Quantitative Skills & Reasoning – Math 1001

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Introduction to Modeling Unit
Exponential Function Models
pp 182-188 in textbook

Exponential Functions

Describe Astounding Growth!
As Well As
Hoped for Quick Decay)

Things We Want To Increase

- **Our Economy**
- **Our Technology**
- **Our Income**
- **Life Expectancy**

Things We Want To Decrease

- **Deaths by War and Violence**
- **Unemployment**
- **Disease**
- **Crime Rate**

BUSINESS

PPING IN ANTA: LIFE IN SLOW LANE

IN LIVING

THIS IS AN ICON?

Another perfectly good word loses its identity in the swamp of pop culture

he Atlanta Journal-Constitution

AT AJC.COM

TUESDAY, OCT. 10, 2006

A7

Mr. n" oo econd me.

.COM ofile Woo

Booming expectations: Arrival of 300-millionth American may reflect our nation's changing face

By EUNICE MOSCOSO
emoscoso@ajc.com

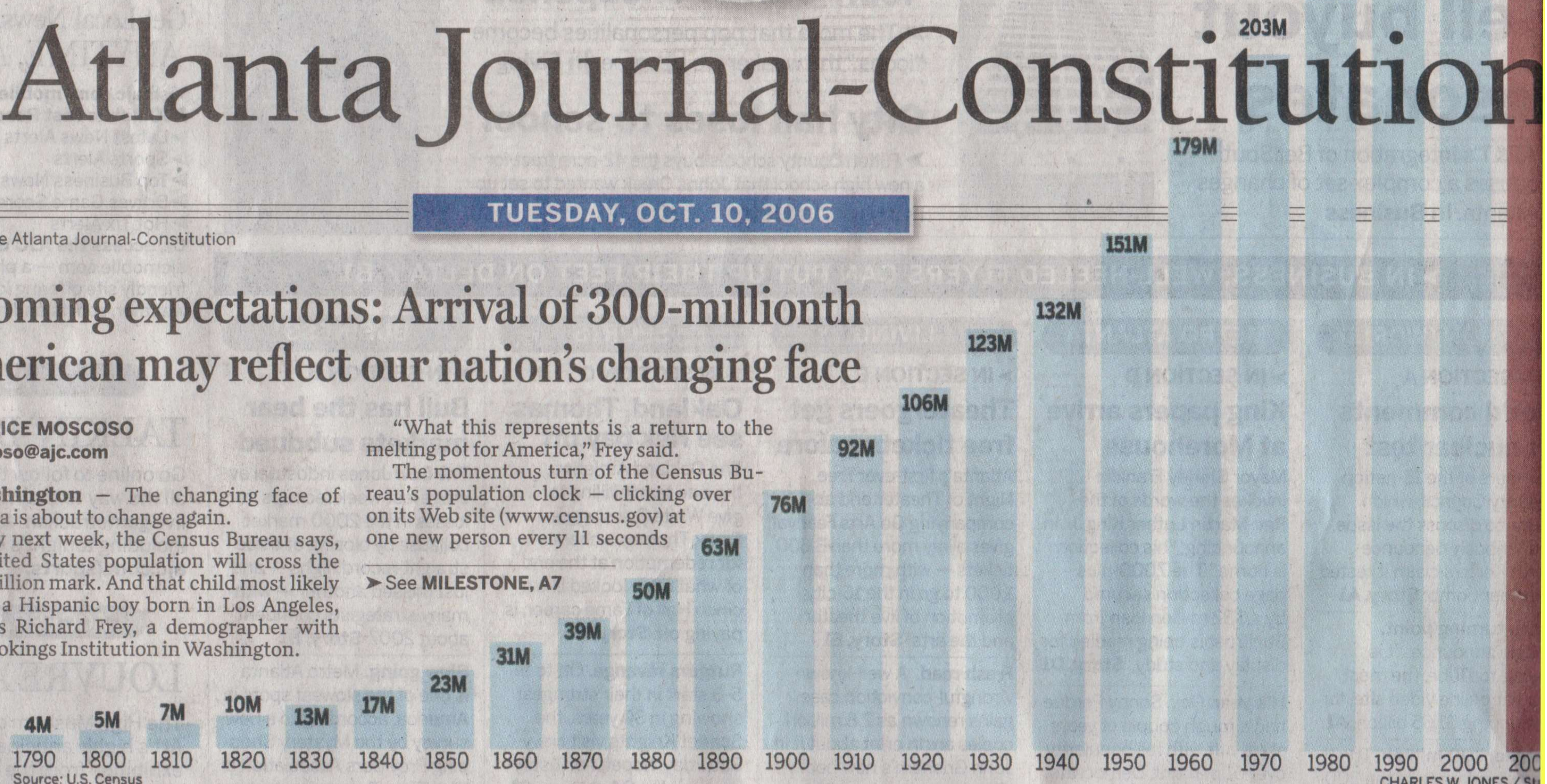
Washington — The changing face of America is about to change again.

