Calculus I – Math 1540

Dr. Bob Brown, Jr.
Dean Emeritus
Professor Emeritus
East Georgia State College

Final Exam Review Video

1. Find the average rate of change of the function over the given interval.

$g(t) = 2 + \tan t, \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

A. $-\frac{4}{\pi}$

B. $\frac{4}{\pi}$

C. -2

D. 0

2. Find the derivative.

$$y = \int_0^{x^8} \cos \sqrt{t}\, dt$$

- A. $\cos\left(x^4\right) - 1$
- B. $8x^7 \cos\left(x^4\right)$
- C. $\sin\left(x^4\right)$
- D. $\cos\left(x^4\right)$

3.

Write the function in the form y = f(u) and u = g(x). Then find $\frac{dy}{dx}$ as a function of x.

$$y = \tan\left(\pi - \frac{8}{x}\right)$$

- A. $y = \tan u;\ u = \pi - \frac{8}{x};\ \frac{dy}{dx} = \sec^2\left(\frac{8}{x^2}\right)$
- B. $y = \tan u;\ u = \pi - \frac{8}{x};\ \frac{dy}{dx} = \frac{8}{x^2} \sec\left(\pi - \frac{8}{x}\right) \tan\left(\pi - \frac{8}{x}\right)$
- C. $y = \tan u;\ u = \pi - \frac{8}{x};\ \frac{dy}{dx} = \frac{8}{x^2} \sec^2\left(\pi - \frac{8}{x}\right)$
- D. $y = \tan u;\ u = \pi - \frac{8}{x};\ \frac{dy}{dx} = \sec^2\left(\pi - \frac{8}{x}\right)$

4. A company knows that the unit cost C and the unit revenue R from the production and sale of x units are related by $C = \frac{R^2}{252{,}000} + 6579$. Find the rate of change of unit revenue when the unit cost is changing by $14 / unit and the unit revenue is $1,000.

- A. $1,764.00/unit
- B. $1,210.95/unit
- C. $70.00/unit
- D. $657.90/unit

5.

Use implicit differentiation to find $\frac{dy}{dx}$.

$$xy + x + y = x^2y^2$$

- A. $\dfrac{2xy^2 - y}{2x^2y + x}$
- B. $\dfrac{2xy^2 + y}{2x^2y - x}$
- C. $\dfrac{2xy^2 - y - 1}{-2x^2y + x + 1}$
- D. $\dfrac{2xy^2 + y + 1}{-2x^2y - x - 1}$

6. Determine all critical points for the following function.

$$f(x) = x^2 - 4\sqrt{x}$$

The critical point(s) of the given function is/are at x = ____________.

(Use a comma to separate answers as needed.)

7. Use l'Hôpital's Rule to find the limit.

$$\lim_{x \to 0} \frac{x^2}{\cos(x) - 1}$$

$$\lim_{x \to 0} \frac{x^2}{\cos(x) - 1} = \_\_\_\_\_\_\_\_\_\_$$ (Type an exact answer.)

8. Evaluate the integral.

$$\int \frac{x\,dx}{\left(7x^2+3\right)^5}$$

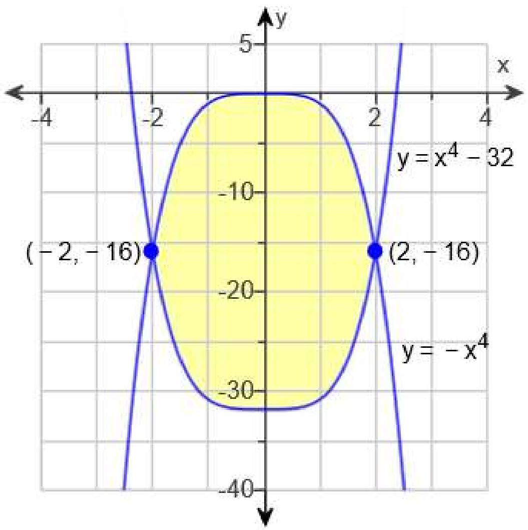
- A. $-\frac{7}{3}\left(7x^2+3\right)^{-4}+C$
- B. $-\frac{7}{3}\left(7x^2+3\right)^{-6}+C$
- C. $-\frac{1}{56}\left(7x^2+3\right)^{-4}+C$
- D. $-\frac{1}{14}\left(7x^2+3\right)^{-6}+C$

