\partial f}{\partial z} = 6 \cos 6x \cos 4y \sin 6z + \frac{dh}{dz} = 6 \cos 6x \cos 4y \sin 6z$$


$$\Rightarrow \frac{dh}{dz} = 0, h = C \quad \text{so } f = -\cos 6x \cos 4y \cos 6z + C, \text{ work} = f\left(\frac{\pi}{3}, \frac{\pi}{2}, \frac{\pi}{6}\right) - f(0, 0, 0)$$

$$= -\cos 2\pi \cos 2\pi \cos \pi + \cos 0 \cos 0 \cos 0 = -1 + 1 = 2$$

Using Green's Theorem, find the outward flux of F across the closed curve C .

- 37) $F = \sin 10y\mathbf{i} + \cos 4x\mathbf{j}$; C is the rectangle with vertices at $(0, 0)$, $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{\pi}{10}, \frac{\pi}{4}\right)$, and $\left(0, \frac{\pi}{4}\right)$
- A) $-\frac{2}{5}\pi$ B) 0 C) $-\frac{1}{5}\pi$ D) $\frac{1}{5}\pi$

outward flux across closed curve C



Use Green's Theorem \rightarrow

$$\oint F \cdot dx = \int_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy = 0 \quad \text{(B)}$$

Calculate the flux of the field F across the closed plane curve C .

38) $F = xi + yj$; the curve C is the counterclockwise path around the circle $x^2 + y^2 = 121$

A) 242π

B) 22π

C) 0

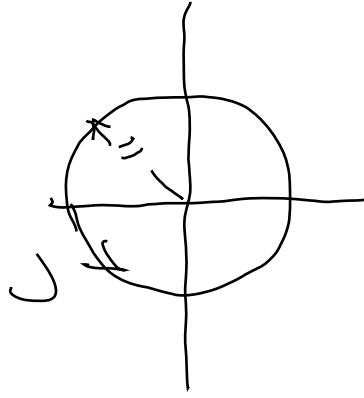
D) 484π

using Green's Theorem

$$F_{LUX} = \iint_R \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} dx dy$$

$$M = x \quad N = y$$

$$\frac{\partial M}{\partial x} = 1 \quad \frac{\partial N}{\partial y} = 1$$



$$= \iint_R 1 + 1 dx dy$$

$$= 2 \iint 1 dx dy$$

$$= 2 (\text{Area of circle})$$

$$= 2 (\pi 11^2) = 242\pi$$

Calculate the flux of the field F across the closed plane curve C .

- 39) $F = y^3 \mathbf{i} + x^3 \mathbf{j}$; the curve C is the closed counterclockwise path formed from the semicircle $\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j}$, $0 \leq t \leq \pi$, and the straight line segment from $(-2, 0)$ to $(2, 0)$

A) 4

B) 8

C) -4

D) 0

$$\text{FLUX} = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy$$

$$M = y^3 \quad N = x^3$$

$$\frac{\partial M}{\partial x} = 0 \quad \frac{\partial N}{\partial y} = 0$$

$$\iint_R 0 \, dx dy = 0$$

Calculate the flux of the field F across the closed plane curve C .

- 40) $F = x^2\mathbf{i} + y^2\mathbf{j}$; the curve C is the closed counterclockwise path around the triangle with vertices at $(0, 0)$, $(3, 0)$, and $(0, 5)$

A) -10

B) 120

C) 40

D) 0

$$\iint \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \quad M = x^2 \quad N = y^2$$

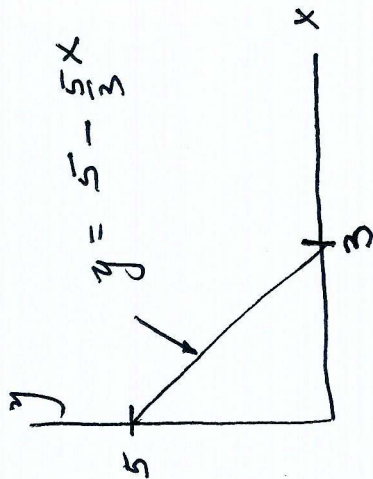
$$\frac{\partial M}{\partial x} = 2x \quad \frac{\partial N}{\partial y} = 2y$$