Early next week, the Census Bureau says, the United States population will cross the 300-million mark. And that child most likely will be a Hispanic boy born in Los Angeles, predicts Richard Frey, a demographer with the Brookings Institution in Washington.

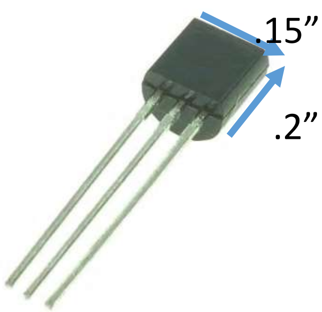
"What this represents is a return to the melting pot for America," Frey said.

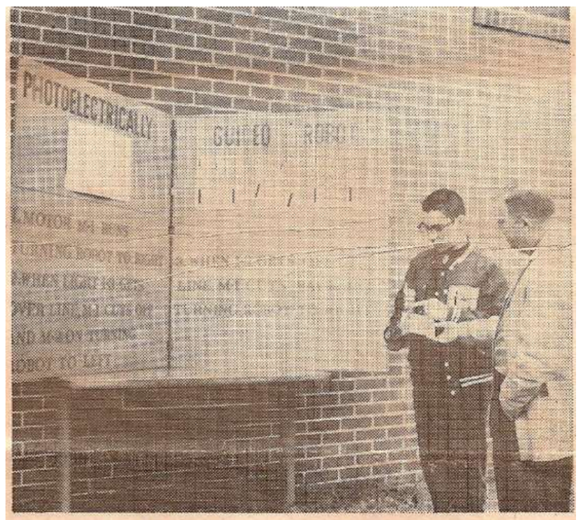
The momentous turn of the Census Bureau's population clock — clicking over on its Web site (www.census.gov) at one new person every 11 seconds

➤ See MILESTONE, A7



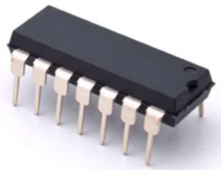
Bell Labs
December 23, 1947



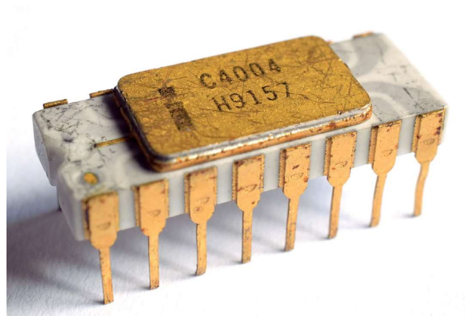


JAMES COLLINS STUDIES A SCIENCE EXHIBIT
Robert Brown's Photoelectric Robot Almost Human

Dr. Brown’s 1st Transistor Used in 1963 to Guide a Robot Along a Programmed Path



Dr. Brown's Motorola Chip Used in 1964 in Design of Digital Circuits – Six Transistors On the Chip!