9. Evaluate the integral.

$$\int 7x^2 \sqrt[4]{6+4x^3}\, dx$$

- A. $7\left(6+4x^3\right)^{5/4} + C$
- B. $\frac{28}{5}\left(6+4x^3\right)^{5/4} + C$
- C. $\frac{7}{15}\left(6+4x^3\right)^{5/4} + C$
- D. $-\frac{14}{3}\left(6+4x^3\right)^{-3/4} + C$

10. Find the area of the shaded region.



- A. $\frac{256}{5}$
- B. $\frac{2816}{5}$
- C. $\frac{512}{5}$
- D. $\frac{516}{5}$

11. If f(1) = 5, must $\lim_{x \to 1} f(x)$ exist? If it does, then must $\lim_{x \to 1} f(x) = 5$? Can we conclude anything about $\lim_{x \to 1} f(x)$? Explain.

If f(1) = 5, must $\lim_{x \to 1} f(x)$ exist?

- ○ **A.** Yes, because $\lim_{x \to a} f(x) = f(a)$.
- ○ **B.** No, because $\lim_{x \to a} f(x)$ does not exist for x = 1.
- ○ **C.** No, because even if a function is defined at a point, the limit may not exist at that point.
- ○ **D.** Yes, because f(x) is defined at 1.

If $\lim_{x \to 1} f(x)$ exists, must $\lim_{x \to 1} f(x) = 5$?

- ○ **A.** Yes, because f(1) = 5.
- ○ **B.** No, because even if a function is defined at a point, the limit may not exist at that point.
- ○ **C.** No, because f(x) could be a piecewise function where the limit approaching 1 from the left and the right are the same, but f(1) is defined as a different value.
- ○ **D.** Yes, because $\lim_{x \to a} f(x) = f(a)$.

What can we conclude about $\lim_{x \to 1} f(x)$?

- ○ **A.** $\lim_{x \to 1} f(x)$ does not exist.
- ○ **B.** $\lim_{x \to 1} f(x)$ exists, but we cannot find the value of the limit.
- ○ **C.** $\lim_{x \to 1} f(x) = 5$
- ○ **D.** We cannot conclude anything about $\lim_{x \to 1} f(x)$.

12. Find the limit, if it exists.

$$\lim_{x \to 4} \frac{x^2 - 9x + 20}{x^2 - 6x + 8}$$

- A. $\frac{9}{2}$
- B. $\frac{1}{2}$
- C. $-\frac{1}{2}$
- D. The limit does not exist.

13. Divide numerator and denominator by the highest power of x in the denominator to find the limit.

$$\lim_{x \to \infty} \frac{-4x^{-1} - 3x^{-3}}{-2x^{-2} + x^{-5}}$$

- A. ∞
- B. 2
- C. $-\infty$
- D. 0

14. Find the derivative.

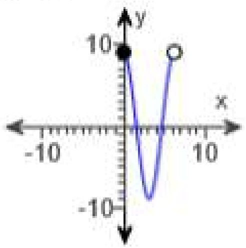
$y = \sqrt[5]{x^4} + x^{4e}$

- A. $\frac{4}{5}x^{-1/5} + 4x^{4e-1}$
- B. $\frac{4}{5}x^{1/5} + 4ex^{4e-1}$
- C. $\frac{4}{5}x^{-1/5} + 4ex^{4e-1}$
- D. $\frac{4}{5}x^{1/5} + 4x^{4e-1}$

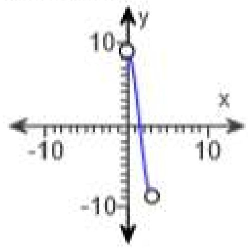
15. Sketch the graph of the function and determine whether it has any absolute extreme values on its domain.

