Quantitative Skills & Reasoning – Math 1001

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Logic Unit - Sets and Venn Diagrams
pp 319-328 in textbook

Sets

It is natural for us to classify items into groups, or sets, and consider how those sets overlap with each other. We can use these sets understand relationships between groups, and to analyze survey data.

A **set** is a collection of distinct objects, called **elements** of the set

A set can be defined by describing the contents, or by listing the elements of the set, enclosed in curly brackets.

Example 1

Some examples of sets defined by describing the contents:

a) The set of all even numbers

b) The set of all books written about travel to Chile

Some examples of sets defined by listing the elements of the set:

a) $\{1, 3, 9, 12\}$

b) {red, orange, yellow, green, blue, indigo, purple}

Notation

Commonly, we will use a variable to represent a set, to make it easier to refer to that set later.

The symbol ∊ means “is an element of”.

A set that contains no elements, { }, is called the **empty set** and is notated Ø

Example 2

Let $A = \{1, 2, 3, 4\}$

To notate that 2 is element of the set, we'd write $2 \in A$

Subset

A **subset** of a set A is another set that contains only elements from the set A, but may not contain all the elements of A.

If B is a subset of A, we write $B \subseteq A$

A **proper subset** is a subset that is not identical to the original set – it contains fewer elements.

If B is a proper subset of A, we write $B \subset A$

Example 3

Consider these three sets

A = the set of all even numbers $B = \{2, 4, 6\}$ $C = \{2, 3, 4, 6\}$

Here $B \subset A$ since every element of B is also an even number, so is an element of A.

More formally, we could say $B \subset A$ since if $x \in B$, then $x \in A$.

It is also true that $B \subset C$.

C is not a subset of A, since C contains an element, 3, that is not contained in A

Example 4

Suppose a set contains the plays "Much Ado About Nothing", "MacBeth", and "A Midsummer's Night Dream". What is a larger set this might be a subset of?

There are many possible answers here. One would be the set of plays by Shakespeare. This is also a subset of the set of all plays ever written. It is also a subset of all British literature.

Union, Intersection, and Complement

The **union** of two sets contains all the elements contained in either set (or both sets).

The union is notated $A \cup B$.

More formally, $x \in A \cup B$ if $x \in A$ or $x \in B$ (or both)

The **intersection** of two sets contains only the elements that are in both sets.

The intersection is notated $A \cap B$.

More formally, $x \in A \cap B$ if $x \in A$ and $x \in B$

The **complement** of a set A contains everything that is *not* in the set A.

The complement is notated A', or A^c, or sometimes $\sim A$.

Example 5

Consider the sets: A = {red, green, blue} B = {red, yellow, orange}

C = {red, orange, yellow, green, blue, purple}

a) Find $A \cup B$

b) Find $A \cap B$

c) Find $A^c \cap C$

Example 5 Solution

A = {red, green, blue} B = {red, yellow, orange}
C = {red, orange, yellow, green, blue, purple}

a) The union contains all the elements in either set: $A \cup B$ = {red, green, blue, yellow, orange}
Notice we only list red once.

b)The intersection contains all the elements in both sets: $A \cap B$ = {red}

c)Here we're looking for all the elements that are *not* in set A and are also in C.
A^c = {orange, yellow, purple} $A^c \cap C$ = {orange, yellow, purple}

Universal Set

A **universal set** is a set that contains all the elements we are interested in. This would have to be defined by the context.

A complement is relative to the universal set, so A^c contains all the elements in the universal set that are not in A.

Example 6

a) If we were discussing searching for books, the universal set might be all the books in the library.

b) If we were grouping your Facebook friends, the universal set would be all your Facebook friends.

c) If you were working with sets of numbers, the universal set might be all whole numbers, all integers, or all real numbers

Example 7

Suppose the universal set is U = all whole numbers from 1 to 9.

If $A = \{1, 2, 4\}$, then

$A^c = \{3, 5, 6, 7, 8, 9\}$.

As we saw earlier with the expression $A^c \cap C$, set operations can be grouped together. Grouping symbols can be used like they are with arithmetic – to force an order of operations.

