Quantitative Skills & Reasoning – Math 1001

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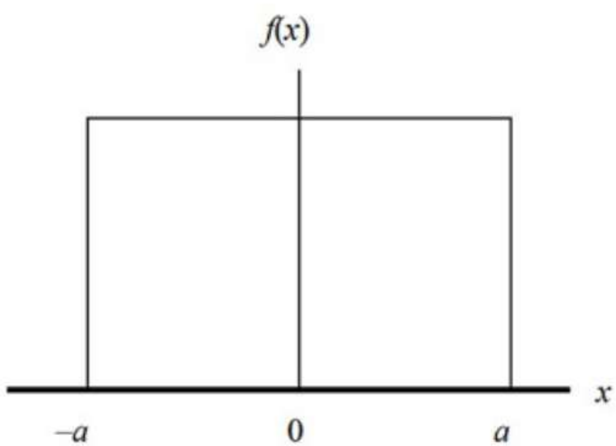
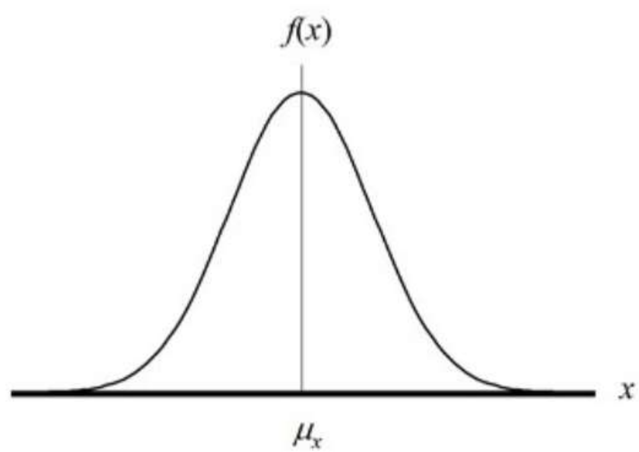
Probability Unit – The Normal Distribution

Probability Distributions In General

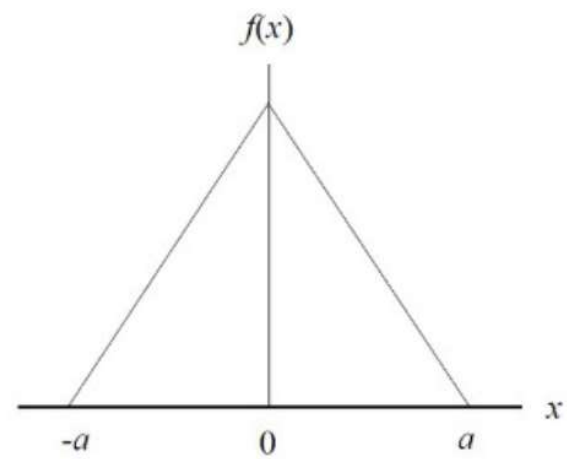
Uniform Distribution



Normal Distribution



Triangle Distribution

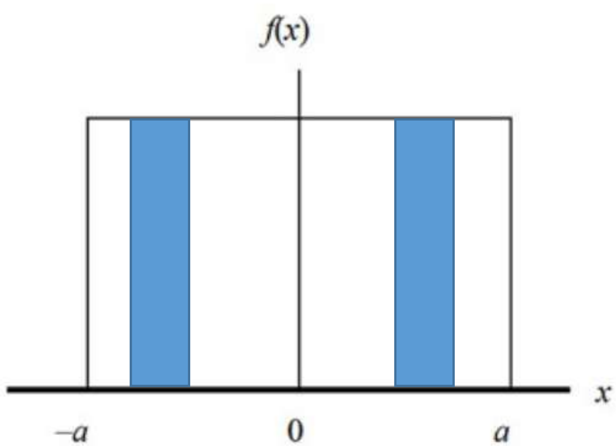


The Area Under The Curve Is 1.0 For All Distributions (100%)

