Quantitative Skills & Reasoning – Math 1001

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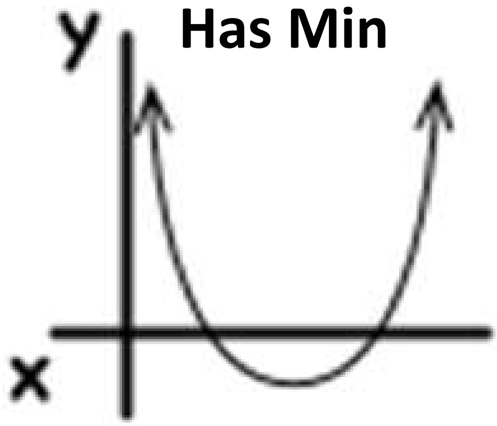
Introduction to Modeling Unit

Quadratic Models

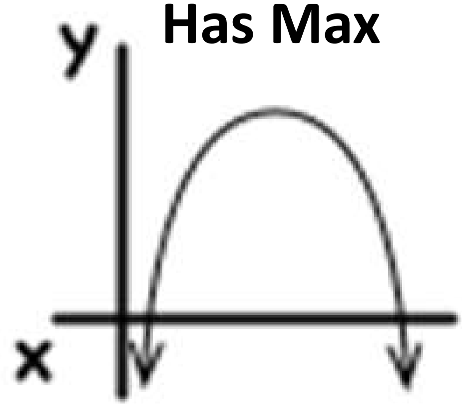
Quadratic Functions

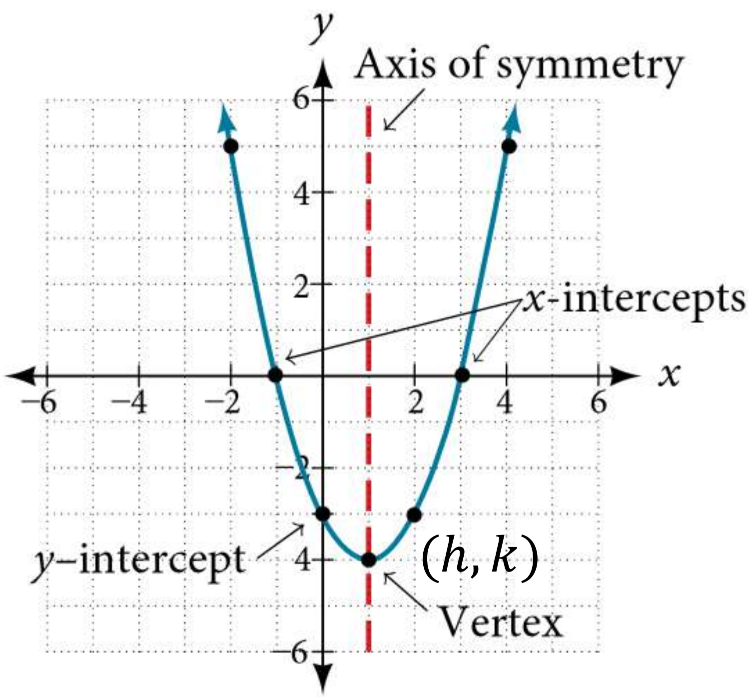
Definition: A quadratic function has the form y = f(x)=ax^2+bx+c, where a, b, and c are real numbers and $a \neq 0$. The domain of quadratic functions is the set of all real numbers. The graph of a quadratic function is called a parabola.

a > 0
Opens Up
Has Min



a < 0
Opens Down
Has Max





The axis of symmetry is defined by

$$x = -\frac{b}{2a}$$

The x intercepts (where y = 0) can be calculated by
The famous Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

An alternate form of the quadratic function is y $= f(x) = a(x-h)^2 + k$

$$where\ h = -\frac{b}{2a}\ and\ \ k\ is\ the\ \min\ or\ \max\ value\ of\ the\ function.$$

$$k = f(h) = f(-\frac{b}{2a})$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Note\ the\ discriminant\ is\ \ b^2 - 4ac$$

If $b^2 - 4ac > 0$, there are two real roots (zeros)

If $b^2 - 4ac < 0$, there are two complex conjugate roots (zeros)

If $b^2 - 4ac = 0$, there are two equal real roots (zeros)

Given $f(x) = x^2 + 4x + 3$

Identify the coefficients:

a = b = c =

Use the discriminant, $b^2 - 4ac$, to determine the type and number of solutions:

$$b^2 - 4ac =$$

Does the function open up or down?

Does the function have a minimum or maximum value?

Given $f(x) = x^2 + 4x + 3$

Find the axis of symmetry.

$f(x) = x^2 + 4x + 3$

The axis of symmetry is defined by

$$x = -\frac{b}{2a}$$

Find the vertex.