Example 8

Suppose H = {cat, dog, rabbit, mouse}, F = {dog, cow, duck, pig, rabbit}
W = {duck, rabbit, deer, frog, mouse}

a) Find $(H \cap F) \cup W$

b) Find $H \cap (F \cup W)$

c) Find $(H \cap F)^c \cap W$

Example 8 Solution

Suppose H = {cat, dog, rabbit, mouse}, F = {dog, cow, duck, pig, rabbit}
W = {duck, rabbit, deer, frog, mouse}

a) Find $(H \cap F) \cup W$
We start with the intersection: $H \cap F$ = {dog, rabbit}
Now we union that result with W: $(H \cap F) \cup W$ = {dog, duck, rabbit, deer, frog, mouse}

b)Find $H \cap (F \cup W)$
We start with the union: $F \cup W$ = {dog, cow, rabbit, duck, pig, deer, frog, mouse}
Now we intersect that result with H: $H \cap (F \cup W)$ = {dog, rabbit, mouse}

c) Find $(H \cap F)^c \cap W$
We start with the intersection: $H \cap F$ = {dog, rabbit},
$(H \cap F)^c$ = {cat, mouse, cow, duck, pig, deer, frog} (all the others in the universal set besides dog, rabbit)
$(H \cap F)^c \cap W$ = {duck, deer, frog, mouse}

Note this is all the elements of W that are *not* in $H \cap F$

Venn Diagram

To visualize the interaction of sets, John Venn in 1880 thought to use overlapping circles, building on a similar idea used by Leonhard Euler in the 18th century. These illustrations are now called **Venn Diagrams**.

A Venn diagram represents each set by a circle, usually drawn inside of a containing box representing the universal set. Overlapping areas indicate elements common to both sets.

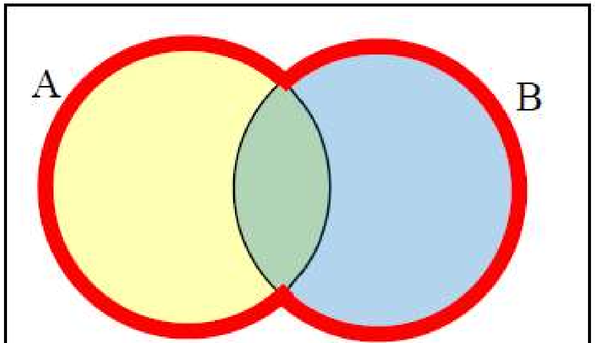
Basic Venn diagrams can illustrate the interaction of two or three sets.

Example 9

Example 9

Create Venn diagrams to illustrate $A \cup B$, $A \cap B$, and $A^c \cap B$

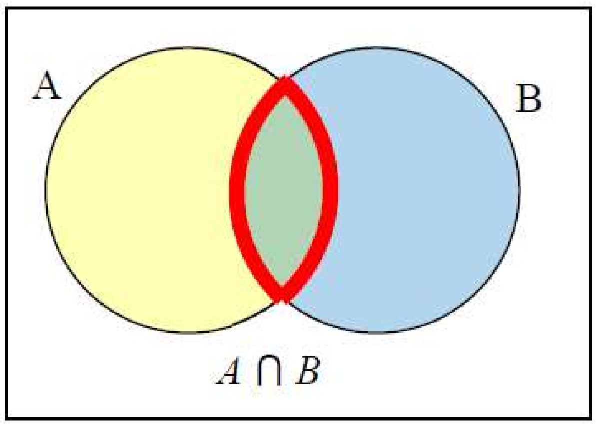
$A \cup B$ contains all elements in *either* set.