$$= \int_0^3 \int_0^{5-\frac{5}{3}x} (2x + 2y) \, dy \, dx$$

$$= \int_0^3 (2xy + y^2) \Big|_0^{5-\frac{5}{3}x} \, dx$$

$$= \int_0^3 2x(5-\frac{5}{3}x) + (5-\frac{5}{3}x)^2 \, dx = \int_0^3 10x - \frac{10x^2}{3} + (5-\frac{5}{3}x)^2 \, dx$$

$$= 5x^2 - \frac{10x^3}{3} + \frac{(5-\frac{5}{3}x)^3}{3(\frac{5}{3})} \Big|_0^3 = 45 - 30 + \frac{125}{5} = 45 - 30 + 25 = 40$$



Calculate the flux of the field F across the closed plane curve C .

41) $F = (x+y)\mathbf{i} + xy\mathbf{j}$; the curve C is the closed counterclockwise path around the rectangle with vertices at $(0, 0)$, $(2, 0)$, $(2, 9)$, and $(0, 9)$

A) 54

B) 36

C) 81

D) 117

$$\text{Flux} = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dy dx$$

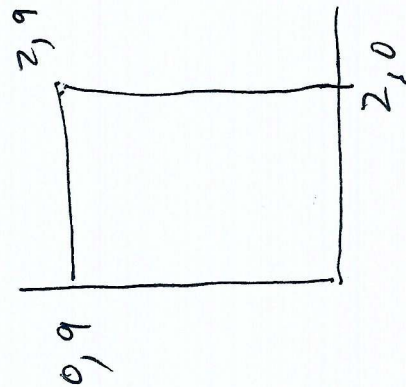
$$M = x + y \quad \frac{\partial M}{\partial x} = 1$$

$$N = xy \quad \frac{\partial N}{\partial y} = x$$

$$\text{Flux} = \int_0^2 \int_0^9 (1 + x) dy dx$$

$$= \int_0^2 (y + xy) \Big|_0^9 dx$$

$$= \int_0^2 (9 + 9x) dx = \left(9x + \frac{9x^2}{2} \right) \Big|_0^2 = 18 + 18 = 36$$



Calculate the circulation of the field F around the closed curve C .

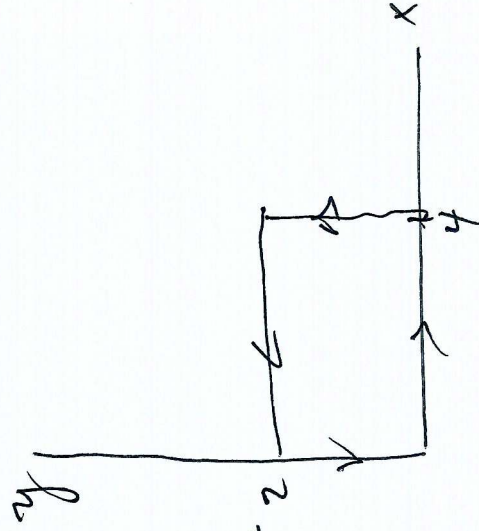
- 42) $F = x^2y^3\mathbf{i} + x^2y^3\mathbf{j}$; curve C is the counterclockwise path around the rectangle with vertices at $(0, 0)$, $(4, 0)$, $(4, 2)$, and $(0, 2)$

A) 0

B) $-\frac{320}{3}$

C) $\frac{704}{3}$

D) -256



$$\text{CIR} = \iint_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dx dy$$

$$u = x^2y^3 \quad v = x^2y^3$$

$$\frac{\partial u}{\partial x} = 2xy^3 \quad \frac{\partial v}{\partial y} = 3x^2y^2$$

$$\int_0^4 \int_0^2 (2xy^3 - 3x^2y^2) dy dx$$

$$= \int_0^4 \left(\frac{2xy^4}{4} - x^2y^3 \right) \Big|_0^2 dx$$

$$\int_0^4 (8x - 8x^2) dx = \left(4x^2 - \frac{8x^3}{3} \right) \Big|_0^4$$

$$= 64 - 64\left(\frac{8}{3}\right) = 64\left(1 - \frac{8}{3}\right) = 64\left(-\frac{5}{3}\right) = -\frac{320}{3}$$

Calculate the circulation of the field F around the closed curve C .