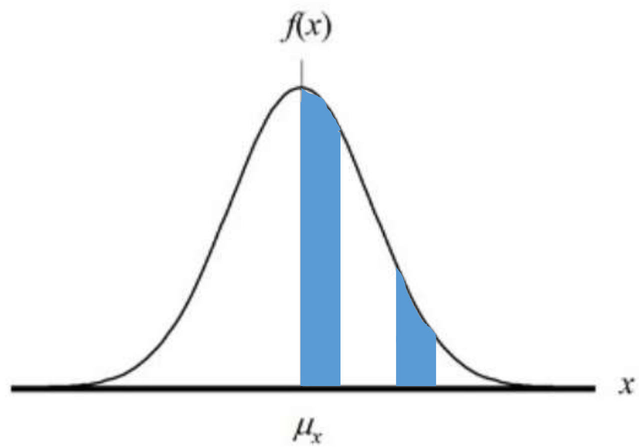
The Probability Of A Number Being Between a_1 and a_2 = Area Under The Curve

Probability Distributions In General

Uniform Distribution



Normal Distribution



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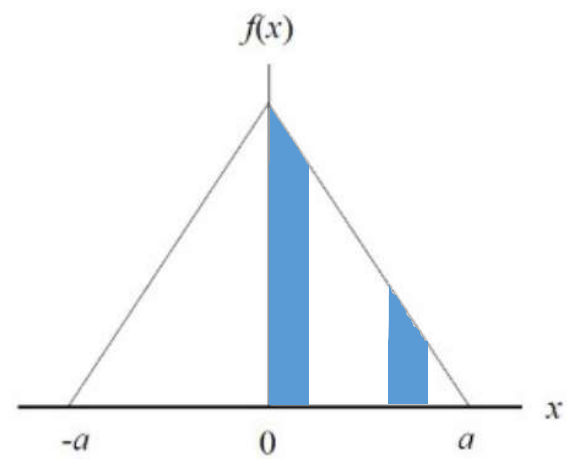

The Area Under The Curve Is 1.0 For All Distributions (100%)

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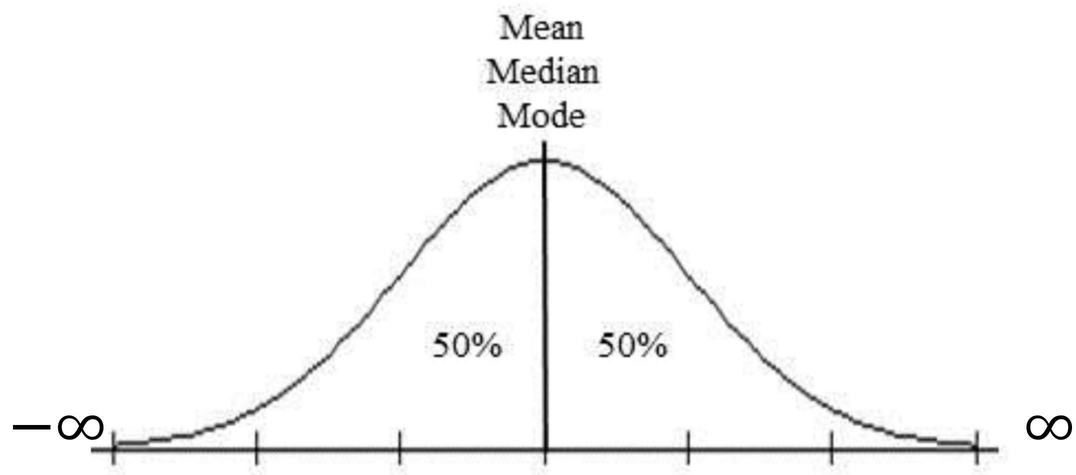
The Normal Distribution

- The Normal Distribution is a symmetric, bell-shaped distribution with a single peak. This peak in the distribution corresponds to the mean, median, and mode.



The Normal Distribution Facts

- Because the distribution is symmetric, 50% of the data values are below the mean, and 50% of the data values are above the mean.

- Data values farther from the mean become increasingly rare.

- The graph of the Normal Distribution is bell-shaped, with tapering tails that approach, but never actually touch the horizontal axis.

- Almost all of the area under a Normal Distribution curve is within three standard deviations of the mean.