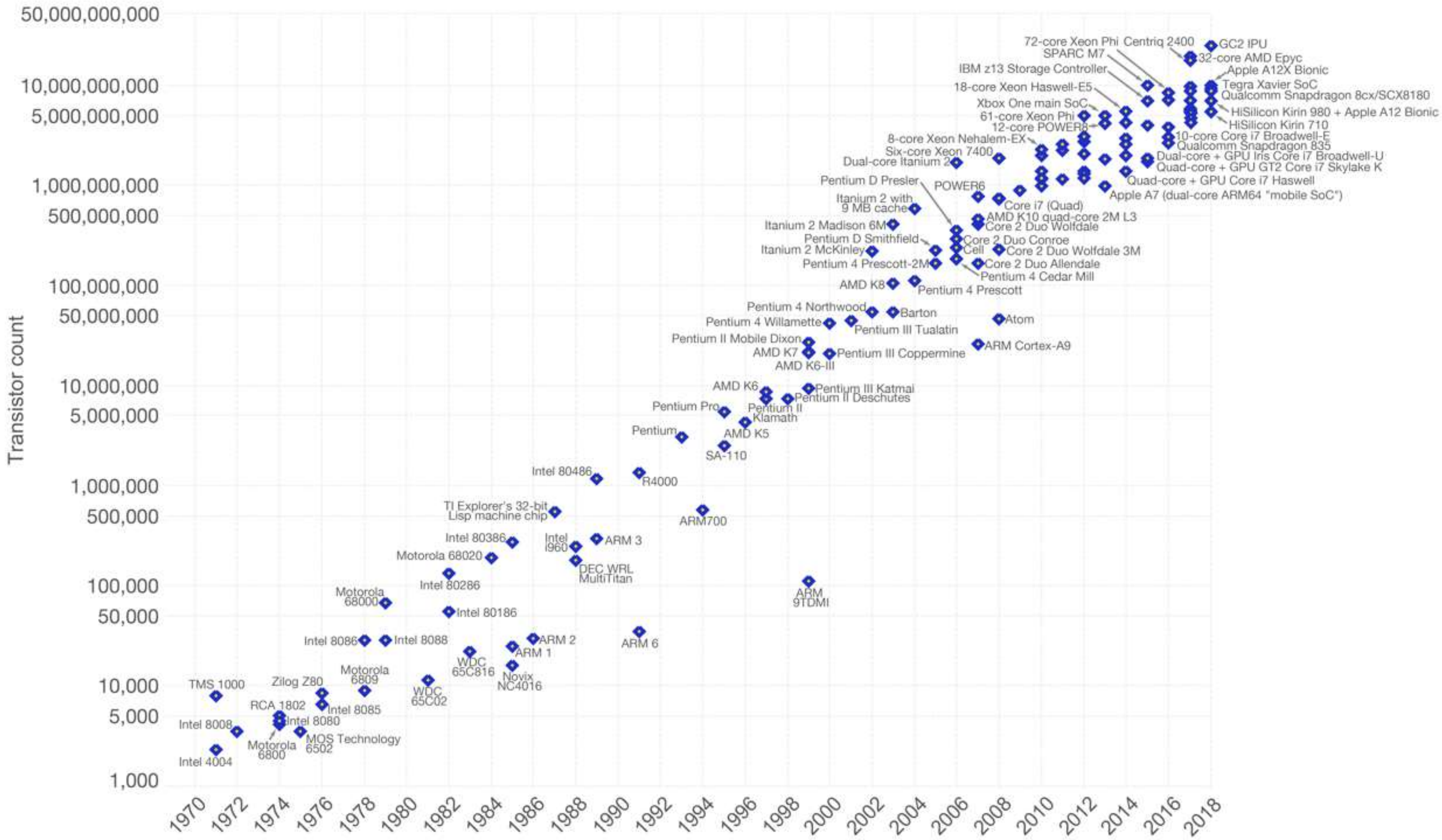


Intel 4004 Chip with 2,250 Transistors Used in First Computer Dr. Brown Used at Bell Labs in 1974

Intel Core I7 Chip 2,100,000,000 Transistors (2.1 Billion) In Dr. Brown's Home Computer – To make/edit videos For our class.

Moore's Law – The number of transistors on integrated circuit chips (1971-2018)

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important as other aspects of technological progress – such as processing speed or the price of electronic products – are linked to Moore's law.



You Already Know About Exponential Growth!

$FV = PV\left(1+\frac{\mathrm{r}}{N}\right)^{(Nt)}$

$FV = PV(1+r)^t$ N = 1

Let $P_0 = Starting\ Value$

$FV = P_0(1+r)^t$ t years (periods)

Amount (t=1) After First Year $= P_0(1+r)$ for growth rate of r per period

This Means Next Period Amount = $(1+r)$ times last Period Amount

Exponential Growth

If a quantity starts at size P_0 and grows by R% (written as a decimal, r) every time period, then the quantity after n time periods can be determined using either of these relations:

Recursive form: $P_n = (1 + r)\, P_{n-1}$

Explicit form: $P_n = P_0(1 + r)^n$

We call *r* the **growth rate**. Can be positive or negative

The term (1 + *r*) is called the **growth multiplier**, or common ratio. Can be > 1 (growth) or < 1 (decay)

Example

Suppose that a lake began with 1,000 fish and 10% of the fish have surviving offspring each year. Complete the following table using the explicit or recursive form of the equation:

