Quantitative Skills & Reasoning – Math 1001

Dr. Bob Brown, Jr.
Dean Emeritus
Professor Emeritus
East Georgia State College

Introduction to Modeling Unit
Functions and Linear Modeling
pp 271-278 in textbook

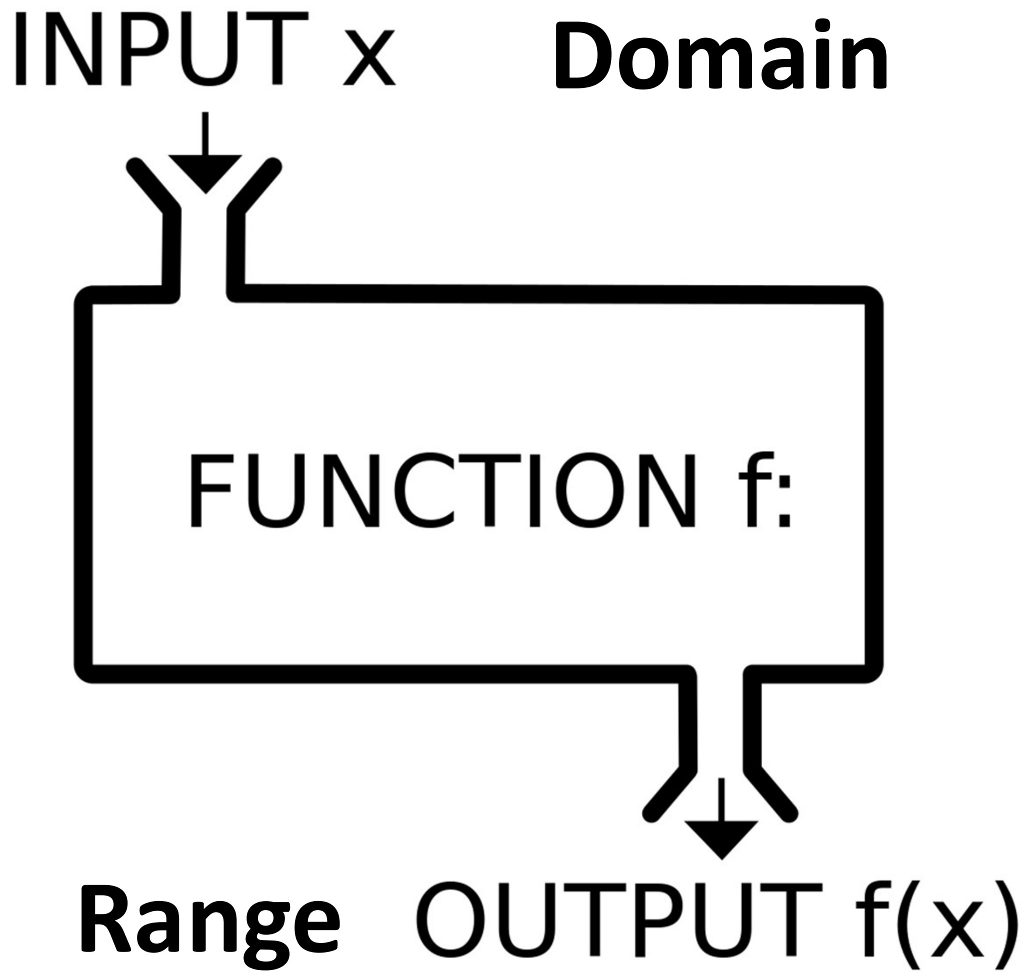
Functions

A **function** is a rule or correspondence that assigns to each element of a set (called the domain) exactly one element of a second set (called the range).

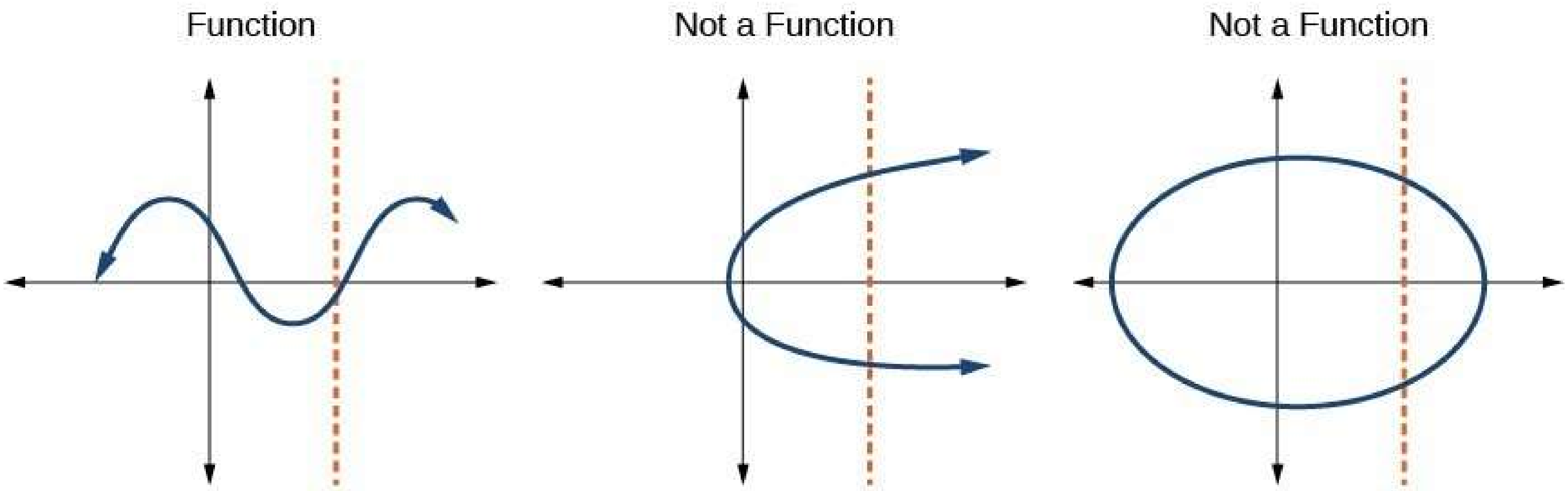
A function may be defined by a set of ordered pairs, a diagram, a table, a graph, an equation, or a verbal description.

The set of all first elements of the function, or the inputs, is called the **domain**. The variable representing elements in the domain is called the **independent variable.**

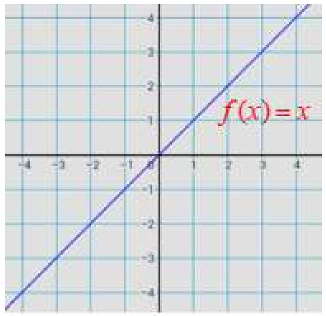
The **range** is the set of all second elements of the function, or the outputs. The **dependent variable** is the variable representing elements in the range.


