Quantitative Skills & Reasoning – Math 1001

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Probability Unit - Expected Value

pp 309-312 in textbook

Definitions

Expected Value is the average gain or loss of an event if the procedure is repeated many times.

$$\mu = E(X) = \sum_i x_i p_i,$$

where x_i = is an outcome and p_i is the probability of that outcome

We can compute the expected value by multiplying each outcome by the probability of that outcome, then adding up the products.

Example

You roll a die. You receive a dollar for each dot. What is the expected value of the amount you receive. This would also be the average of what you would receive over many attempts.

$$P(1) = P(2) = P(3) = \cdots\cdots = P(6) = \frac{1}{6}$$

$$\text{Expected Value} = \$1\left(\frac{1}{6}\right)+\$2\left(\frac{1}{6}\right)+\$3\left(\frac{1}{6}\right) + \$4\left(\frac{1}{6}\right) + \$5\left(\frac{1}{6}\right)+\$6\left(\frac{1}{6}\right) = \$21\left(\frac{1}{6}\right) = \$3.50$$

Example

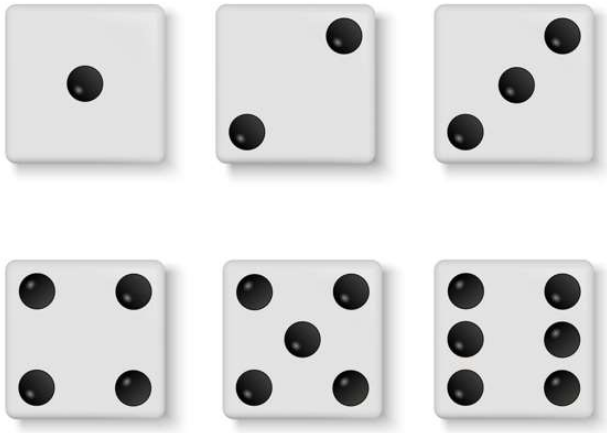
You purchase a raffle ticket to help out a the high school football team. The raffle ticket costs $20 and 1,500 tickets are sold. One of them will be drawn and the winner receives $1,000. Compute the expected value for this raffle.

Outcome	Gain or Loss	Probability
Win Prize	$980	$\frac{1}{1500}$
Lose	-$20	$\frac{1499}{1500}$

Expected Value = $\$980 \frac{1}{1500} - \$20 \frac{1499}{1500} = -\$19.33$

Example

A game involves rolling two dice. If the sum is 12, you win $10, otherwise if the sum is greater than 8, you win $5. It costs $2 to play. What is the expected value? Should you play?



Example (cont.)

Possible Outcomes

		Dice #2					
		1	2	3	4	5	6
Dice #1	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Roll 12 8<Roll<12

Example (cont.)

Outcome	Gain or Loss	Probability
Roll 12	\$8	$\frac{1}{36}$
8<Roll<12	\$3	$\frac{9}{36} = \frac{1}{4}$
Roll 8 or less	-\$2	$\frac{26}{36} = \frac{13}{18}$

Expected Value = $\$8\,\frac{1}{36} + \$3\,\frac{1}{4} - \$2\,\frac{13}{18} = -\$.47$

Example (cont.)

What if instead it cost $1 to play? What is the expected value? Should you play?

Outcome	Gain or Loss	Probability
Roll 12	$9	$\frac{1}{36}$
8<Roll<12	$4	$\frac{9}{36} = \frac{1}{4}$
Roll 8 or less	-$1	$\frac{26}{36} = \frac{13}{18}$

Expected Value = $\$9\frac{1}{36} + \$4\frac{1}{4} - \$1\frac{13}{18} = \$.53$ Yes. The House Will Lose

Example

An insurance company estimates the probability of an earthquake in the next year to be 0.0013. The average damage done by an earthquake it estimates to be $60,000. If the company offers earthquake insurance for $100, what is their expected value of the policy?

Outcome	Gain or Loss	Probability
Earthquake	$59900	.0013
No Earthquake	-$100	.9987

Expected Value = $59900(.0013) -100(.9987) = -$22

If > 0 The insurance company would go out of business