

Errors and the Algebra of Mathematics

Knowing How To Avoid These Common Errors

Will Help Improve Your Grades!

Errors Involving Parentheses

<i>Potential Error</i>	<i>Correct Form</i>	<i>Comment</i>
$a - (x - b) = a - x - b$	$a - (x - b) = a - x + b$	Change all signs when distributing minus sign.
$(a + b)^2 = a^2 + b^2$	$(a + b)^2 = a^2 + 2ab + b^2$	Remember the middle term when squaring binomials.
$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) = \frac{1}{2}(ab)$	$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) = \frac{1}{4}(ab) = \frac{ab}{4}$	$\frac{1}{2}$ occurs twice as a factor.
$(3x + 6)^2 = 3(x + 2)^2$	$(3x + 6)^2 = [3(x + 2)]^2$ $= 3^2(x + 2)^2$	When factoring, apply exponents to all factors.

Errors Involving Fractions

Potential Error

$$\cancel{\frac{a}{x+b} = \frac{a}{x} + \frac{a}{b}}$$

$$\cancel{\left(\frac{x}{a}\right) \div \frac{bx}{a}}$$

$$\cancel{\frac{1}{a} + \frac{1}{b} = \frac{1}{a+b}}$$

$$\cancel{\frac{1}{3x} = \frac{1}{3}x}$$

$$\cancel{(1/3)x = \frac{1}{3x}}$$

$$\cancel{(1/x) + 2 = \frac{1}{x+2}}$$

Correct Form

Leave as $\frac{a}{x+b}$.

$$\left(\frac{x}{a}\right) \div \frac{bx}{a} = \left(\frac{x}{a}\right)\left(\frac{1}{b}\right) = \frac{x}{ab}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$$

$$\frac{1}{3x} = \frac{1}{3} \cdot \frac{1}{x}$$

$$(1/3)x = \frac{1}{3} \cdot x = \frac{x}{3}$$

$$(1/x) + 2 = \frac{1}{x} + 2 = \frac{1+2x}{x}$$

Comment

Do not add denominators when adding fractions.

Multiply by the reciprocal when dividing fractions.

Use the property for adding fractions.

Use the property for multiplying fractions.

Be careful when using a slash to denote division.

Be careful when using a slash to denote division and be sure to find a common denominator before you add fractions.

Errors Involving Exponents

Potential Error	Correct Form	Comment
$(x^2)^3 = x^5$	$(x^2)^3 = x^{2 \cdot 3} = x^6$	Multiply exponents when raising a power to a power.
$x^2 \cdot x^3 = x^6$	$x^2 \cdot x^3 = x^{2+3} = x^5$	Add exponents when multiplying powers with like bases.
$2x^3 = (2x)^3$	$2x^3 = 2(x^3)$	Exponents have priority over coefficients.
$\frac{1}{x^2 - x^3} = x^{-2} - x^{-3}$	Leave as $\frac{1}{x^2 - x^3}$.	Do not move term-by-term from denominator to numerator.

Errors Involving Radicals

Potential Error	Correct Form	Comment
$\sqrt{5x} = 5\sqrt{x}$	$\sqrt{5x} = \sqrt{5}\sqrt{x}$	Radicals apply to every factor inside the radical.
$\sqrt{x^2 + a^2} = x + a$	Leave as $\sqrt{x^2 + a^2}$.	Do not apply radicals term-by-term.
$\sqrt{-x + a} = -\sqrt{x - a}$	Leave as $\sqrt{-x + a}$.	Do not factor minus signs out of square roots.

Errors Involving Dividing Out

Potential Error	Correct Form	Comment
$\frac{a + bx}{a} = 1 + bx$	$\frac{a + bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{b}{a}x$	Divide out common factors, not common terms.
$\frac{a + ax}{a} = a + x$	$\frac{a + ax}{a} = \frac{a(1 + x)}{a} = 1 + x$	Factor before dividing out.
$1 + \frac{x}{2x} = 1 + \frac{1}{x}$	$1 + \frac{x}{2x} = 1 + \frac{1}{2} = \frac{3}{2}$	Divide out common factors.