INPUT x
Domain
FUNCTION f:
Range
OUTPUT f(x)

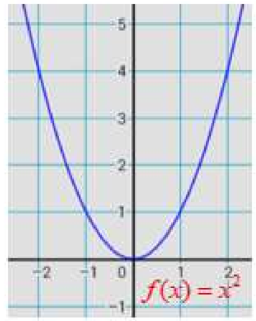
Must Pass Vertical Line Test



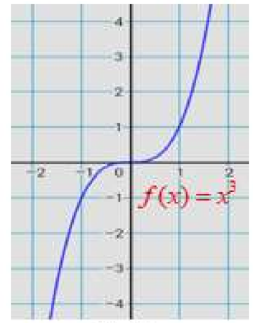
Functions



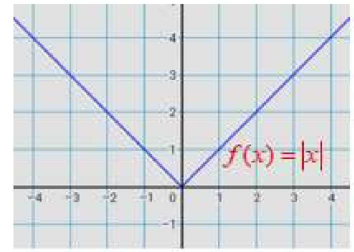
Linear



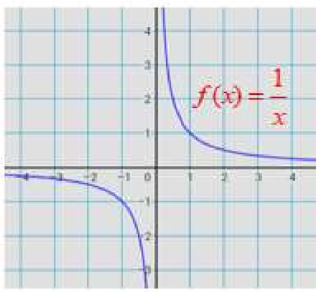
Quadratic



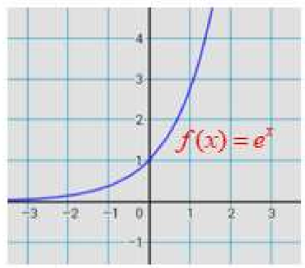
Cubic



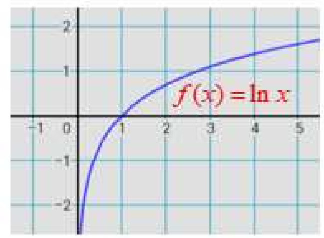
Absolute



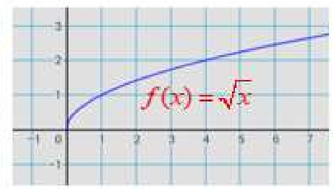
Reciprocal



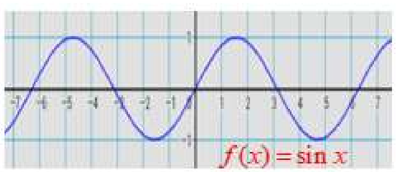
Exponential



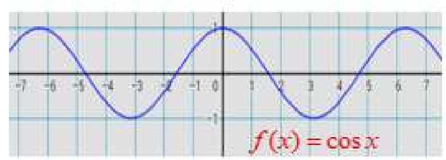
Logarithmic



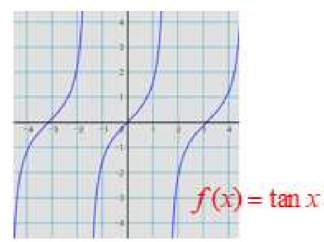
Square Root



Sine



Cosine



Tangent

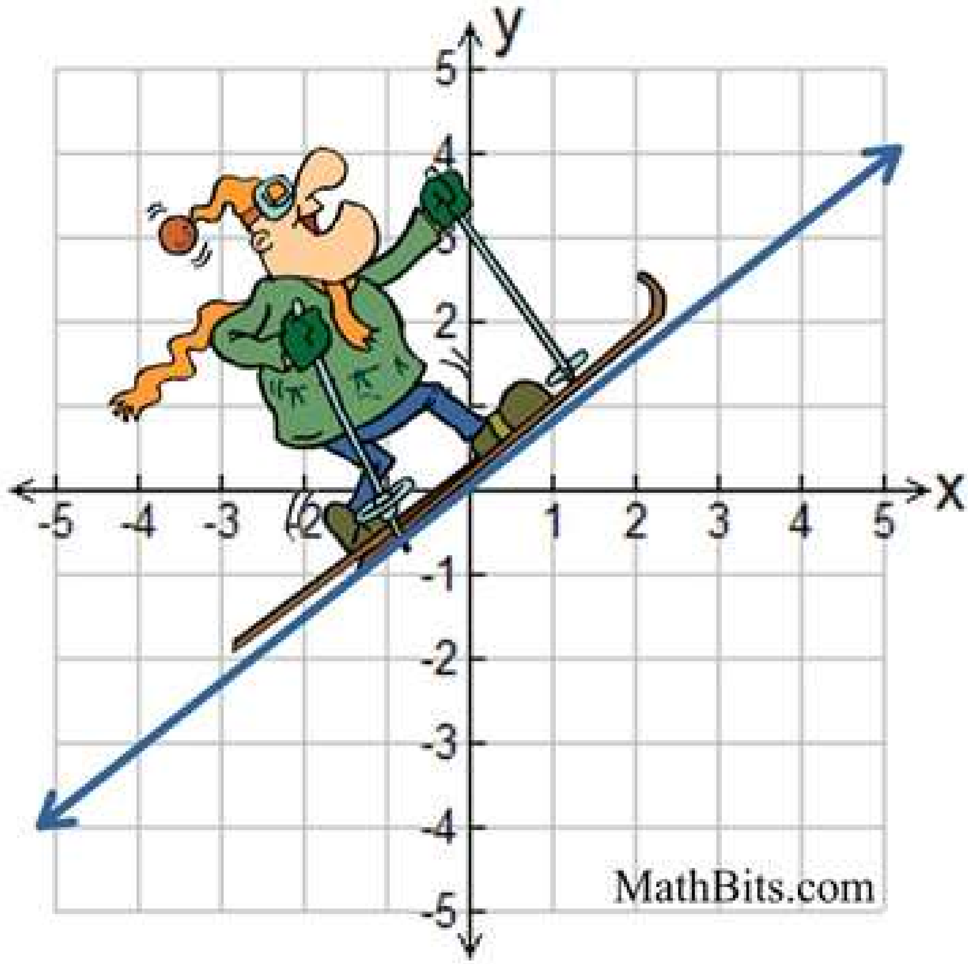
Before We Get Into Linear Function Details