43) $F = xy^2\mathbf{i} + x^2y\mathbf{j}$; curve C is the counterclockwise path around $C_1 \cup C_2$: $C_1: \mathbf{r}(t) = 2 \cos t\mathbf{i} + 2 \sin t\mathbf{j}$, $0 \leq t$

$\leq \pi$

$C_2: \mathbf{r}(t) = t\mathbf{i}$, $-2 \leq t \leq 2$

A) 0

B) 4

C) 2

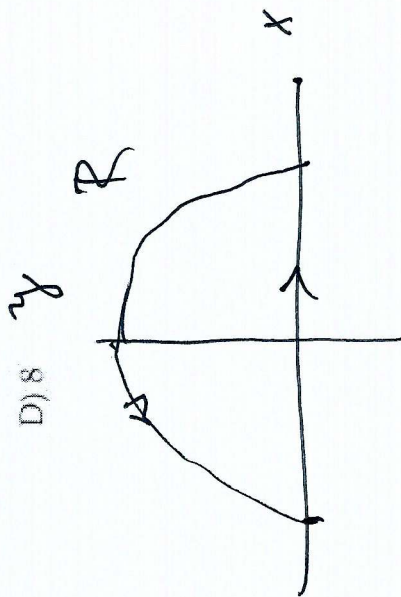
D) 8

$$dR = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dy dx$$

$$N = x^2y \quad M = xy^2$$

$$\frac{\partial N}{\partial x} = 2xy \quad \frac{\partial M}{\partial y} = 2xy$$

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dy dx = \iint_R 0 \, dy dx = 0$$



Calculate the circulation of the field F around the closed curve C .

44) $F = (-x - y)\mathbf{i} + (x + y)\mathbf{j}$, curve C is the counterclockwise path around the circle with radius 4 centered at $(10, 3)$ ~~with radius 4~~ ✓

A) ~~64π~~

B) ~~32(1 + π) + 208~~

C) ~~32(1 + π)~~

D) 32π ✓

$$\text{circ} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dy dx$$

$$N = x + y \quad M = -x - y$$

$$\frac{\partial N}{\partial x} = 1 \quad \frac{\partial M}{\partial y} = -1$$

$$\begin{aligned} \text{circ} &= \iint_R (1 - (-1)) dy dx = \iint_R 2 dy dx = 2 \text{ area of circle} \\ &= 2(16\pi) = 32\pi \end{aligned}$$

Calculate the circulation of the field F around the closed curve C .

- 45) $F = y^3 \mathbf{i} + x^2 \mathbf{j}$; curve C is the counterclockwise path around the triangle with vertices at $(0, 0)$, $(3, 0)$, and $(0, 5)$

I
A) 0 B) $\frac{435}{4}$ C) $\frac{315}{4}$ D) $-\frac{315}{4}$

$$\text{circ} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dy dx$$

$$M = y^3, N = x^2$$

$$\frac{\partial M}{\partial y} = 3y^2, \frac{\partial N}{\partial x} = 2x$$

$$\text{circ} = \iint_R (2x - 3y^2) dy dx$$

$$= \int_0^3 \int_0^{5-5/3x} (2x - 3y^2) dy dx = \int_0^3 (2xy - y^3) \Big|_0^{5-5/3x} dx = \int_0^3 2x(5-5/3x) - (5-5/3x)^3 dx$$

$$= \int_0^3 \left(10x - \frac{10x^2}{3} - (5-5/3x)^3 \right) dx = 5x^2 - \frac{10x^3}{9} + \frac{(5-5/3x)^4}{4(5/3)} \Big|_0^3$$

$$= 45 - 30 - \frac{625}{27} = 15 - \frac{1875}{270} = \frac{300 - 1875}{270} = -\frac{1575}{270} = -\frac{5(315)}{5(4)} = -\frac{315}{4}$$

