Math 2012 Calculus II Final Practice Spring 2008

Name: Last _____, First _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

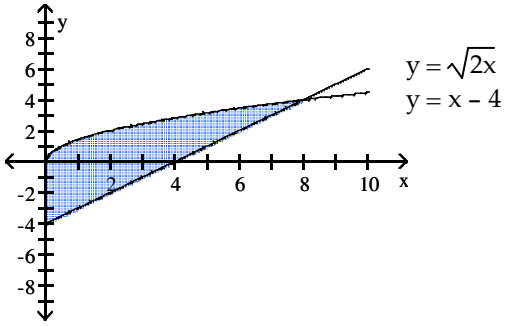
Find the length of the curve.

1) $y = (16 - x^{2/3})^{3/2}$ from $x = 1$ to $x = 64$ 1) _____

A) 96 B) 90 C) 180 D) 60

Find the area of the shaded region.

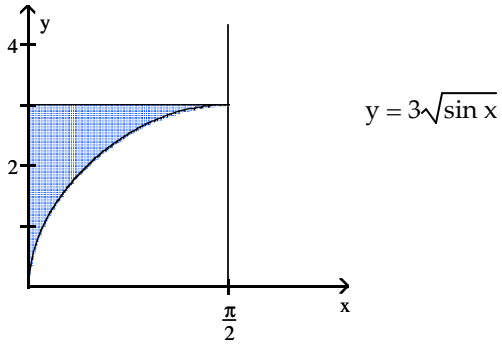
2) 2) _____



A) 32 B) $\frac{32}{3}$ C) $\frac{128}{3}$ D) $\frac{64}{3}$

Find the volume of the solid generated by revolving the shaded region about the given axis.

3) About the x–axis 3) _____



A) $\frac{9}{2}\pi^2 - 3\pi$ B) $\frac{9}{2}\pi^2 - 9\pi$ C) $\frac{9}{2}\pi^2$ D) $\frac{9}{2}\pi^2 + 9\pi$

Solve the problem.

4) A rescue cable attached to a helicopter weighs 2 lb/ft. A 160–lb man grabs the end of the rope and is pulled from the ocean into the helicopter. How much work is done in lifting the man if the helicopter is 50 ft above the water? 4) _____

A) 13,000 ft • lb B) 2660 ft • lb C) 8100 ft • lb D) 10,500 ft • lb

Integrate the function.

5) $\int \frac{dx}{(x^2 + 16)^{3/2}}$ 5) _____

A) $\frac{4}{x\sqrt{16 - x^2}} + C$

B) $\frac{x}{16\sqrt{16 - x^2}} + \frac{\sqrt{16 - x^2}}{x} + C$

C) $\frac{x}{4\sqrt{16 + x^2}} + C$

D) $\frac{x}{16\sqrt{16 + x^2}} + C$

Express the integrand as a sum of partial fractions and evaluate the integral.

6) $\int \frac{x + 8}{x^2 + 6x}\, dx$ 6) _____

A) $\frac{4}{3} \ln \left|x^8(x + 6)^2\right| + C$

B) $\frac{1}{6} \ln \left|x^8(x + 6)^2\right| + C$

C) $\frac{1}{6} \ln \left|\frac{x^8}{(x + 6)^2}\right| + C$

D) $\ln \left|\frac{x^8}{(x + 6)^2}\right| + C$

Evaluate the integral.

7) $\int_0^{\pi/2} x^3 \cos 5x\, dx$ 7) _____

A) $\frac{1}{5}x^3 \sin 5x - \frac{3}{25}x^2 \cos 5x + \frac{6}{125}x \sin 5x + \frac{6}{625} \cos 5x + C$

B) $\frac{1}{5}x^3 \sin 5x + \frac{3}{5}x^2 \cos 5x - \frac{6}{5}x \sin 5x - \frac{6}{5} \cos 5x + C$

C) $\frac{1}{5}x^3 \cos 5x + \frac{3}{25}x^2 \sin 5x - \frac{6}{125}x \cos 5x - \frac{6}{625} \sin 5x + C$

D) $\frac{1}{5}x^3 \sin 5x + \frac{3}{25}x^2 \cos 5x - \frac{6}{125}x \sin 5x - \frac{6}{625} \cos 5x + C$

A recursion formula and the initial term(s) of a sequence are given. Write out the first five terms of the sequence.

