Quantitative Skills & Reasoning – Math 1001

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Counting and Probability
pp 297-309 in textbook

Definitions

- **Fundamental Counting Principle**

If there are *m* items in one category and *n* items in another category, then the total number of available choices is *m*n*.

This principle can be extended to more than two categories as well.

Example

Suppose at a particular restaurant you have eight choices for an appetizer, eleven choices for a main course and five choices for dessert. If you are allowed to choose exactly one item from each category for your meal, how many different meal options do you have?

Number of Choices = (8)(11)(5) = 440

Definitions

The **factorial** of a non-negative integer n, is the product of all positive integers less than or equal to n.

Notation

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Note: 0! = 1

Example

5! = 5*4*3*2*1 = 120

Example

A charity benefit is attended by 25 people and three gift certificates are given away as door prizes: one gift certificate is in the amount of $100, the second is worth $25 and the third is worth $10. Assuming that no person receives more than one prize, how many different ways can the three gift certificates be awarded?

Number of Ways = (25)(24)(23) = 13,800

Definitions

Permutations

We say that there are ${}_nP_r$ permutations of size *r* that may be selected from among *n* choices **without replacement <u>when order matters</u>**.

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Definitions

Combinations

We say that there are ${}_nC_r$ combinations of size *r* that may be selected from among *n* choices **without replacement <u>when order doesn't matter</u>**.

$$ {}_nC_r = \frac{n!}{(n-r)!r!} $$

Example

The United States Senate Appropriations Committee consists of 29 members; the Defense Subcommittee of the Appropriations Committee consists of 19 members. Disregarding party affiliation or any special seats on the Subcommittee, how many different 19-member subcommittees may be chosen from among the 29 Senators on the Appropriations Committee?

$$_{29}C_{19} = \frac{29!}{10!19!} = 20{,}030{,}010$$

Probability Using Permutations and Combinations

Example

In a certain state's lottery, 48 balls numbered 1 through 48 are placed in a machine and six of them are drawn at random. If the six numbers drawn match the numbers that a player had chosen, the player wins $1,000,000. In this lottery, the order the numbers are drawn in doesn't matter. Compute the probability that you win the million-dollar prize if you purchase a single lottery ticket.

Number of ways to pick 6 of 48 = ${}_{48}C_6$ = 12,271,512

$$\text{Probability} = \frac{1}{12{,}271{,}512} = .0000000815$$

Probability Using Permutations and Combinations

Example

Compute the probability of randomly drawing five cards from a standard deck of cards and getting three Aces and two Kings.

number of ways you can get 3 aces out of 4 aces is ${}_4C_3 = 4$ 1st Requirement to win

number of ways you can get 2 kings out of 4 kings is ${}_4C_2 = 6$ 2nd Requirement to win

the number of ways you can get 2 kings and 3 aces is (4)(6) = 24 possible ways to win

number of ways you can get 5 cards out of a deck of 52 cards is ${}_{52}C_5 = 2598960$

probability of getting 3 aces and 2 kings when you draw 5 cards from the deck is

$$\frac{24}{2598960} = .000009234 = 9.234 \text{ X } 10^{-6}$$

Probability Using Permutations and Combinations

Example

A jury pool consists of 27 people, 14 men and 13 women. Compute the probability that a randomly selected jury of 12 people is all male.

From the 14 men, pick 12 to make an all male jury ${}_{14}C_{12} = 91$

Total ways from the 27 people of picking "any" 12 people = ${}_{27}C_{12} = 17383860$

$$P(\text{all male}) = \frac{91}{17383860} = .000005235 = 5.235 \times 10^{-6}$$