

Math 1540 Quiz 1 Study Guide

1. Calculate the average rate of change of a function $y = f(x)$ over interval ARC

Example: Calculate ARC of $y = f(x) = \frac{10}{x^2+1}$ over interval $(1, 3)$

$$ARC = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{10}{3^2+1} - \frac{10}{1^2+1}}{3-1} = \frac{1-5}{2} = \boxed{-2}$$

2. Calculate limits, $\lim_{x \rightarrow x_0} f(x) = L$ using various methods

Example a) Plug-In $\lim_{x \rightarrow 2} x^3 - 2x^2 + 3 = 2^3 - 2(2^2) + 3 = 8 - 8 + 3 = 3$

Example b) Function with a "hole" $\lim_{x \rightarrow -5} \frac{x^2 + 13x + 40}{x + 5} = \lim_{x \rightarrow -5} \frac{(x+8)(\cancel{x+5})}{(\cancel{x+5})} = \lim_{x \rightarrow -5} x + 8 = 3$

Example c) Function With Radical $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{(x-1)}} = \boxed{2}$

Example D: Functions with $\sin x$, remember $\sin(\text{anything}) = \text{anything}$
anything $\rightarrow 0$

$$\lim_{x \rightarrow 0} \frac{6x + \sin 10x}{x} = \lim_{x \rightarrow 0} \frac{6x}{x} + \frac{\sin 10x}{x} = \lim_{x \rightarrow 0} 6 + \frac{10x}{x} = 16$$

$$\lim_{x \rightarrow 0} \frac{6x^2 + \sin 5x}{x} = \lim_{x \rightarrow 0} \frac{6x^2}{x} + \frac{\sin 5x}{x} = \lim_{x \rightarrow 0} 6x + \frac{5x}{x} = 0 + 5 = \boxed{5}$$

Example e, Limits as $x \rightarrow \infty$

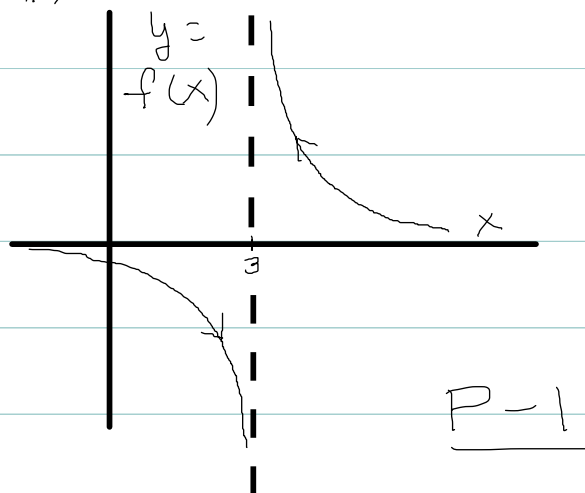
$$\lim_{x \rightarrow \infty} \frac{3x^3 + 4x^2 - 2x + 1}{2x^3 - 6x^2 + x - 2} = \lim_{x \rightarrow \infty} \frac{3x^3}{2x^3} = \frac{3}{2}, \quad \lim_{x \rightarrow \infty} \frac{10x^2 - 4x + 3}{5x^3 + 2x + 12} = \lim_{x \rightarrow \infty} \frac{10x^2}{5x^3} = \lim_{x \rightarrow \infty} \frac{2}{x} = 0$$

$$\lim_{x \rightarrow \infty} \frac{10x^4 + 3x^3 - 2x + 1}{2x^3 - 5x^2 + 6x - 10} = \lim_{x \rightarrow \infty} \frac{10x^4}{2x^3} = \lim_{x \rightarrow \infty} 5x = \infty$$

Example f. Infinite Limits

$$\lim_{x \rightarrow 3^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 3^+} f(x) = +\infty$$



P-1

3. Calculate the slope M and the equation of the tangent line at the point (x_0, y_0)

Example: $y = f(x) = 3x^2 - 2x + 5$ at $(2, 13)$, slope $M = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$M = \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 2(x+h) + 5 - (3x^2 - 2x + 5)}{h}$$

$$M = \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 5 - 3x^2 + 2x - 5}{h}$$

$$M = \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h + \cancel{5} - \cancel{3x^2} + \cancel{2x} - \cancel{5}}{h} = \lim_{h \rightarrow 0} \frac{6xh + 3h^2 - 2h}{h}$$