The vertex is (h, k) $\quad where\ h = -\frac{b}{2a}\ and$

$$k\ is\ the \min or \max value\ of\ the\ function.$$

$$k = f(h) = f(-\frac{b}{2a})$$

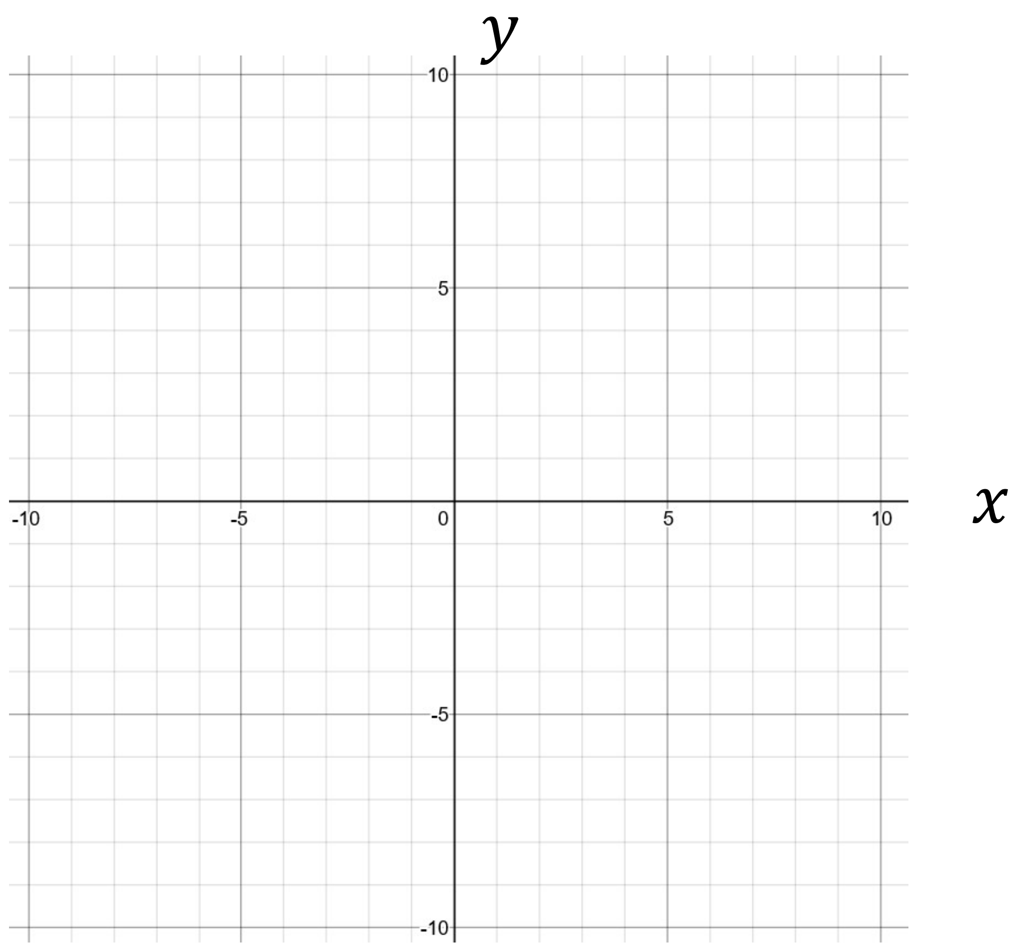
Find the intercepts. $f(x) = x^2 + 4x + 3$