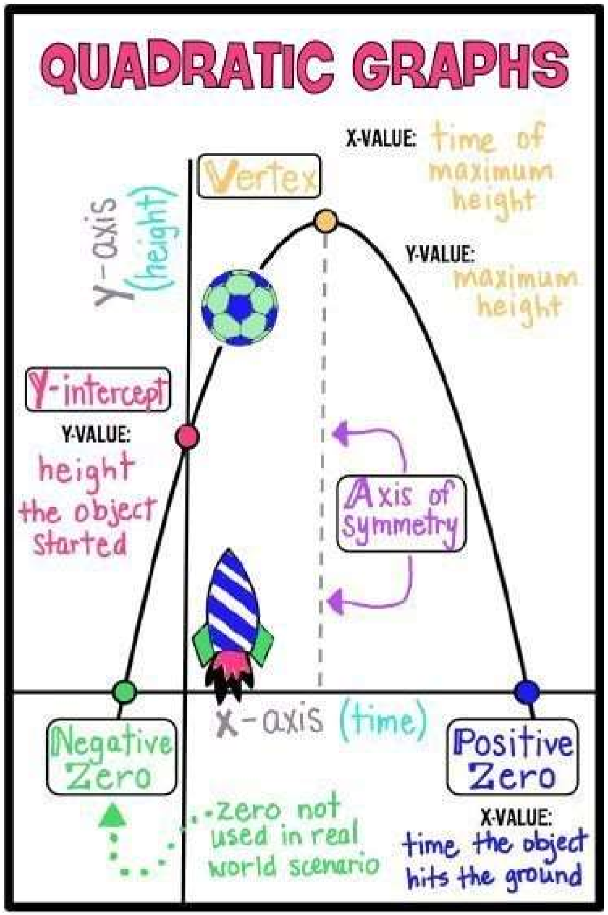
Let Me Tell You About The Four Most Important Functions

In The World!

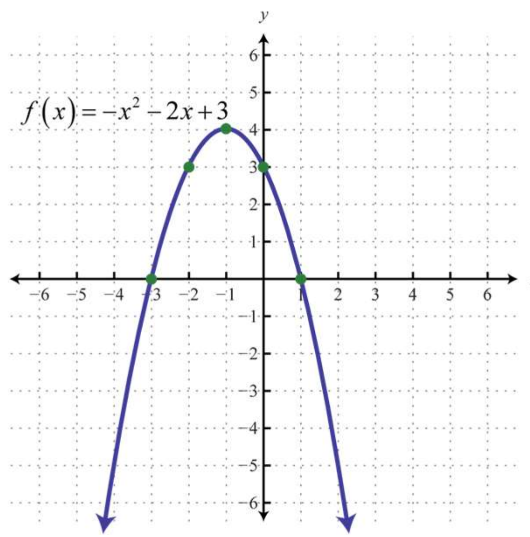
Linear

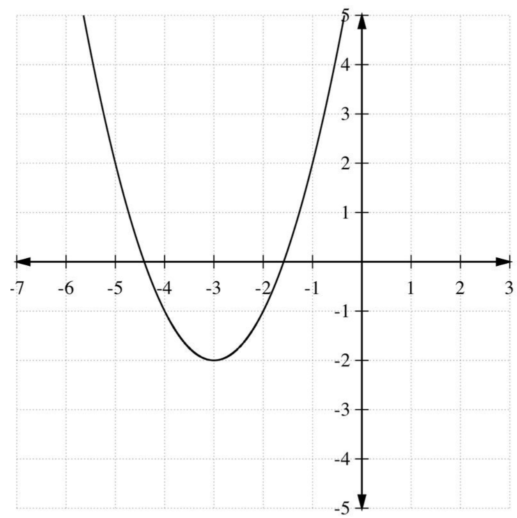
$$y = mx+b$$



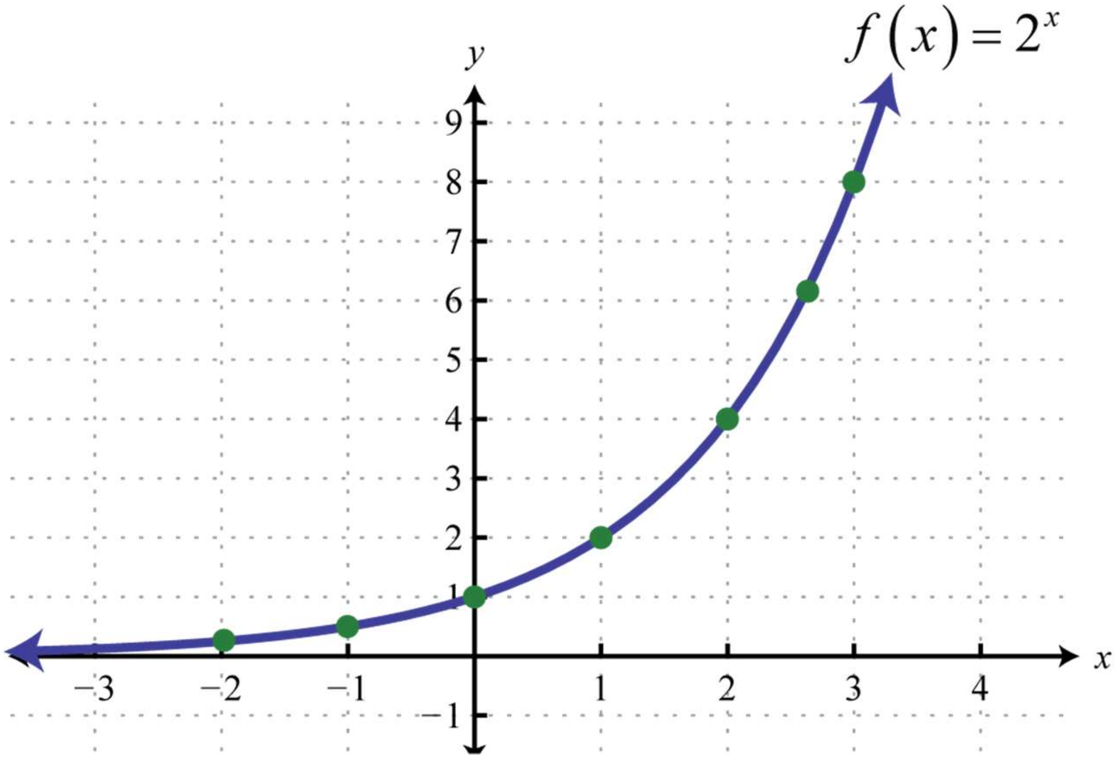

QUADRATIC GRAPHS
X-VALUE: time of maximum height
Vertex
Y-axis (height)
Y-VALUE: maximum height
Y-intercept
Y-VALUE: height the object started
Axis of Symmetry
Negative Zero
X-axis (time)
Positive Zero
zero not used in real world scenario
X-VALUE: time the object hits the ground