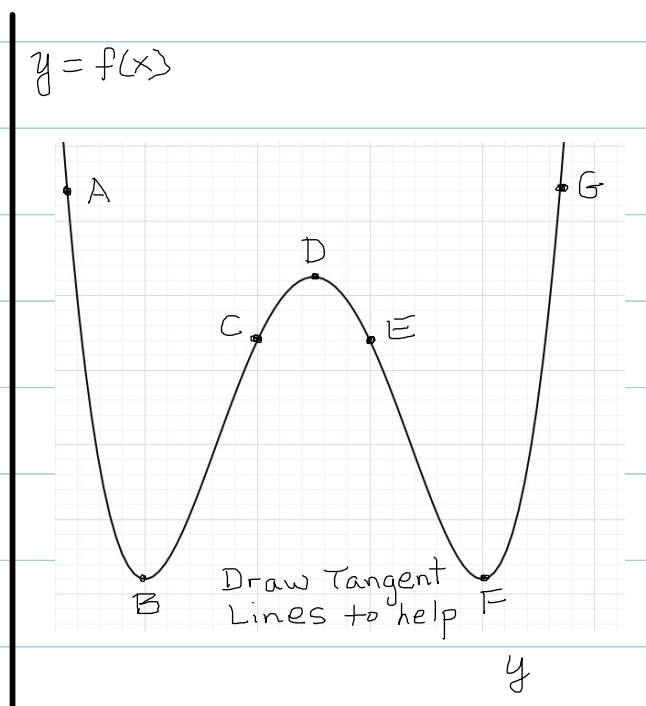
$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}} = 6x - 2, \text{ at } x=2, M = 6(2) - 2 = 10$$

so $y = Mx + b = 10x + b$ (Plug In 2, 13)

so $13 = 10(2) + b, b = -7$

$y = 10x - 7$ Equation of tangent line.

4. Evaluate the relative values of slope (instantaneous rate of change) at points on graph.



which points have the slope described

Zero Slope _____

Large Negative Slope _____

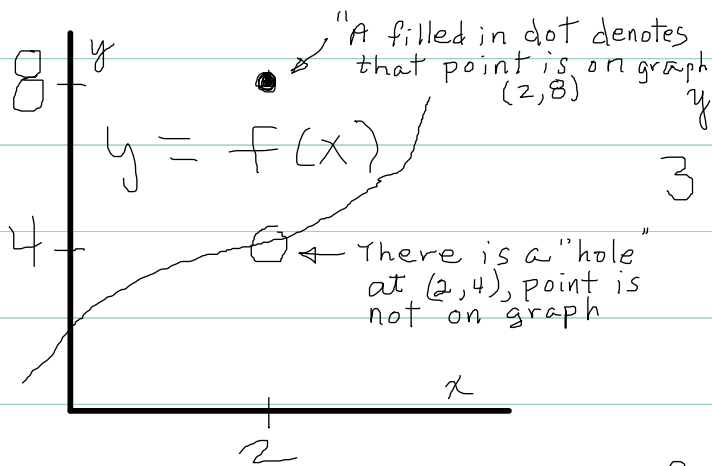
Large Positive Slope _____

Moderate Positive Slope _____

Moderate Negative Slope _____

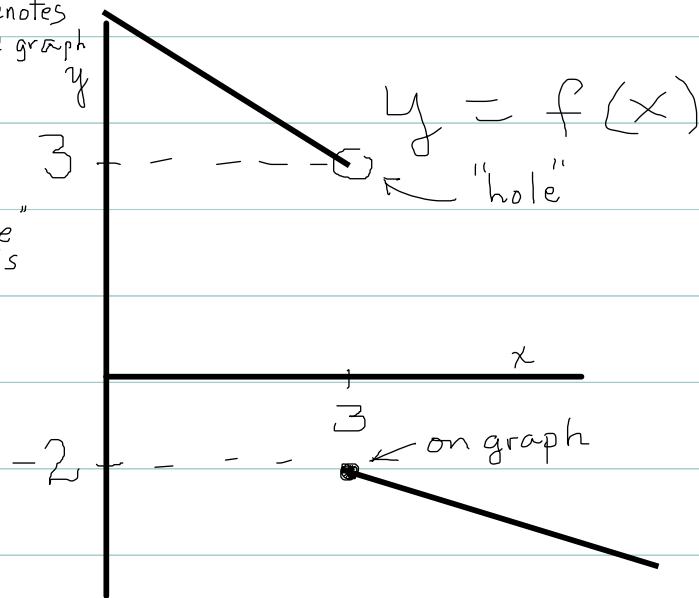
Large Positive Slope _____

5. Use graph to evaluate function values and info on limits



$$f(2) = 8$$

$$\lim_{x \rightarrow 2} f(x) = 4$$



$$f(3) = -2$$

$$\lim_{x \rightarrow 3^+} f(x) = -2$$

$$\lim_{x \rightarrow 3^-} f(x) = 3$$

6. More Avg. and Inst. Rate of Change

On Earth an object falls a distance given by $s = 16t^2$, s in feet, t in seconds

a) What is Avg. Rate of Change (ARC) from 1 to 5 seconds?

$$ARC = \frac{16(5)^2 - 16(1)^2}{5 - 1} = \frac{400 - 16}{4} = \frac{384}{4} = 96 \text{ ft/sec}$$

b) What is inst. rate of Change at $t = 5$?

$$IRC = \lim_{h \rightarrow 0} \frac{S(t+h) - S(t)}{h} = \lim_{h \rightarrow 0} \frac{16(t+h)^2 - 16t^2}{h} = \lim_{h \rightarrow 0} \frac{16t^2 + 32th + 16h^2 - 16t^2}{h} = \lim_{h \rightarrow 0} \frac{h(32t + 16h)}{h} = 32t$$

at $t = 5$, $IRC = 160 \text{ ft/sec}$.