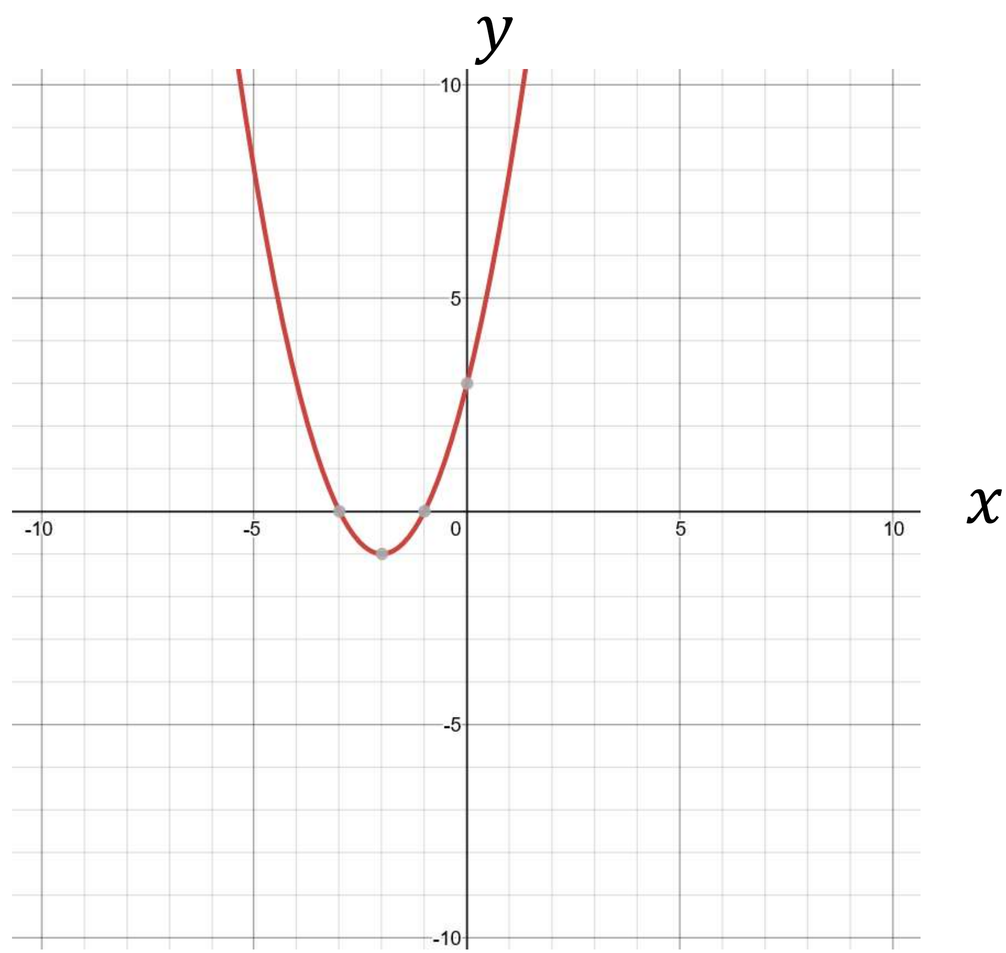
y-intercept

x-intercepts

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graph the function using its vertex, A.0.S., and intercepts.





y
x
10
5
0
-5
-10
-10
-5
5
10

Application Problems

A ball is thrown in the air and the equation $H(t) = -16t^2 + 60t + 6$ is used to model its height, in feet, as a function of time. Calculate the maximum height the ball achieves by hand and showing all work. Also, find the time when the ball will hit the ground.

Use the discriminant, $b^2 - 4ac$, to determine the type and number of solutions:

Note the $-16t^2$ in the equation is due to the force of gravity causing the ball to return to earth

$H(t) = -16t^2 + 60t + 6$

Does the function open up or down?

Does the function have a minimum or maximum value?

$H(t) = -16t^2 + 60t + 6$

Find the axis of symmetry (time the ball reaches its maximum height).

Find the vertex.

H(t) = −16t² + 60t + 6

The vertex is (h, k) $\quad where\ h = -\dfrac{b}{2a}\ and$

$$k\ is\ the\ \min\ or\ \max\ value\ of\ the\ function.$$

$$k = f(h) = f(-\frac{b}{2a})$$

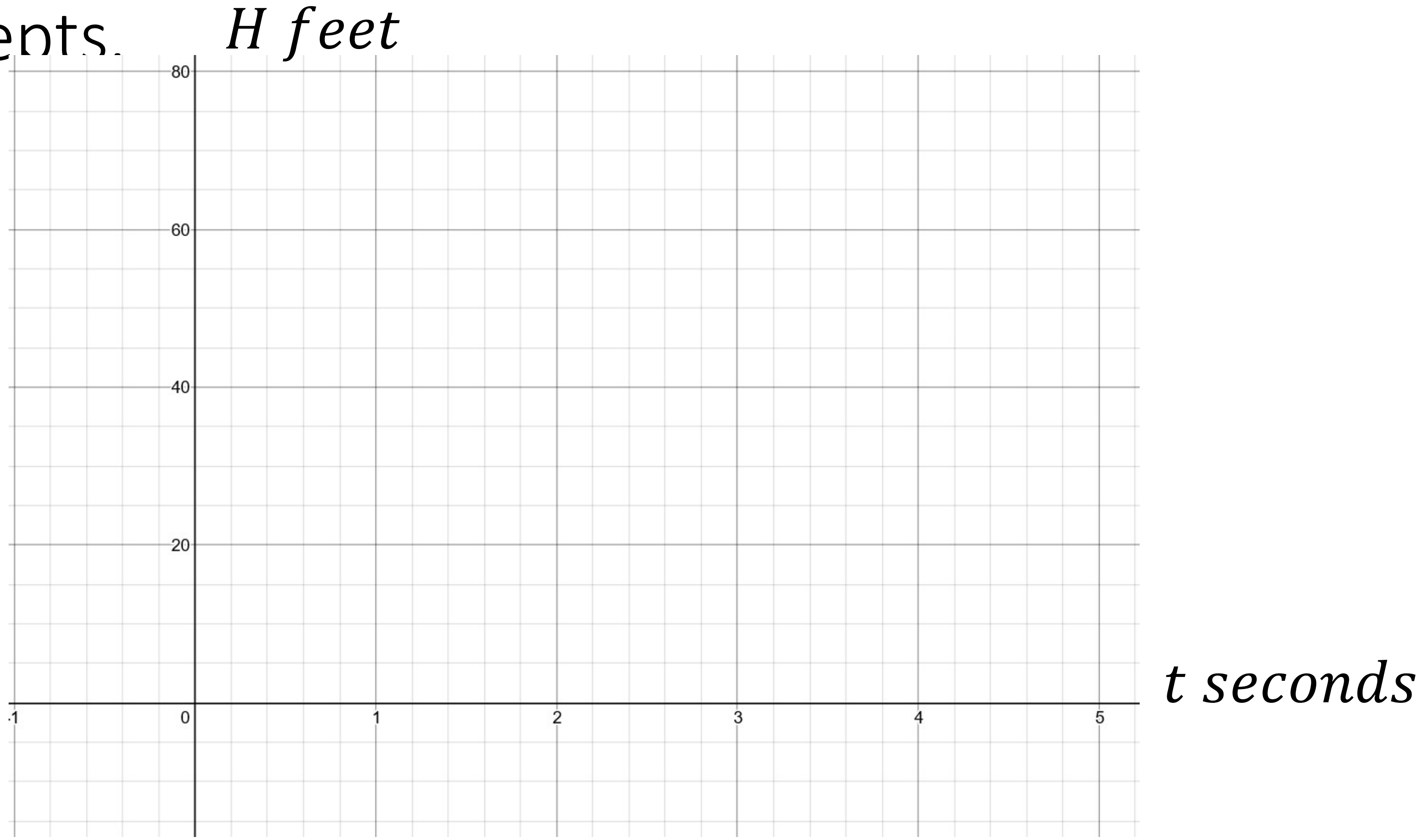
Find the intercepts.