Year	Population
Year 0 – starting Population	1,000
Year 1	
Year 2	
Year 3	
Year 4	
Year 5	
Year 10	
Year 20	

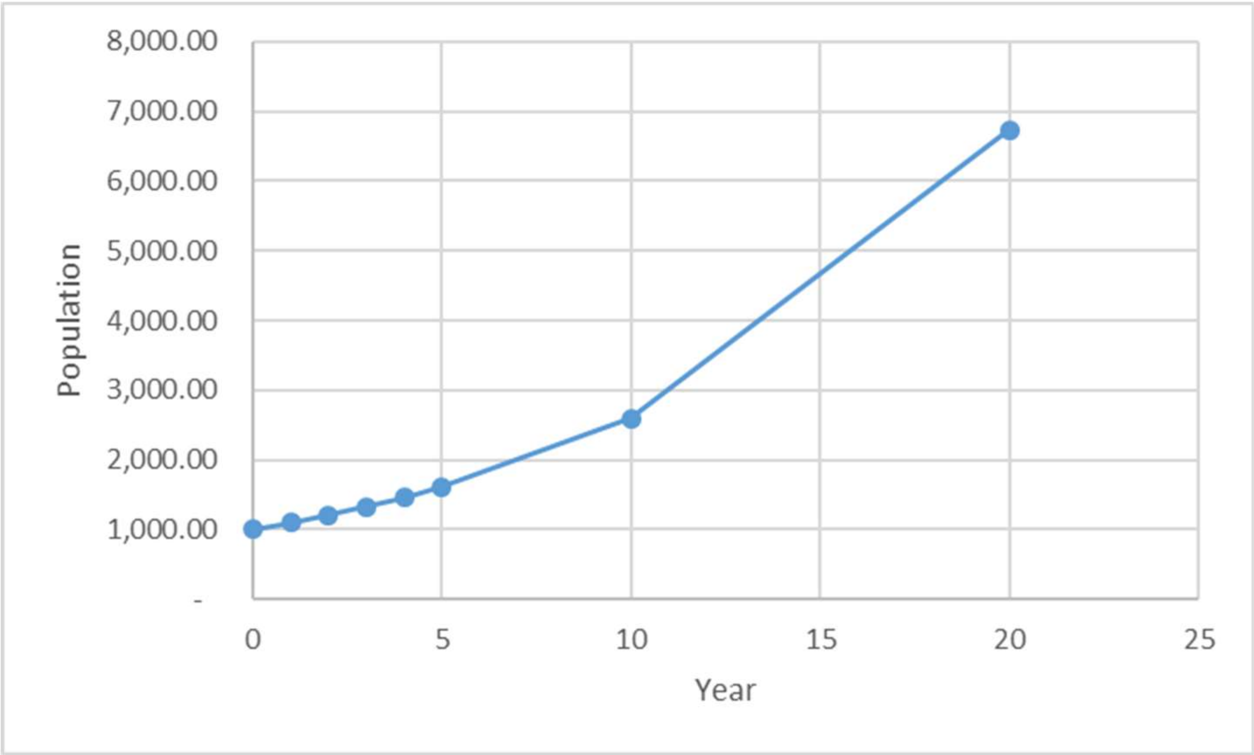
Example

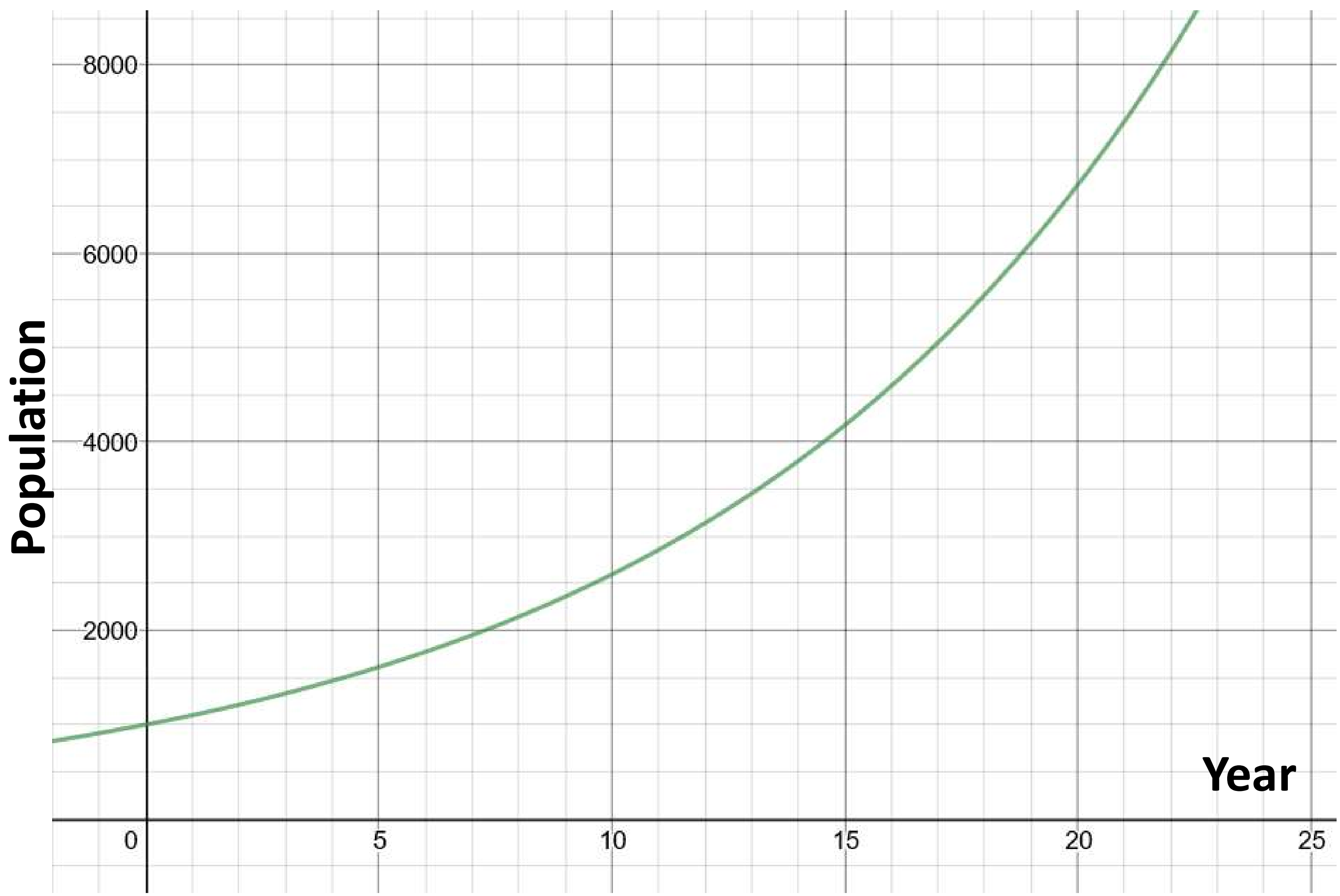
Suppose that a lake began with 1,000 fish and 10% of the fish have surviving offspring each year. Complete the following table using the explicit or recursive form of the equation:

Year	Population
Year 0 – starting Population	1,000
Year 1	1,100.00
Year 2	1,210.00
Year 3	1,331.00
Year 4	1,464.10
Year 5	1,610.51
Year 10	2,593.74
Year 20	6,727.50

Example (cont.)

Use your data from the table to sketch your data:





Population
Year
8000
6000
4000
2000
0
5
10
15
20
25

Example

A friend is using the equation P_n = $4,600(1.072)^n to predict the annual tuition at a local college. She says the formula is based on years after 2010. What does this equation tell us?

The equation tells us the local college's tuition has constantly increased by 0.072 or 7.2% every year since 2010.

Example

Tacoma's population in 2000 was about 200,000, and had been growing by about 9% each year.

a) Write a recursive formula for the population of Tacoma.

growth rate = 9% = 0.09

growth multiplier = 1 +.09 = 1.09

$P_n = 1.09\, P_{n-1}$

b) Write an explicit formula for the population of Tacoma.

$P_n = 200{,}000(1 + r)^n = 200{,}000(1.09)^n$

c) If this trend continued, what was Tacoma's population in 2016?

n = 2016 – 2000 = 16

$P_n = 200{,}000(1.09)^{16} = 794{,}061.18$ ----- $794{,}061$

Example

Suppose that you have a bowl of 500 M&M candies, and each day you eat ¼ of the candies you have. Is the number of candies left changing linearly or exponentially?

a) Write an equation to model the number of candies left after n days.

$P_n = 500(1 - 0.25)^n = 500(.75)^n$

b) How many candies will you have left after one week?