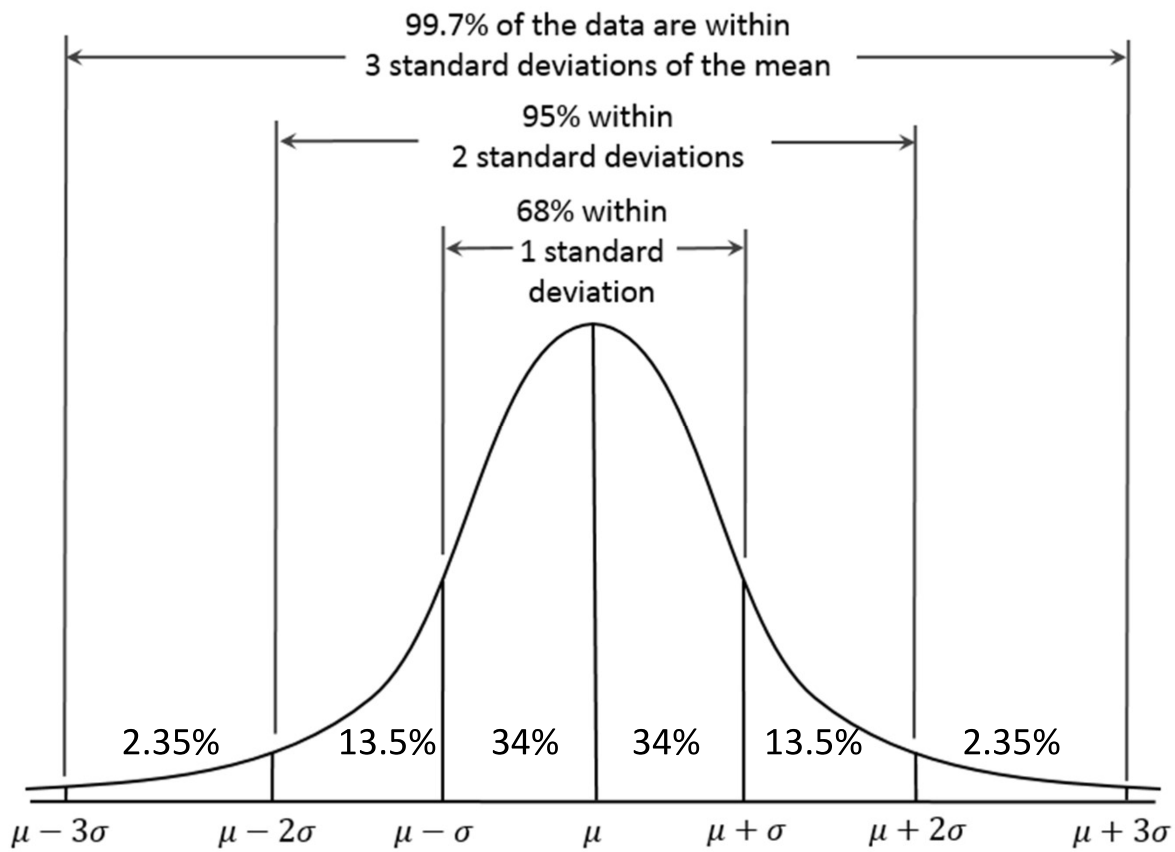
- The total area under the curve is 1 (100%)

The Empirical Rule (68-95-99.7 Rule)