8) $a_1 = 1, a_2 = 3, a_{n+2} = a_{n+1} - a_n$ 8) _____

A) 1, –3, 4, –5, 6 B) 1, 3, 2, 1, 0 C) 1, –1, 2, –3, 5 D) 1, 3, 2, –1, –3

Determine if the series converges or diverges; if the series converges, find its sum.

9) $\sum_{n=1}^{\infty} \frac{4^{n+1}}{5^{n-1}}$ 9) _____

A) Converges; 100 B) Converges; 20 C) Converges; 80 D) Diverges

Find the Taylor polynomial of order 3 generated by f at a.

10) $f(x) = x^3, a = 7$ 10) _____

A) $343 + 49(x - 49) + 49(x - 49)^2 + (x - 49)^3$

B) $6 + 3(x - 49) + (x - 49)^2 + (x - 49)^3$

C) $343 + 147(x - 49) + 21(x - 49)^2 + (x - 49)^3$

D) $1372 + 147(x - 49) + 14(x - 49)^2 + (x - 49)^3$

Answer Key
Testname: MATH 2012-FINAL PRACTICE-SP-08UPDATE

1) B
2) D
3) B
4) D
5) D
6) C
7) D
8) D
9) C
10) C

11. Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the *x*-axis.

$y = x,\ y = 0,\ x = 4$

A) $V = 2\pi \int_0^2 (y)(2+y)dy = \frac{64}{3}\pi$

B) $V = 2\pi \int_0^2 (y)(4+y)dy = \frac{32}{3}\pi$

C) $V = 2\pi \int_0^2 (y)(4-y)dy = \frac{64}{3}\pi$

D) $V = 2\pi \int_0^4 (y)(2+y)dy = \frac{64}{3}\pi$

E) $V = 2\pi \int_0^4 (y)(4-y)dy = \frac{64}{3}\pi$

12. Find the arc length of the graph of the function $y = \frac{2}{3}x^{\frac{3}{2}} + 2$ over the interval [0,7].

A) $\frac{2}{3}\sqrt{512}$

B) $\frac{2}{3}\left(\sqrt{343}-1\right)$

C) $\frac{2}{3}\left(\sqrt{512}+1\right)$

D) $\frac{2}{3}\sqrt{343}$

E) $\frac{2}{3}\left(\sqrt{512}-1\right)$

13. Find the definite integral.

$$\int_0^2 x\, e^{-x^2}\, dx$$

A) $\frac{1}{2}\left[1-e^{-4}\right]$

B) $2\left[1-e^{-4}\right]$

C) $\left[1-e^{-4}\right]$

D) $-\frac{1}{2}\left[1+e^{-4}\right]$

E) $-2\left[1-e^{-4}\right]$

14. Find the indefinite integral.

$$\int \sin^3 \frac{x}{7}\, dx$$

A) $-\frac{7\left(3-\cos^2\frac{x}{7}\right)\cos\frac{x}{7}}{3}+C$

B) $-\frac{7\left(2-\cos^2\frac{x}{7}\right)\cos\frac{x}{7}}{3}+C$

C) $\frac{7\left(3-\cos^2\frac{x}{7}\right)\cos\frac{x}{7}}{3}+C$

D) $\frac{7\left(1+\cos^2\frac{x}{7}\right)\cos\frac{x}{7}}{3}+C$

E) $\frac{7\left(2-\cos^2\frac{x}{7}\right)\cos\frac{x}{7}}{3}+C$

15. Write the first five terms of the sequence.

$a_n = (-1)^{n+2}\left(\frac{13}{n}\right)$

A) $-13, \frac{13}{2}, -\frac{13}{3}, \frac{13}{4}, -\frac{13}{5}$

B) $13, -\frac{13}{2}, \frac{13}{3}, -\frac{13}{4}, \frac{13}{5}$

C) $-13, -\frac{13}{2}, -\frac{13}{3}, -\frac{13}{4}, -\frac{13}{5}$

D) $13, \frac{13}{2}, \frac{13}{3}, \frac{13}{4}, \frac{13}{5}$

E) $-13, -\frac{13}{2}, \frac{13}{3}, \frac{13}{4}, -\frac{13}{5}$

16. Determine the convergence or divergence of the sequence with the given *n*th term. If the sequence converges, find its limit.