$P_7 = 500(1 - 0.25)^n = 500(.75)^7 = 66.74$

c) How many candies will you have left after ten days?

$P_{10} = 500(.75)^{10} = 28.16$

d) How many candies will you have left after two weeks?

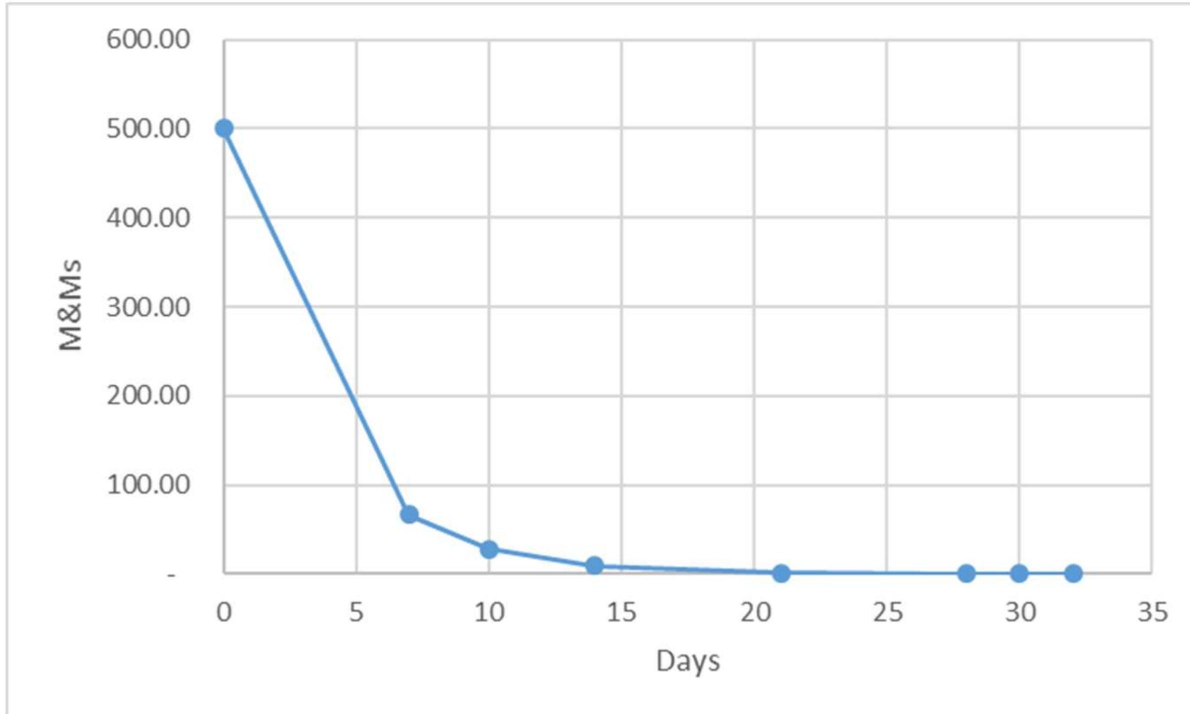
$P_{14} = 500(.75)^{14} = 8.91$

e) How many candies will you have left after three weeks?