$A \cup B$

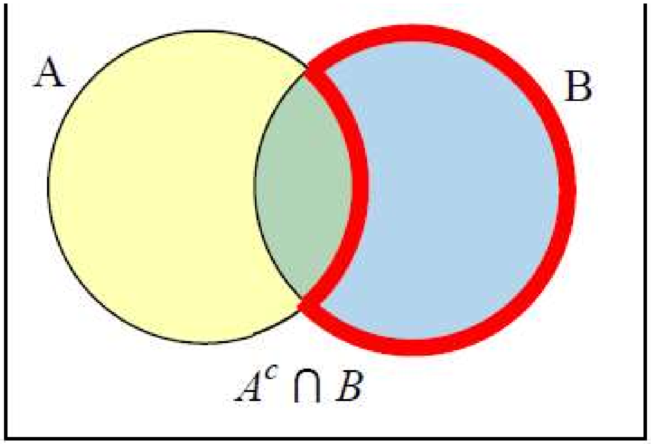
Example 9 (cont)

$A \cap B$ contains only those elements in both sets – in the overlap of the circles.



Example 9 (cont)

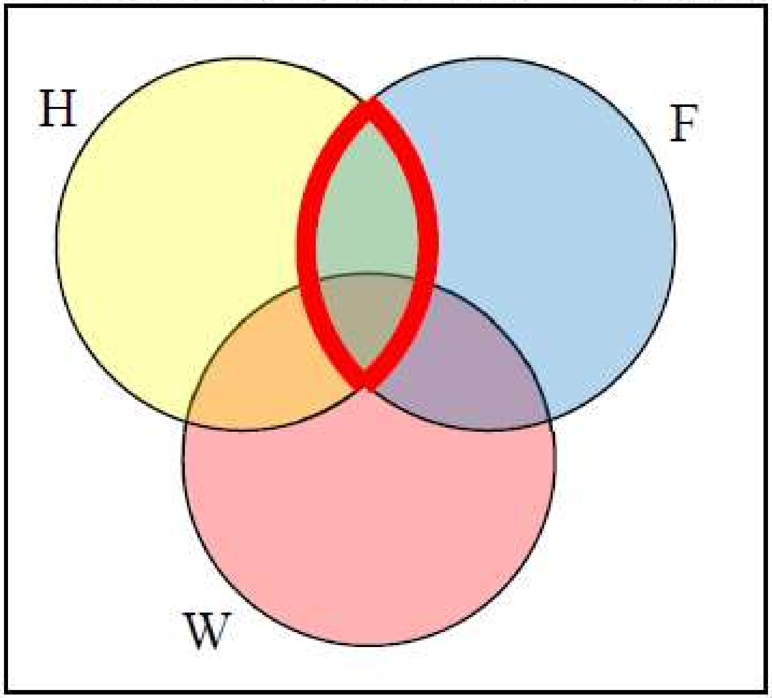
A^c will contain all elements *not* in the set A. $A^c \cap B$ will contain the elements in set B that are not in set A.



Example 10

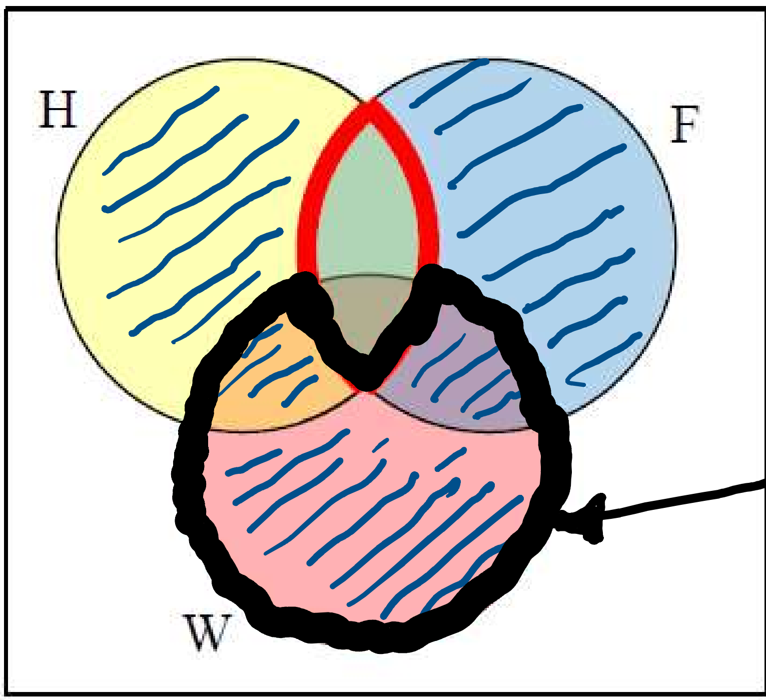
Use a Venn diagram to illustrate $(H \cap F)^c \cap W$