$$y = a(x - k)^2 + b$$


$f(x) = -x^2 - 2x + 3$



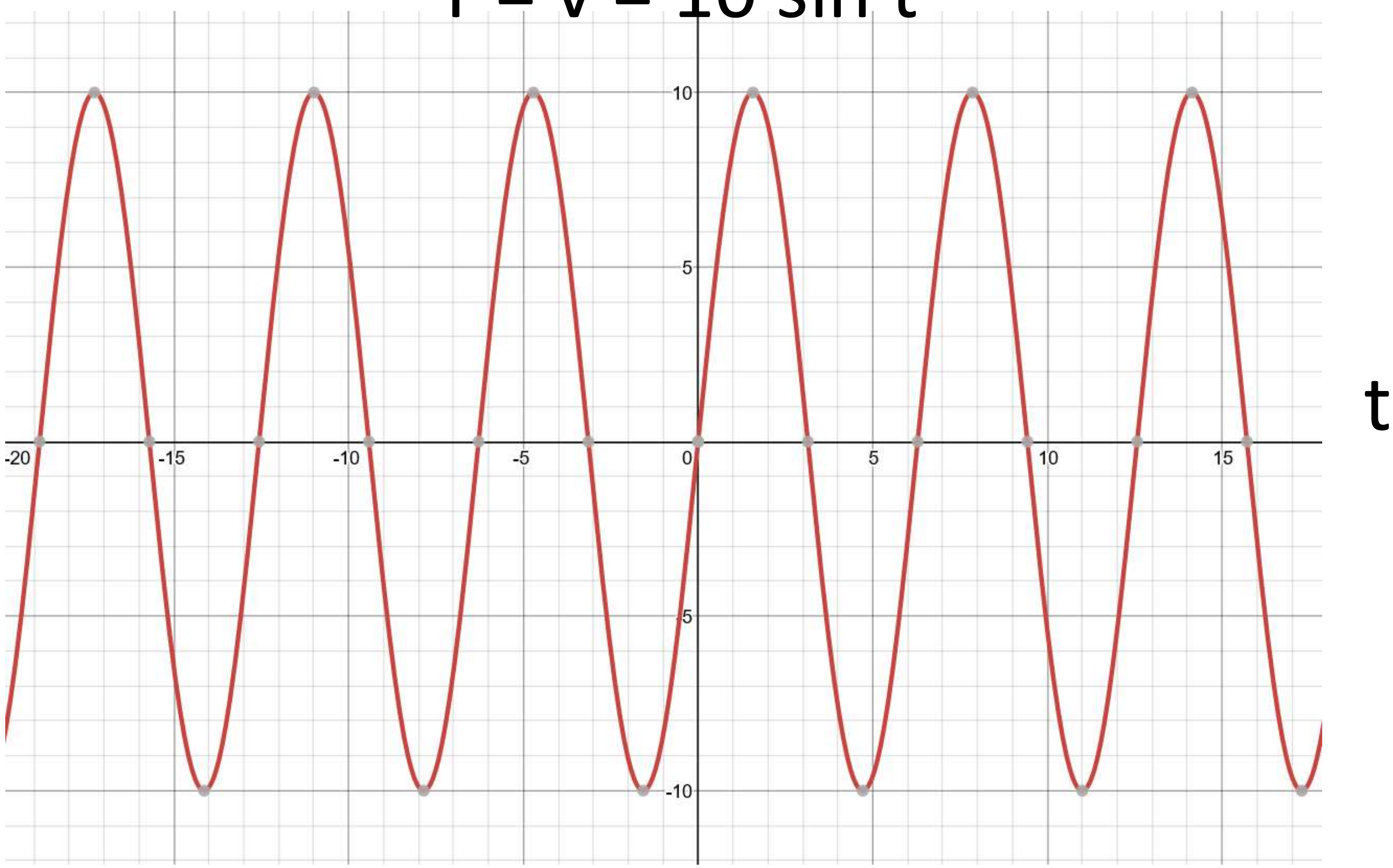
Exponential



$$FV = PV\left(1 + \frac{r}{n}\right)^{(nt)}$$

Sinusoidal

$Y = v = 10 \sin t$

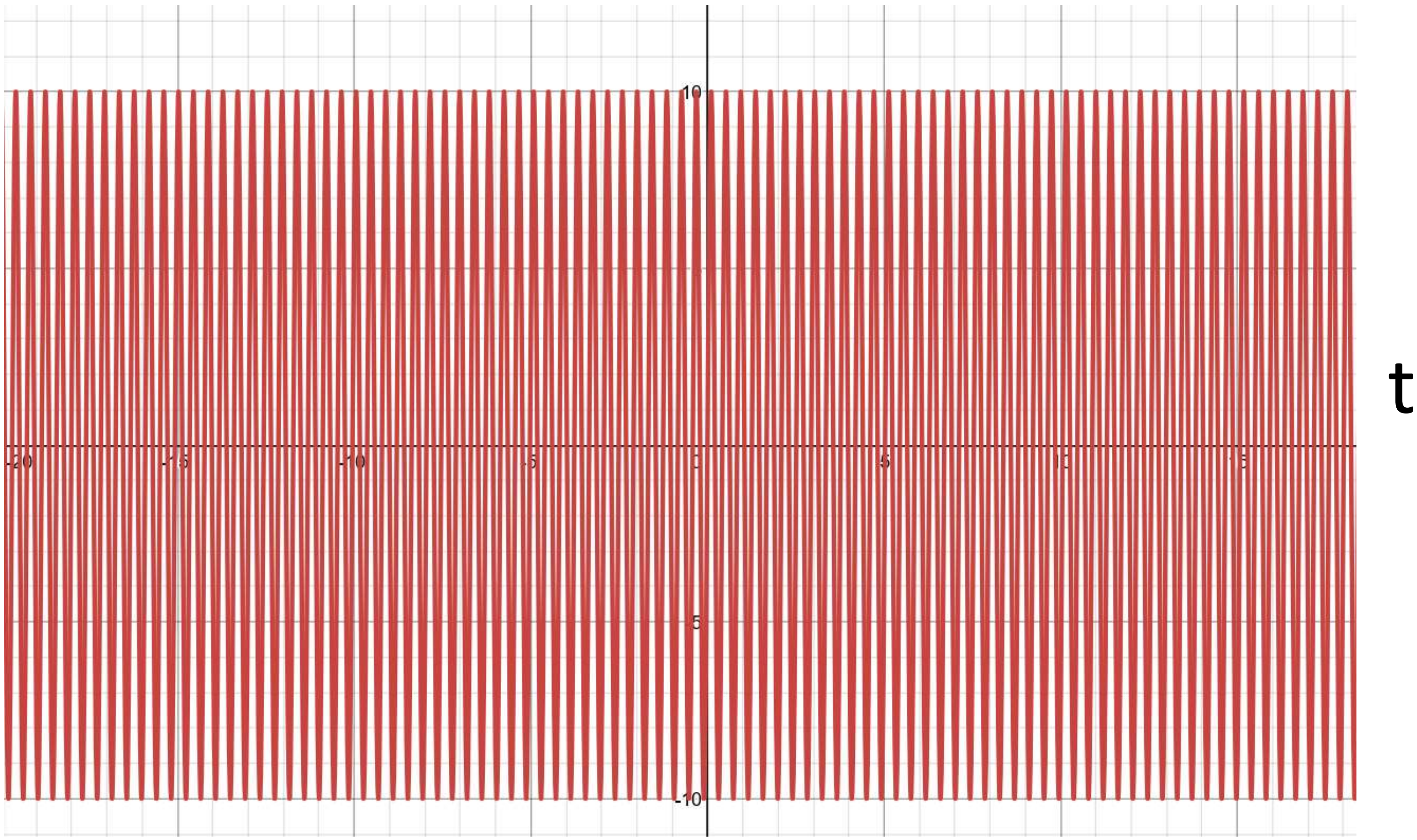


Sinusoidal

$Y = v = 10 \sin 3t$

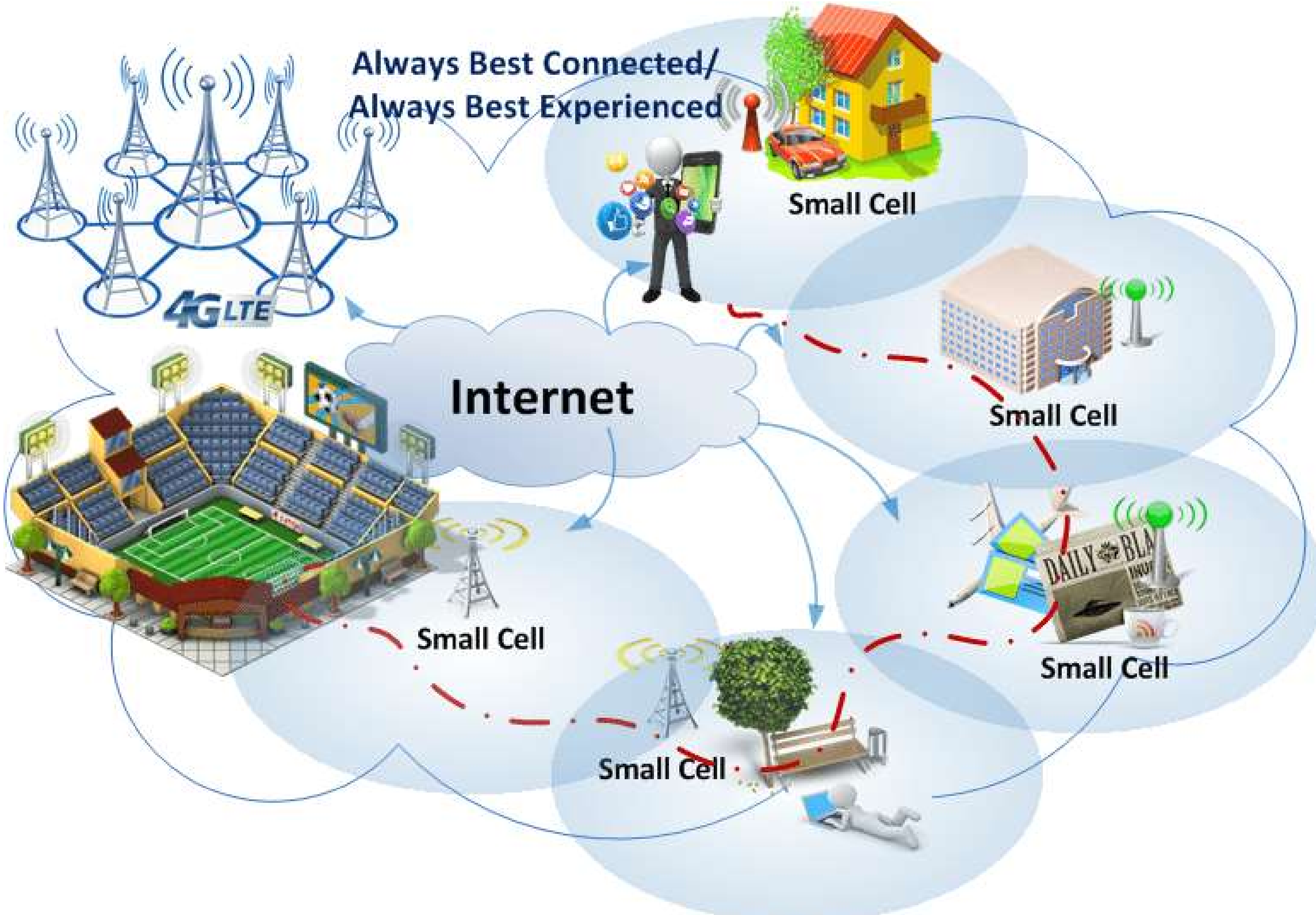
Sinusoidal

$Y = v = 10 \sin 15t$



Your Cellphone Signal Which Carries Your Texts, Phone Calls, Videos, Movies, Facetime Calls
800 Million Cycles Per Second

$$Y = v = 10 \sin 2\pi(800000000)t$$



Always Best Connected/
Always Best Experienced
4G LTE
Internet
Small Cell
Small Cell
Small Cell
Small Cell
Small Cell

Linear Functions

A function whose graph is a line is a **linear function**. A linear function is a function that can be written in the form

$$y = mx + b \qquad\qquad f(x) = mx + b$$

where *m* and *b* are constants.

f(*x*) is the notation we use to symbolize a function.

Recall that *x* and *y* can be replaced by any other variables.