Notation

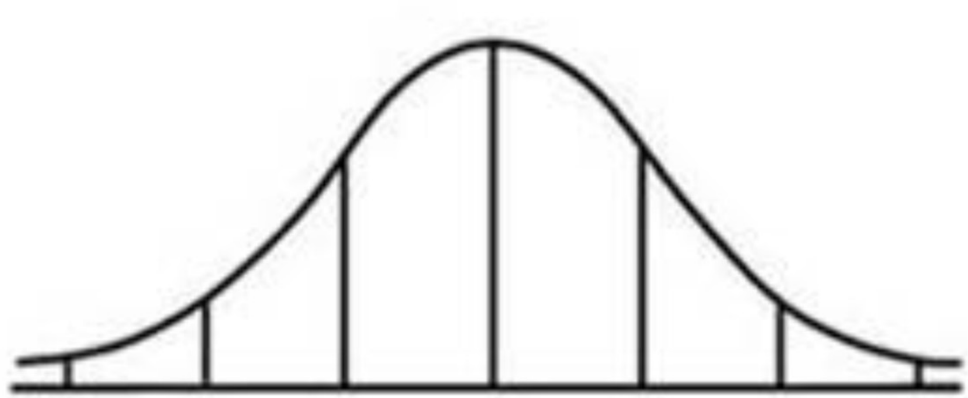
μ = mean

σ = standard deviation



Example

The test scores on a math exam are approximately normally distributed with mean 72 and standard deviation 8. Draw the associated normal distribution curve, and label the axis appropriately.



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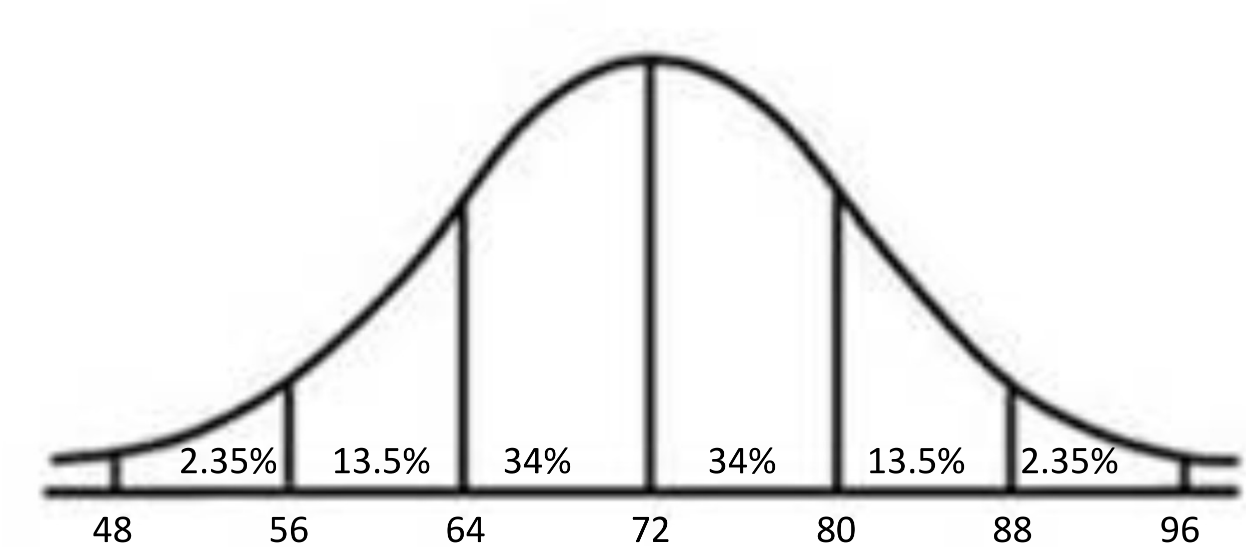


Example

Scores on a standardized test were normally distributed with a mean of 510 and a standard deviation of 95. Use the Empirical Rule to complete following statements. Also, draw and label a sketch of the distribution.

- 68% of the students taking this exam scored between _____ and _____.

- 95% of the students taking this exam scored between _____ and _____.

- 99.7% of the students taking this exam scored between _____ and _____.