y-intercept (height where the ball is thrown from).

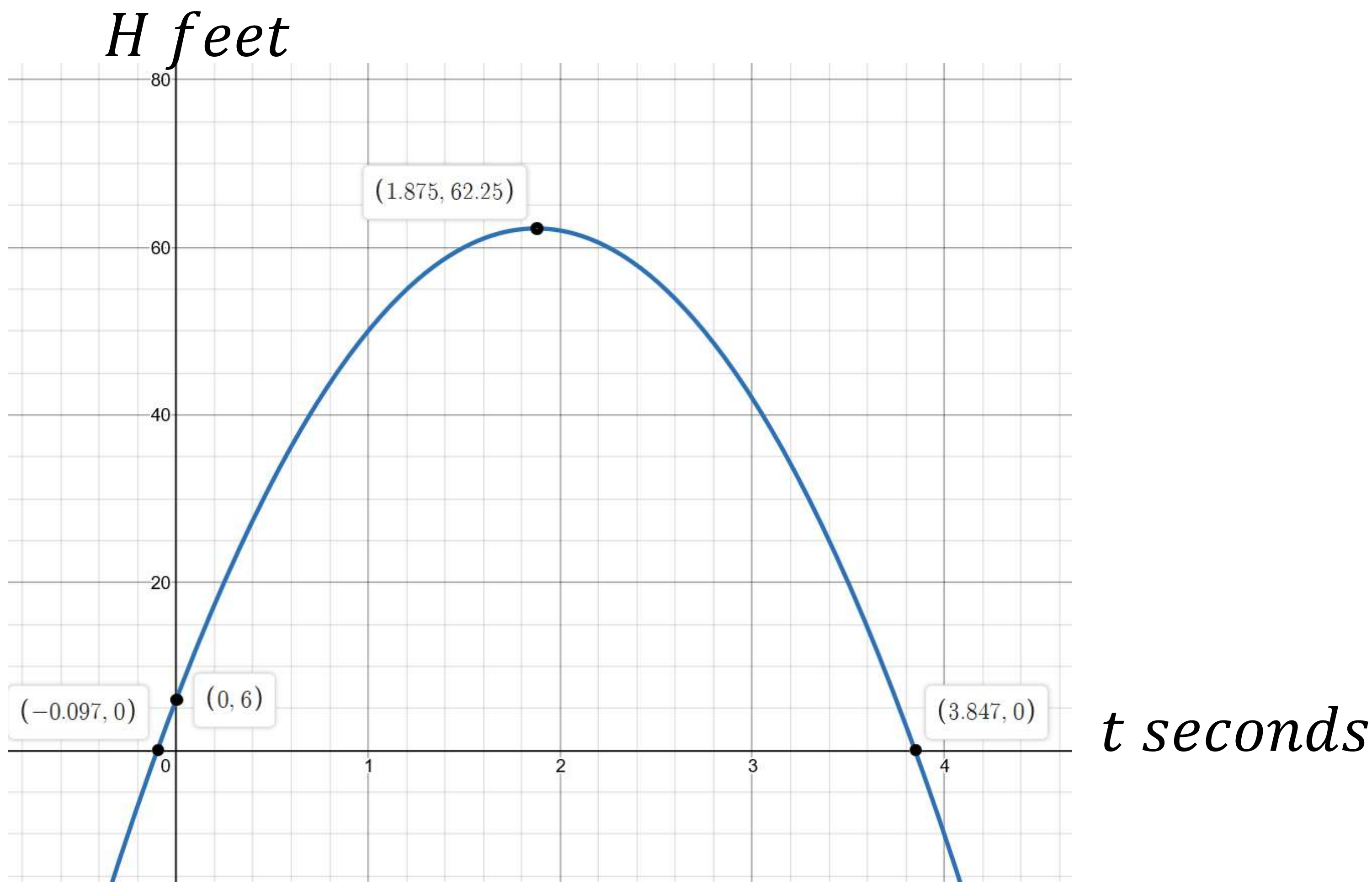
x-intercepts (when the ball reaches the ground).