$y = 9 \cos x, \quad 0 < x < 2\pi$

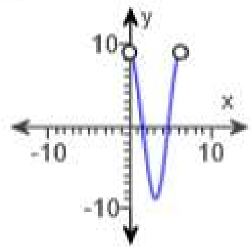
A. absolute maximum at $x = 0$; absolute minimum at $x = \pi$



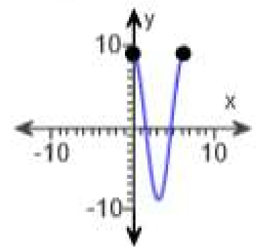
B. there is no absolute maximum; there is no absolute minimum



C. there is no absolute maximum; absolute minimum at $x = \pi$



D. absolute maximum at $x = 0$ and $x = 2\pi$; absolute minimum at $x = \pi$



16. Find the absolute extreme values of the function on the interval.

$f(x) = x^{\frac{8}{3}}, \ -1 \leq x \leq 8$

- A. absolute maximum is 256 at $x = 8$; absolute minimum is 1 at $x = -1$
- B. absolute maximum is 256 at $x = 8$; absolute minimum does not exist
- C. absolute maximum is 256 at $x = 8$; absolute minimum is 0 at $x = 0$
- D. absolute maximum is 512 at $x = 8$; absolute minimum is 0 at $x = 0$

17. The driver of a car traveling at 60 ft/sec suddenly applies the brakes. The position of the car is $s = 60t - 3t^2$, t seconds after the driver applies the brakes. How far does the car go before coming to a stop?

- A. 10 ft
- B. 1,200 ft
- C. 600 ft
- D. 300 ft

18. A rock is thrown vertically upward from the surface of an airless planet. It reaches a height of $s = 120t - 3t^2$ meters in t seconds. How high does the rock go? How long does it take the rock to reach its highest point?

- **A.** 2,400 m, 40 sec
- **B.** 1,200 m, 20 sec
- **C.** 4,680 m, 40 sec
- **D.** 2,380 m, 20 sec

19. Find the tangent to $y = \mathbf{cot}\, x$ at $x = \frac{\pi}{4}$.

- A. $y = 2x - \frac{\pi}{2} + 1$
- B. $y = -2x + \frac{\pi}{2}$
- C. $y = 2x + 1$
- D. $y = -2x + \frac{\pi}{2} + 1$

20. Find the number of units that must be produced and sold in order to yield the maximum profit, given the equations for revenue and cost shown below.

$R(x) = 40x - 0.5x^2$
$C(x) = 9x + 9$

- A. 49 units
- B. 32 units
- C. 40 units
- D. 31 units

21. The following equation gives the position s = f(t) of a body moving on a coordinate line (s in meters, t in seconds). Find the body's acceleration at time $t = \frac{\pi}{3}$ sec.

$s = -6 + 7 \cos t$

A. $\frac{7\sqrt{3}}{2}$ m/sec^2

B. $\frac{7}{2}$ m/sec^2

C. $-\frac{7}{2}$ m/sec^2

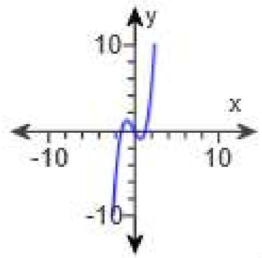
D. $-\frac{7\sqrt{3}}{2}$ m/sec^2

22. Find the second derivative.

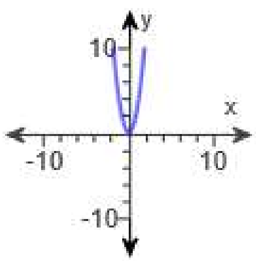
$$y = \frac{1}{11x^2} + \frac{1}{9x}$$

- A. $\frac{6}{11x^4} + \frac{2}{9x^3}$
- B. $-\frac{2}{11x^3} - \frac{1}{9x^2}$
- C. $-\frac{2}{11x^4} + \frac{1}{9x^3}$
- D. $\frac{6}{11x^4} - \frac{2}{9x^3}$

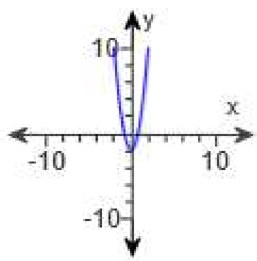
23. The graph of a function is given. Choose the answer that represents the graph of its derivative.