Intercepts

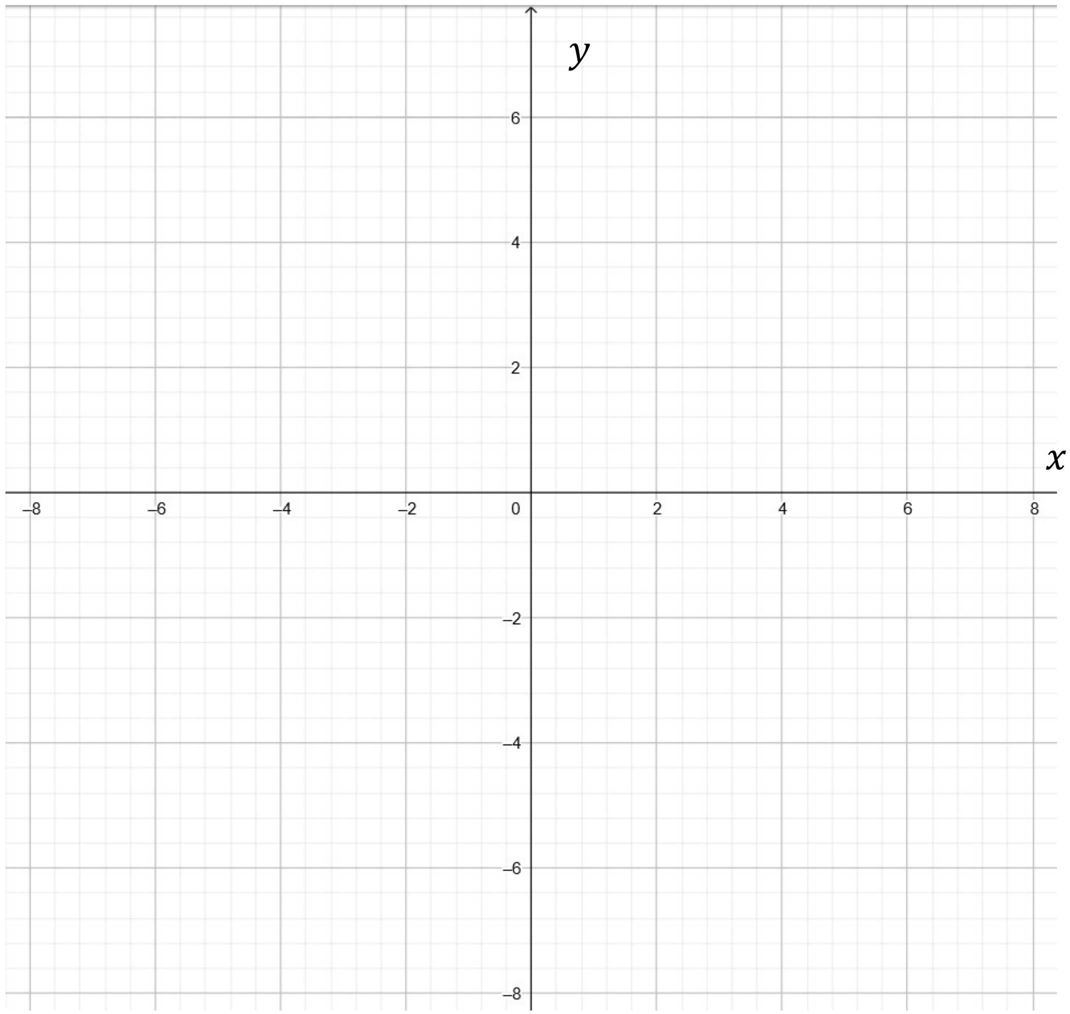
The points where a graph crosses or touches the *x*-axis (where $y = 0$) or *y*-axis (where $x = 0$) are called the ***x*-intercepts** and ***y*-intercepts,** respectively, of the graph.

The *x*-intercept of the graph of a function can be found by substituting 0 for y and solving for x.

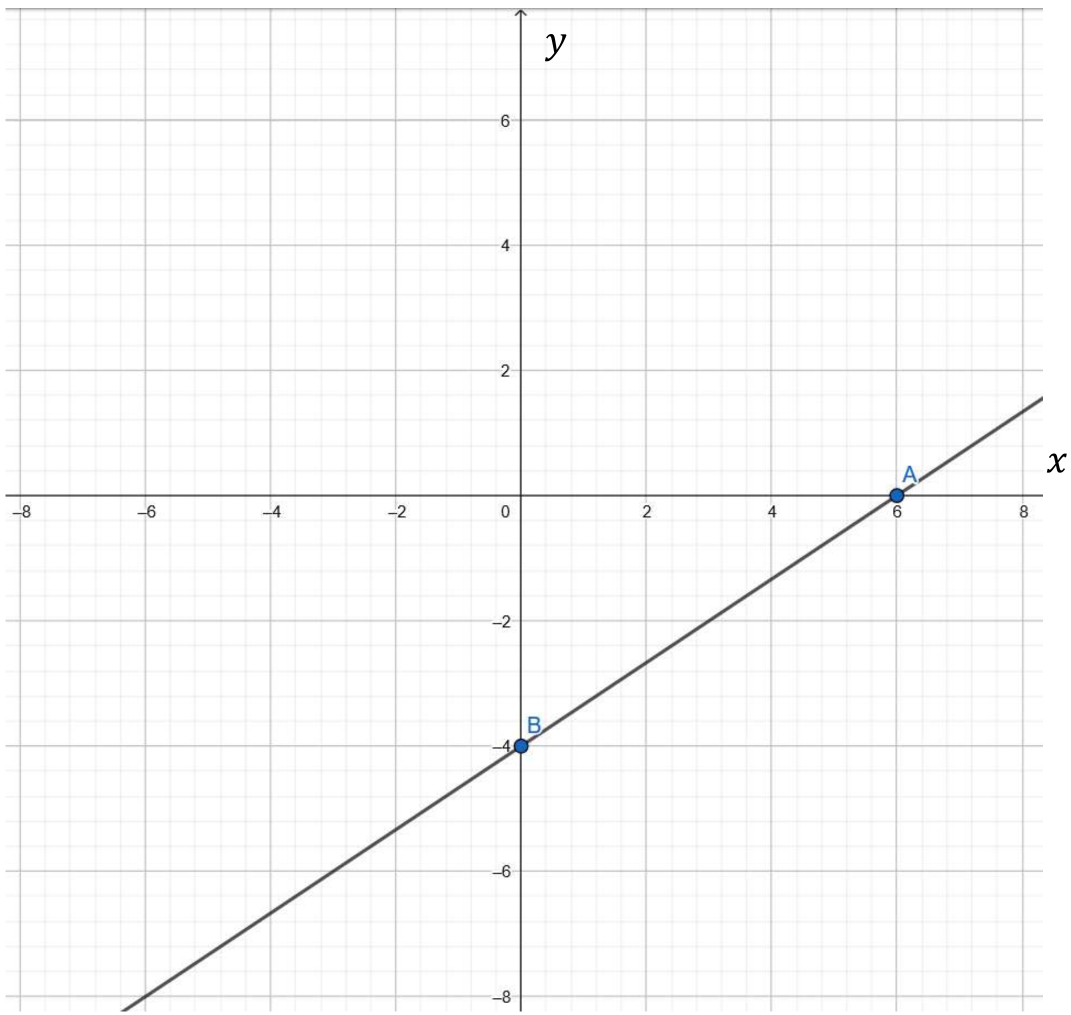
The *y*-intercept of the graph of a function can be found by substituting 0 for x and solving for y.