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graph the function using its vertex, A.0.S., and intercepts.



Graph the function using its vertex, A.0.S., and intercepts.



Application Problems

A bottle rocket that takes off at a velocity of 200ft/sec from a height of 2 feet. The constant for gravity is −16 feet/second squared. Create a quadratic equation to model the height as a function of time.

$\mathbf{H(t) = -16t^2 + 200t + 2}$

Use the discriminant, $b^2 - 4ac$, to determine the type and number of solutions:

Does the function open up or down?

$H(t) = -16t^2 + 200t + 2$

Does the function have a minimum or maximum value?

$H(t) = -16t^2 + 200t + 2$

Find the axis of symmetry (time the rocket reaches its maximum height). **$H(t) = -16t^2 + 200t + 2$**

$$\mathrm{x} = h = -\frac{b}{2a}$$

Find the vertex.

H(t) = −16t² + 200t + 2

The vertex is (h, k) $\quad where\ h = -\frac{b}{2a}\ and$
$k\ is\ the \min or \max value\ of\ the\ function.$

$$k = f(h) = f(-\frac{b}{2a})$$

Find the intercepts.

$H(t) = -16t^2 + 200t + 2$

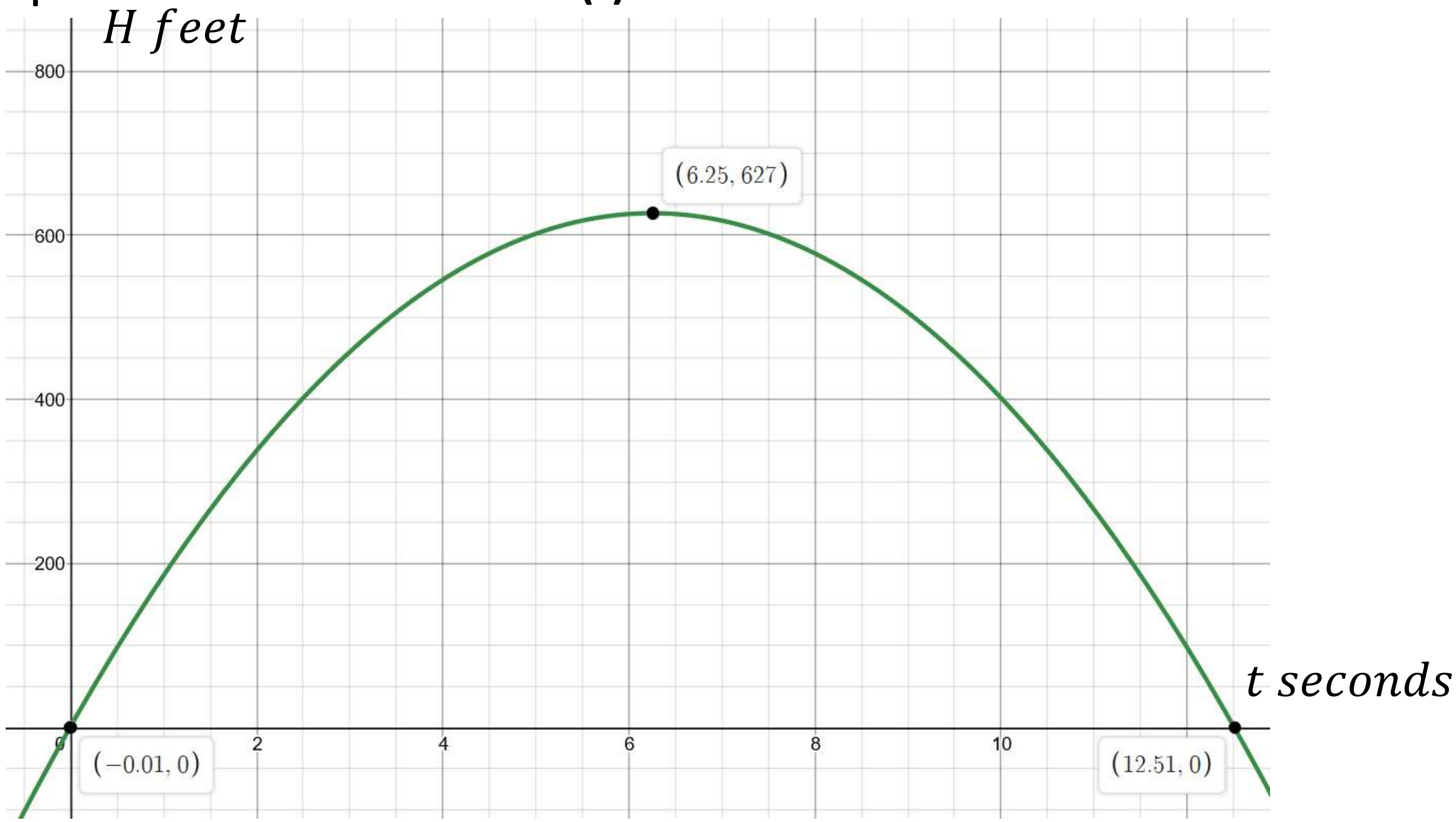
y-intercept (height where the rocket is launched from).