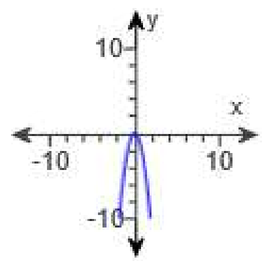
○ **A.**



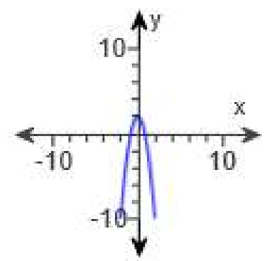
○ **B.**



○ **C.**



○ **D.**



24. Evaluate the integral.

$$\int \frac{e^{\sin^{-1}x}\,dx}{\sqrt{1-x^2}}$$

$$\int \frac{e^{\sin^{-1}x}\,dx}{\sqrt{1-x^2}} = \underline{\hspace{3cm}}$$

(Use C as the arbitrary constant.)

25. A spherical baloon is being inflated at a rate of 4860π cm^3 per min. At what rate is the radius r changing when the radius is 9 cm? $V = \frac{4}{3}\pi r^3$

- A. 50 cm/min
- B. 12.5 cm/min
- C. 15 cm/min
- D. 30 cm/min

26. Find the derivative of the function.

$$g(x) = \frac{x^2 + 5}{x^2 + 6x}$$

- A. $g'(x) = \dfrac{6x^2 - 10x - 30}{x^2(x + 6)^2}$
- B. $g'(x) = \dfrac{4x^3 + 18x^2 + 10x + 30}{x^2(x + 6)^2}$
- C. $g'(x) = \dfrac{2x^3 - 5x^2 - 30x}{x^2(x + 6)^2}$
- D. $g'(x) = \dfrac{x^4 + 6x^3 + 5x^2 + 30x}{x^2(x + 6)^2}$

1. B. $\frac{4}{\pi}$

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2. B. $8x^7 \cos\left(x^4\right)$

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3. C. $y = \tan u$; $u = \pi - \frac{8}{x}$; $\frac{dy}{dx} = \frac{8}{x^2} \sec^2\left(\pi - \frac{8}{x}\right)$

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4. A. $1,764.00/unit

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5. C. $\frac{2xy^2 - y - 1}{-2x^2y + x + 1}$

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6. 1

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7. -2

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8. C. $-\frac{1}{56}\left(7x^2 + 3\right)^{-4} + C$

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9. C. $\frac{7}{15}\left(6 + 4x^3\right)^{5/4} + C$

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10. C. $\frac{512}{5}$

11. C. No, because even if a function is defined at a point, the limit may not exist at that point.

C.
No, because f(x) could be a piecewise function where the limit approaching 1 from the left and the right are the same, but f(1) is defined as a different value.

D. We cannot conclude anything about $\lim_{x \to 1} f(x)$.

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12. C. $-\frac{1}{2}$

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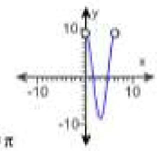
13. A. ∞

14. C. $\frac{4}{5}x^{-1/5} + 4e\,x^{4e-1}$

15.



C. there is no absolute maximum;absolute minimum at $x = \pi$

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16. C. absolute maximum is 256 at $x = 8$; absolute minimum is 0 at $x = 0$

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17. D. 300 ft

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18. B. 1,200 m, 20 sec

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19. D. $y = -2x + \frac{\pi}{2} + 1$

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20. D. 31 units

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21. C. $-\frac{7}{2}$ m/sec^2

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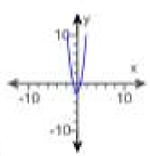
22. A. $\frac{6}{11x^4} + \frac{2}{9x^3}$

23.



B.

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24. $e^{\sin^{-1} x} + C$

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25. C. 15 cm/min

26. A. $g'(x) = \frac{6x^2 - 10x - 30}{x^2(x+6)^2}$