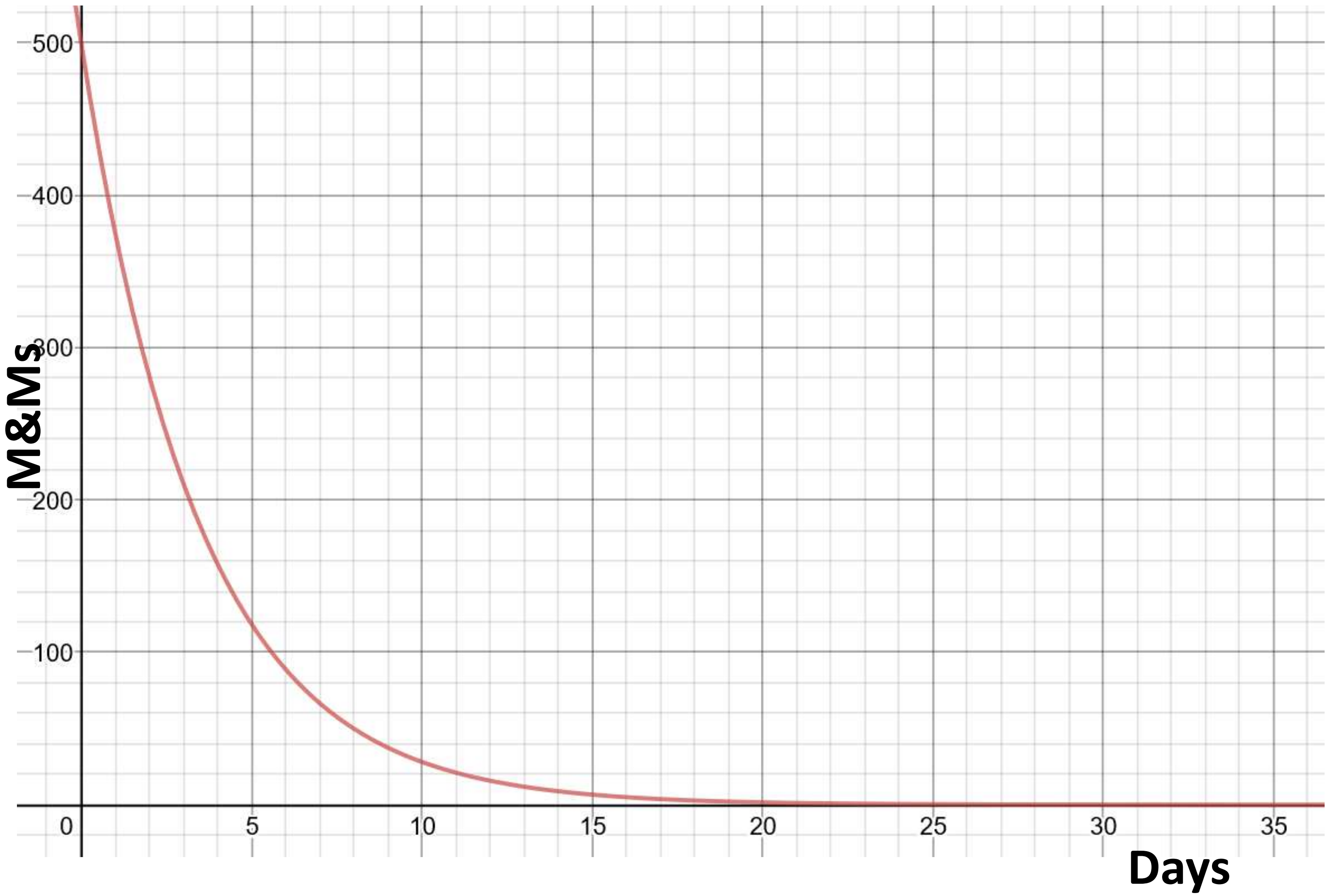
$P_{21} = 500(.75)^{21} = 1.19$

Example (cont.)

e) Draw a sketch of your data.



f) Based on your data, when can you expect all of the M&M candies be eaten?
Approximately 22 days



M&Ms
500
400
300
200
100
0
5
10
15
20
25
30
35
Days

Example

In **<u>1990</u>**, the residential energy use in the US was responsible for 962 million metric tons of carbon dioxide emissions. By the year **<u>2000</u>**, that number had risen to 1182 million metric tons. If the emissions grow exponentially and continue at the same rate, what will the emissions grow to by 2050?

Similar to before, we will correspond n = 0 with 1990, as that is the year for the first piece of data we have.

That will make $P_0 = 962$ (million metric tons of CO2). In this problem, we are not given the growth rate, but instead are given that $P_{10} = 1182$.

Example

When n = 10, the explicit equation looks like:

$P_{10} = (1+r)^{10} P_0$

We know the value for P_0, so we can put that into the equation:

$P_{10} = (1+r)^{10} 962$

We also know that $P_{10} = 1182$, so substituting that in, we get

$1182 = (1+r)^{10} 962$

Example

1182 = $(1+r)^{10}$ 962

We can now solve this equation for the growth rate, r. Start by dividing by 962.

$$\frac{1182}{962} = (1+r)^{10} \qquad\qquad 1 + r = \sqrt[10]{\frac{1182}{962}}$$

$$r = \sqrt[10]{\frac{1182}{962}} \; - 1 \;\; = \;\; .020808 \quad \text{or} \;\; 2.08\%$$

In 2050, $n = 60$

$P_{60} = (1+r)^{60} P_0 = (1.020808)^{60}\ 962 = 3209.93$

3210 to nearest whole number.