x-intercepts (when the rocket reaches the ground).

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graph the function using its vertex, A.0.S., and intercepts.

H(t) = −16t² + 200t + 2



Application Problems

Suppose the cost of producing *x* crates of pencils is given by

$$\boldsymbol{C(x) = \frac{1}{2}x^2 - 10x + 1000}$$

Use the discriminant, $b^2 - 4ac$, to determine the type and number of solutions:

Does the function open up or down?

$$\boldsymbol{C(x) = \frac{1}{2}x^2 - 10x + 1000}$$

Does the function have a minimum or maximum value?

$$\boldsymbol{C(x) = \frac{1}{2}x^2 - 10x + 1000}$$

Find the axis of symmetry (i.e. number of pencils that produce the minimum cost).

$$\boldsymbol{C(x) = \frac{1}{2}x^2 - 10x + 1000}$$

Find the vertex.

$$\boldsymbol{C(x) = \frac{1}{2}x^2 - 10x + 1000}$$

$$\mathrm{x} = h = -\frac{b}{2a}$$

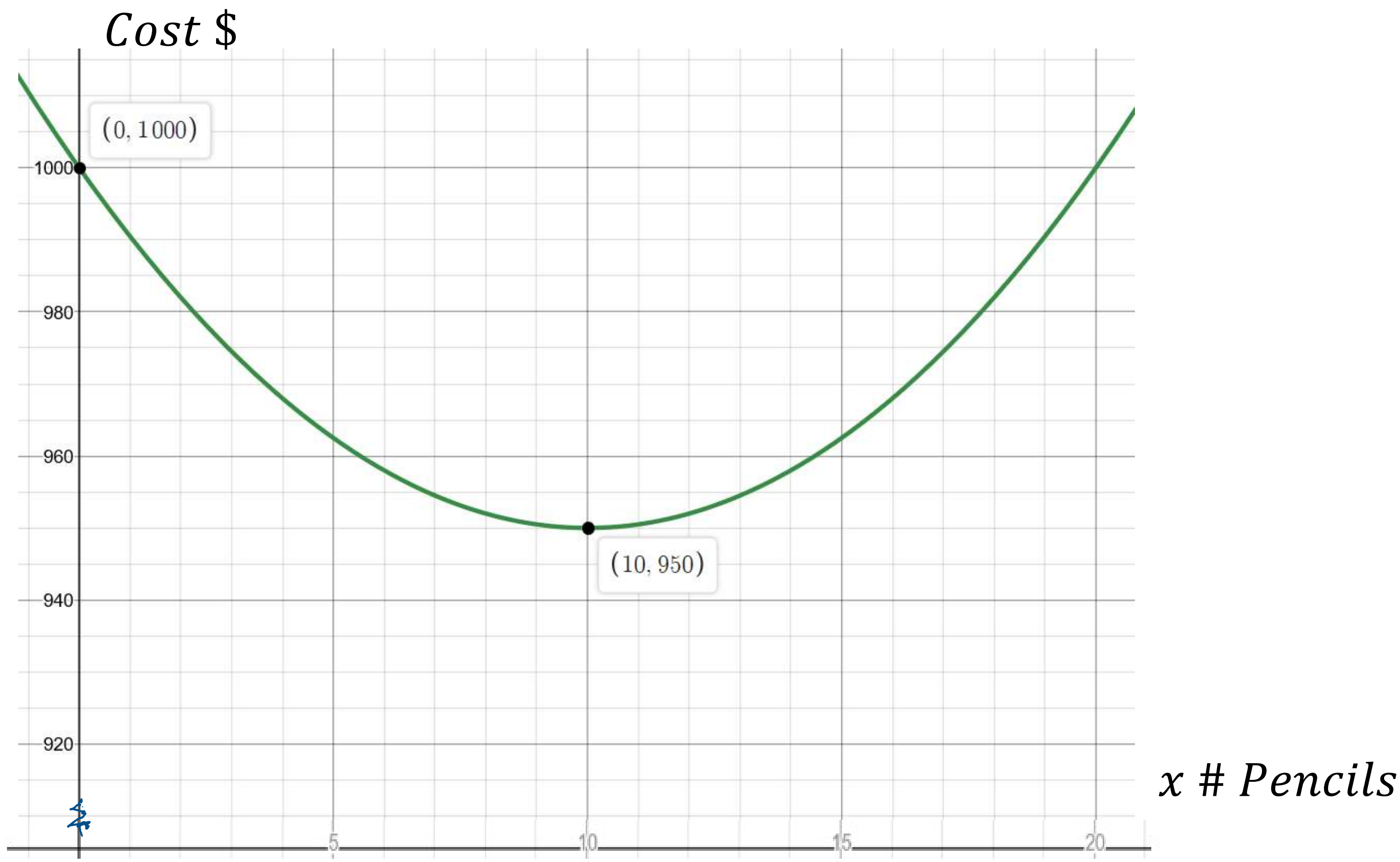
Find the intercepts.