Example 1

Find the intercepts of the graph of $2x - 3y = 12$
and graph the function.



y
x
6
4
2
0
−2
−4
−6
−8
2
4
6
8


y
x
A
B
6
4
2
0
–2
–4
–6
–8
–8
–6
–4
–2
2
4
6
8

Slope of a Line

The **slope** of a line is a number that measures the *steepness* of the line. One definition of slope is

$m =$ Slope = $\frac{vertical\ change}{horizontal\ change} = \frac{rise}{run}$

If a nonvertical line passes through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$,
Its slope is found using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 2

Find the slope of the line passing through the points $(-3, 2)$ and $(5, -4)$. What does the slope mean?

Slope and y-Intercept

The slope of the graph of the equation $y = mx + b$ is *m*, and the *y*-intercept of the graph is *b*.

Example 4

Determine the slope and *y*-intercept of the line.

a. $y = 7x - 12$

b. $2x - 3y = 12$

Example 3

A business property is purchased with a promise to pay off a \$60,000 loan plus the \$16,500 interest on this loan by making 60 monthly payments of \$1275. The amount of money, *y*, remaining to be paid on \$76,500 (the loan plus interest) is reduced by \$1275 each month.
Although the amount of money remaining to be paid changes every month, it can be modeled by the linear equation

$$y = 76{,}500 - 1275x$$

a. Find the x-intercept and the y-intercept of the graph of this linear equation.

b. Interpret the intercepts in the context of this problem situation.

c. How should x and y be limited in this model so that they make sense in the application?

d. Use the intercepts and the results of part (c) to sketch the graph of the given equation.

e. Find the slope of the line using the intercepts.

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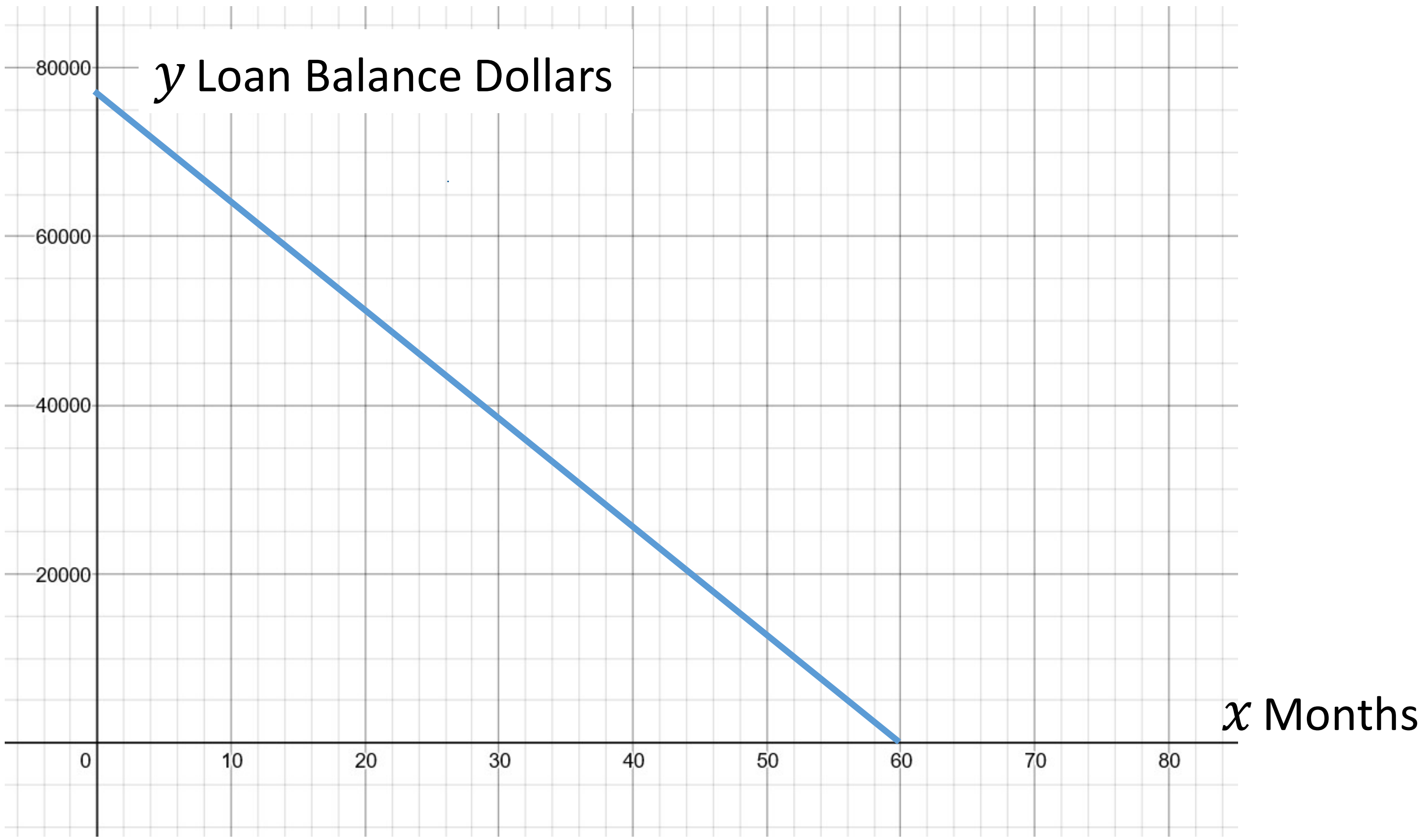
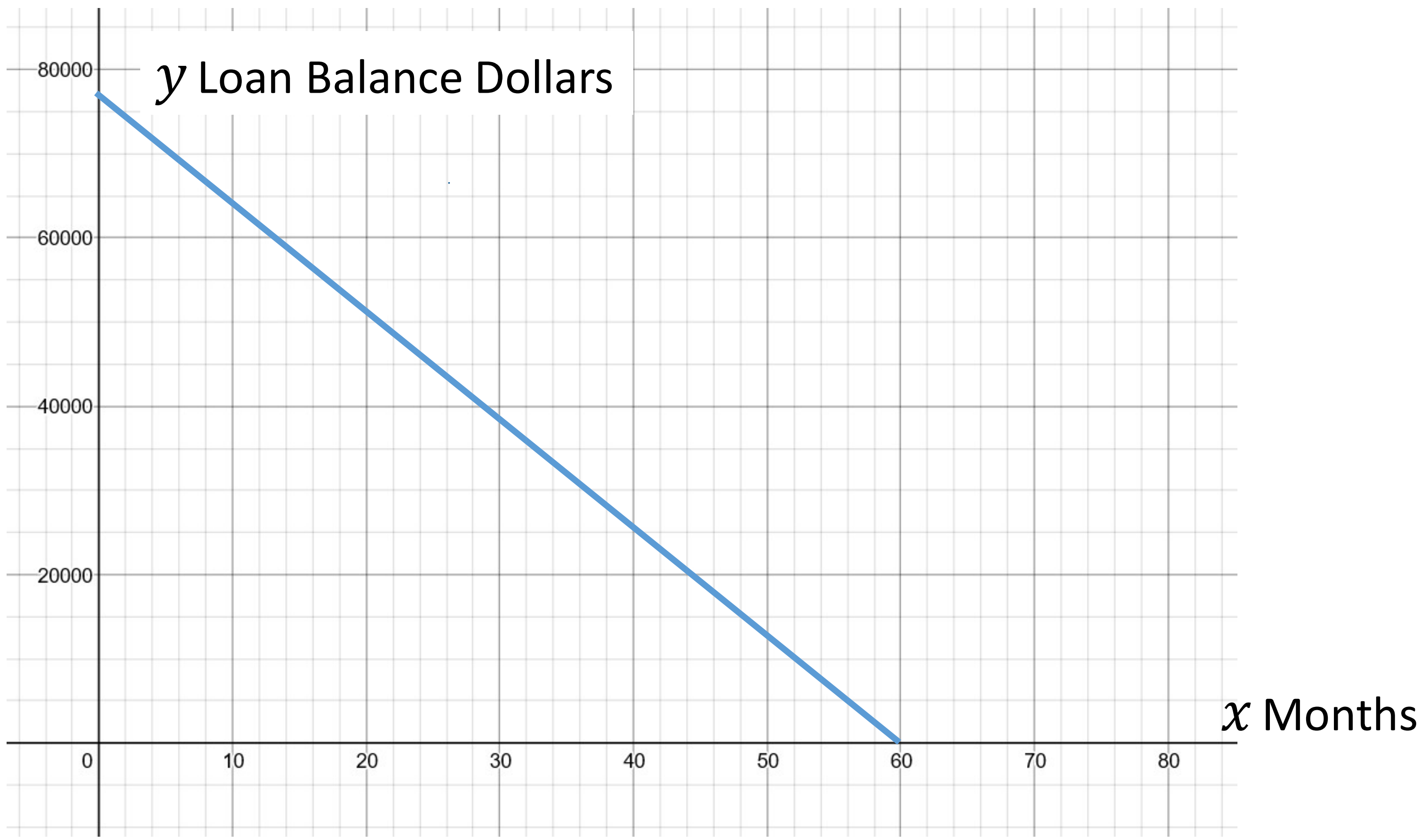