We'll start by identifying everything in the set $H \cap F$



Example 10

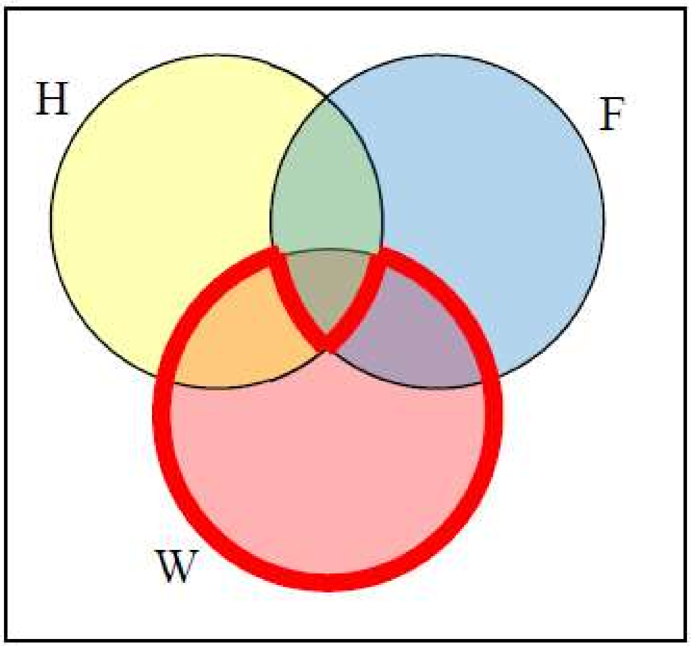
Use a Venn diagram to illustrate $(H \cap F)^c \cap W$



$(H \cap F)^c$

$(H \cap F)^c \cap W$

Now, $(H \cap F)^c \cap W$ will contain everything *not* in the set identified above that is also in set W.



Example 11

A survey asks 200 people “What beverage do you drink in the morning”, and offers choices:

Tea only

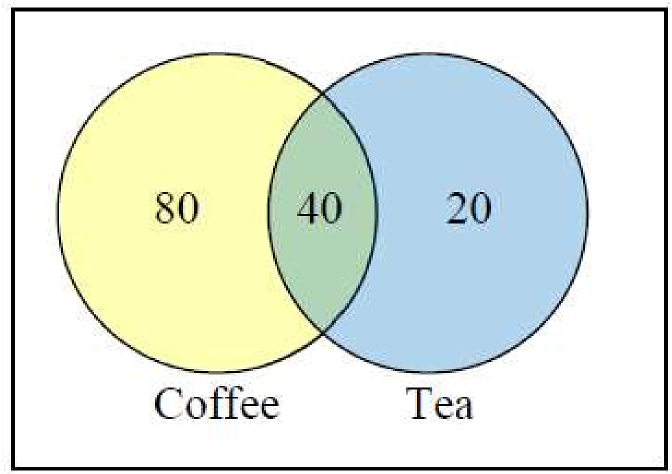
Coffee only

Both coffee and tea

Suppose 20 report tea only, 80 report coffee only, 40 report both. How many people drink tea in the morning? How many people drink neither tea or coffee?

This question can most easily be answered by creating a Venn diagram. We can see that we can find the people who drink tea by adding those who drink only tea to those who drink both: 60 people.

We can also see that those who drink neither are those not contained in the any of the three other groupings, so we can count those by subtracting from the cardinality of the universal set, 200. 200 – 20 – 80 – 40 = 60 people who drink neither.



Fifty students were surveyed, and asked if they were taking a social science (SS), humanities (HM) or a natural science (NS) course the next quarter.

21 were taking a SS course	26 were taking a HM course
19 were taking a NS course	9 were taking SS and HM
7 were taking SS and NS	10 were taking HM and NS
3 were taking all three	7 were taking none

How many students are only taking a SS course?