y-intercept (starting cost of production).

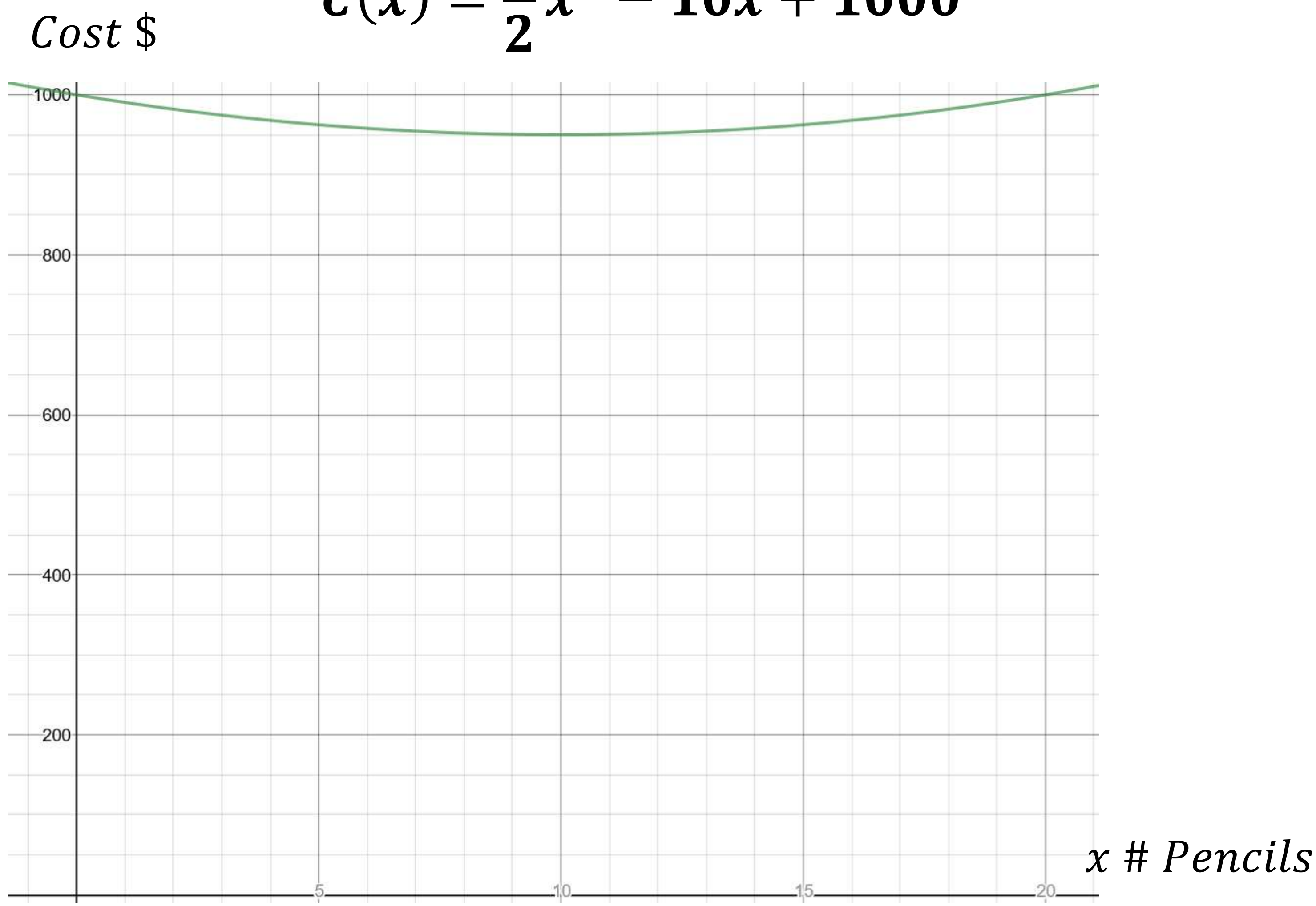
x-intercepts (when cost is zero).

