Quantitative Skills & Reasoning – Math 1001

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Combining Probabilities
pp 282-293 in textbook

Definitions

- **Independent Events**
  - Events *A* and *B* are independent events if the probability of Event *B* occurring is the same whether or not Event *A* occurs.

Formula (P(*A* ***and*** *B*) for independent events)

If events *A* and *B* are independent, then the probability of both *A* and *B* occurring is

$$P(A \textbf{ and } B) = P(A) \cdot P(B)$$

- where P(*A* and *B*) is the probability of events *A* and *B* both occurring, P(*A*) is the probability of event A occurring, and P(*B*) is the probability of event *B* occurring.

Are these events independent?

a) A fair coin is tossed two times. The two events are (1) first toss is a head and (2) second toss is a head.

b) The two events (1) "It will rain tomorrow in Houston" and (2) "It will rain tomorrow in Galveston" (a city near Houston).

c) You draw a card from a deck, then draw a second card without replacing the first.

Are these events independent?

a) A fair coin is tossed two times. The two events are (1) first toss is a head and (2) second toss is a head. **Independent**

b) The two events (1) "It will rain tomorrow in Houston" and (2) "It will rain tomorrow in Galveston" (a city near Houston). **Not Independent**

c) You draw a card from a deck, then draw a second card without replacing the first. **Not Independent**

Example

In your drawer you have 10 pairs of socks, 6 of which are white, and 7 tee shirts, 3 of which are white. If you reach in and randomly grab a pair of socks and a tee shirt, what the probability you grab a white shirt and white socks?

$$\frac{6}{10} \times \frac{3}{7} = \frac{18}{70} = \frac{9}{35}$$

Example

You roll a fair six-sided die twice. What is the probability that you roll two ones?

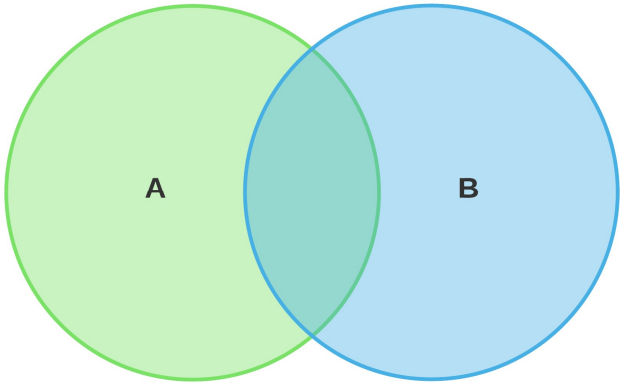
$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Example

What is the probability that you roll an even number and then a 5?

$$\frac{3}{6} \times \frac{1}{6} = \frac{3}{36} = \frac{1}{12}$$

Probability of Either Event



Formula (P(*A or B*))

- The probability of either *A* or *B* occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

Note: Two events are ***mutually exclusive*** if they do not share any common outcomes.

Example

In your drawer you have **10 pairs of socks, 6 of which are white**, and **7 tee shirts, 3 of which are white**. If you reach in and randomly grab a pair of socks and a tee shirt, what is the probability at least one is white?

Let **A Event** be drawing a white sock and **B Event** be drawing a white tee shirt

$$P(A) = \frac{6}{10} \quad P(B) = \frac{3}{7}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A)P(B)$$

$$= \frac{6}{10} + \frac{3}{7} - \frac{6}{10}\frac{3}{7} = \frac{72}{70} - \frac{18}{70} = \frac{54}{70} = \frac{27}{35}$$

Suppose we draw one card from a standard deck. What is the probability that we get a red card or a King?

Half the cards are red, so $P(\text{red}) = \frac{26}{52}$

There are four kings, so $P(\text{King}) = \frac{4}{52}$

There are two red kings, so $P(\text{Red and King}) = \frac{2}{52}$

We can then calculate

$$P(\text{Red or King}) = P(\text{Red}) + P(\text{King}) - P(\text{Red and King}) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$$

Example

Giving a test to a group of students, the grades and gender are summarized below.

	A	**B**	**C**	**Total**
Male	8	18	13	**39**
Female	10	4	12	**26**
Total	**18**	**22**	**25**	**65**

If one student was chosen at random, find the probability that the student was female or got a B.

Example

Giving a test to a group of students, the grades and gender are summarized below.

	A	B	C	Total
Male	8	18	13	**39**
Female	10	4	12	**26**
Total	**18**	**22**	**25**	**65**

If one student was chosen at random, find the probability that the student was female or got a B.

$$P(\text{B or Female}) = \frac{18+10+4+12}{65} = \frac{44}{65}$$

Conditional Probability

The probability the event *B* occurs, given that event *A* has happened, is represented as $P(B|A)$. This is read as "the probability of *B* given *A*."

Example

Giving a test to a group of students, the grades and gender are summarized below.

	A	B	C	Total
Male	8	18	13	**39**
Female	10	4	12	**26**
Total	**18**	**22**	**25**	**65**

If one student was chosen at random, find the probability that the student got an A given that he is male.

$$P(\text{Student Got A} \mid \text{Was Male}) = \frac{8}{39}$$

Example

Giving a test to a group of students, the grades and gender are summarized below.

	A	B	C	Total
Male	8	18	13	**39**
Female	10	4	12	**26**
Total	**18**	**22**	**25**	**65**

Find the probability that a student is male given that he got an A.

$$P(\text{Was Male} \mid \text{Got A}) = \frac{8}{18} = \frac{4}{9}$$

Conditional Probability for Dependent Events

- **Independent Events**
  - Events *A* and *B* are **dependent** events if the probability of Event *B* is affected by Event *A* occurring.

Formula (P(*A* ***and*** *B*) for dependent events)

If events *A* and *B* are dependent, then the probability of both *A* and *B* occurring is

$$P(A \textbf{ and } B) = P(A) \cdot P(B|A)$$

Conditional Probability

If you pull 2 cards out of a deck, what is the probability that both are twos?

Events A – The First Card is a Two $\quad P(A) = \frac{4}{52} = \frac{1}{13}$

Event B – The Second Card is a two $\quad P(B|A) = \frac{3}{51} = \frac{1}{17}$

$$P(A \textbf{ and } B) = P(A) \cdot P(B|A) = \frac{1}{13}\frac{1}{17} = \frac{1}{221}$$