e. Find the slope of the line using the intercepts.



Slope-Intercept Form; Constant Rate of Change

The equation of a line with slope m and y-intercept b is $y = mx + b$.

The rate of change of the function $y = mx + b$ is m, the slope of the graph of the function.

Rate of change describes by how much the output changes (increases or decreases) for every input unit.

In an applied context, m is the rate of change and b is initial value (when $x = 0$).

Example 4

Using data and projections from 1990 through 2050, the percent of Hispanics in the U.S. population can be modeled by

$H(x) = 0.224x + 9.01$

(Source: US Census Bureau)

a. What is the slope of the graph of this function?

b. What does this slope tell us about the annual rate of change in the percent of Hispanics in the U.S.?

Example 5

An appliance repairman charges \$60 for a service call plus \$25 per hour for each hour spent on the repair. Assuming his service call charges can be modeled by a linear function of the number of hours spent on the repair, write the equation of the function.

Point-Slope Form of the Equation of a Line

The equation of the line with slope *m* and passing through a known point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

Example 6

Write the equation of a line through the point (3, -7) having slope -2.

Example 7

The table gives the number of drinks and the resulting blood alcohol percent for a 180-lb man.

("One drink" is equal to 1.25 oz of 80 proof liquor, 12 oz of a regular beer, or 5 oz of table wine, and many states have set 0.08% as the legal limit for driving under the influence.)

Number of Drinks (x)	3	4	5	6	7	8	9	10
Blood Alcohol Percent (y)	0.07	0.09	0.11	0.13	0.15	0.17	0.19	0.21

Write an equation for y (blood alcohol content) versus x (number of drinks)

Write an equation for y (blood alcohol content) versus x (number of drinks)

Number of Drinks (x)	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Blood Alcohol Percent (y)	0.07	0.09	0.11	0.13	0.15	0.17	0.19	0.21

Example 8

The number of people (in millions) in prisons or jails in the U.S. grew at a constant rate from 2001 to 2013, with 1.345 million people incarcerated in 2001 and 1.570 million incarcerated in 2013.

(Source: Bureau of Justice Statistics)

a. What is the rate of growth of people incarcerated from 2001 to 2013?

b. Write the linear equation that models the number N of prisoners as a function of the year x.

c. Use the function to predict the inmate population in 2017.

Example 8

The number of people (in millions) in prisons or jails in the U.S. grew at a constant rate from 2001 to 2013, with 1.345 million people incarcerated in 2001 and 1.570 million incarcerated in 2013.

(Source: Bureau of Justice Statistics)

a. What is the rate of growth of people incarcerated from 2001 to 2013?

x	2001	(x_1)	2013 (x_2)
y	1.345	(y_1)	1.570 (y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 8

The number of people (in millions) in prisons or jails in the U.S. grew at a constant rate from 2001 to 2013, with 1.345 million people incarcerated in 2001 and 1.570 million incarcerated in 2013.

(Source: Bureau of Justice Statistics)

b. Write the linear equation that models the number *N* of prisoners as a function of the year *x*.

$$y - y_1 = m(x - x_1)$$

x	2001	(x_1)	2013	(x_2)
y	1.345	(y_1)	1.570	(y_2)

Example 8

The number of people (in millions) in prisons or jails in the U.S. grew at a constant rate from 2001 to 2013, with 1.345 million people incarcerated in 2001 and 1.570 million incarcerated in 2013.

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