$$\mathrm{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graph the function $C(x) = \frac{1}{2}x^2 - 10x + 1000$



$$C(x) = \frac{1}{2}x^2 - 10x + 1000$$



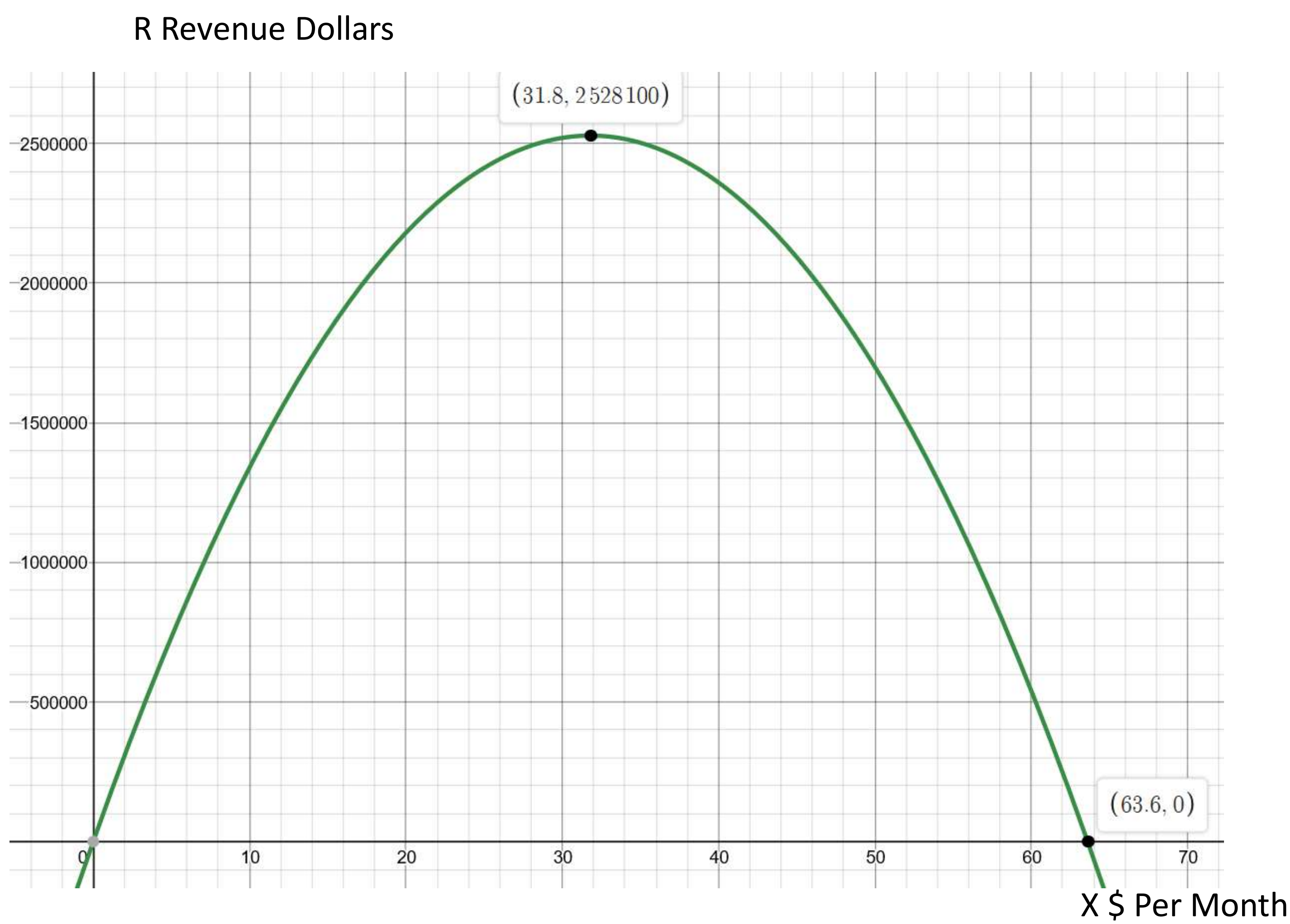
A Newspaper developed a formula to calculate the revenue R

as a function of the quarterly subscription fee x.

$$R(x) = -2500x^2 + 159000x$$

Determine the optimum fee x that will maximize revenue

and sketch a graph of $R(x)$. *What is the maximun revenue?*


R Revenue Dollars
(31.8, 2528100)
2500000
2000000
1500000
1000000
500000
(63.6, 0)
0
10
20
30
40
50
60
70
X $ Per Month