$a_n = \frac{\ln\left(n^{10}\right)}{8n}$

A) Sequence converges to 0
B) Sequence diverges
C) Sequence converges to 1
D) Sequence converges to -1
E) Sequence diverges to 1

17. Determine the convergence or divergence of the series.

$\sum_{n=1}^{\infty} \frac{6^n}{n^6}$

A) Diverges
B) Converges
C) Cannot be determined from the methods in the chapter

18. Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{8}{9^n}$$

A) Diverges
B) Converges
C) Cannot be determined from the methods in the chapter

19. Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{4}{n^{\frac{5}{8}}}$$

A) Inconclusive
B) Converges
C) Diverges

20. Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \left(\frac{10}{7}\right)^n$$

A) Inconclusive
B) Converges
C) Diverges

21. Use the Ratio Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{n^5}{6^{-n}}$$

A) Diverges
B) Converges
C) Ratio Test is inconclusive

22. Use the Root Test to determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty}\left(\frac{10n^2+1}{6n^2-1}\right)^n$$

A) Root Test is inconclusive
B) Converges
C) Diverges

23. Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty}\frac{10n}{n+5}$$

A) Diverges; Ratio Test
B) Diverges; Integral Test
C) Converges; *p*-series
D) Converges; Integral Test
E) Both A and B
F) Both C and D

24. Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

$$\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$$

A) $[-2,2)$
B) $(-2,2)$
C) $[-2,2]$
D) $\left[-\frac{1}{2},\frac{1}{2}\right)$
E) $\left(-\frac{1}{2},\frac{1}{2}\right)$

25. Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

$$\sum_{n=0}^{\infty} \frac{(7x)^n}{(5n)!}$$

A) $(-\infty, \infty)$
B) $(-1,1)$
C) $[-1,1)$
D) $(-7,7)$
E) $\left(-\frac{1}{7}, \frac{1}{7}\right)$

26. Find the indefinite integral by making the substitution $x = 3\sin\theta$.

$$\int \frac{x}{\left(9-x^2\right)^{3/2}} dx$$

A) $\frac{1}{\sqrt{9-x^2}} + C$
B) $-\frac{3}{2\sqrt{9-x^2}} + C$
C) $\frac{3}{2\sqrt{9-x^2}} + C$
D) $-\frac{1}{\sqrt{9-x^2}} + C$
E) None of the above

27. Find the area of the surface generated by revolving the curve about the *x*-axis.

$y = \frac{1}{3}x^3, \quad 0 \le x \le 3$

A) $\frac{1\left(82^{\frac{3}{2}} - 1\right)}{18}\pi$

B) $\frac{\left(82^{\frac{3}{2}} - 1\right)\pi}{9}$

C) $\frac{1\left(82^{\frac{3}{2}} + 1\right)}{9}\pi$

D) $\left(82^{\frac{3}{2}} + 1\right)\pi$

E) $\frac{1\left(28^{\frac{3}{2}} + 1\right)}{18}\pi$

Answer Key 2nd 10

11. E
12. E
13. A
14. B
15. A
16. A
17. A
18. B
19. C
20. C
21. A
22. C
23. B
24. B
25. A
26. A
27. B