Example

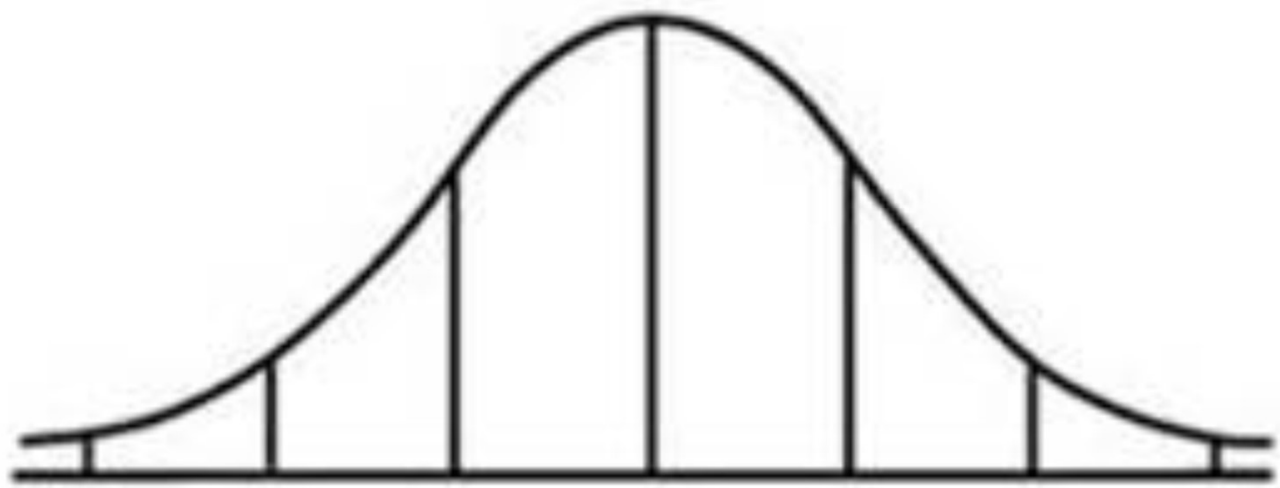
Scores on a standardized test were normally distributed with a mean of 510 and a standard deviation of 95. Use the Empirical Rule to complete following statements. Also, draw and label a sketch of the distribution.

- 68% of the students taking this exam scored between 415 and 605.

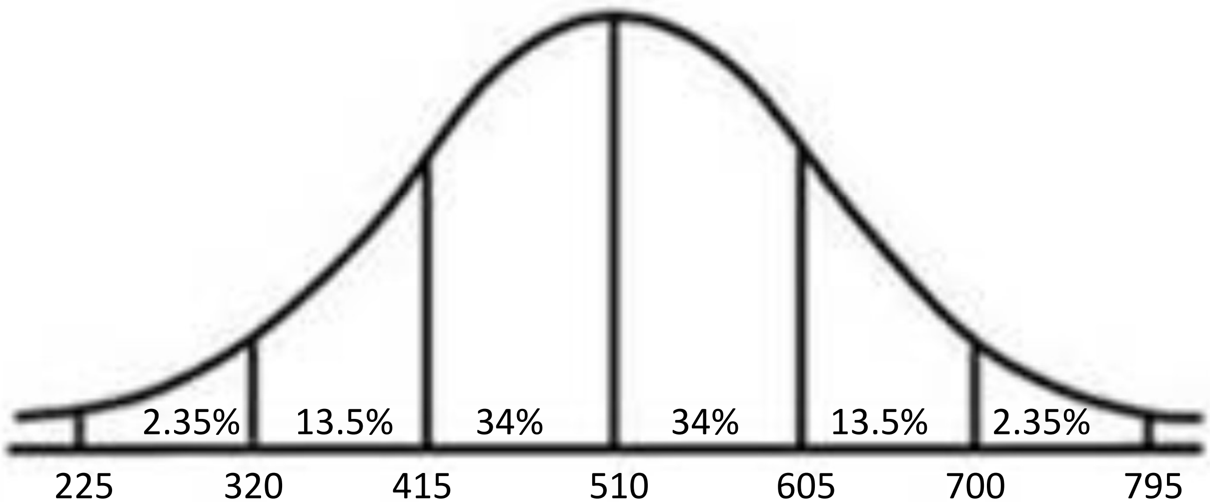
- 95% of the students taking this exam scored between 320 and 700.

- 99.7% of the students taking this exam scored between 225 and 795.

Example (cont.)



Example (cont.)



Example

Gear circumferences for a manufactured bicycle part were normally distributed with a mean of 34 inches and a standard deviation of 0.04 inches.

- 68% of the gear circumferences were between _____ and _____.

- 95% of the gear circumferences were between _____ and _____.

- 99.7% of the gear circumferences were between _____ and _____.

Example

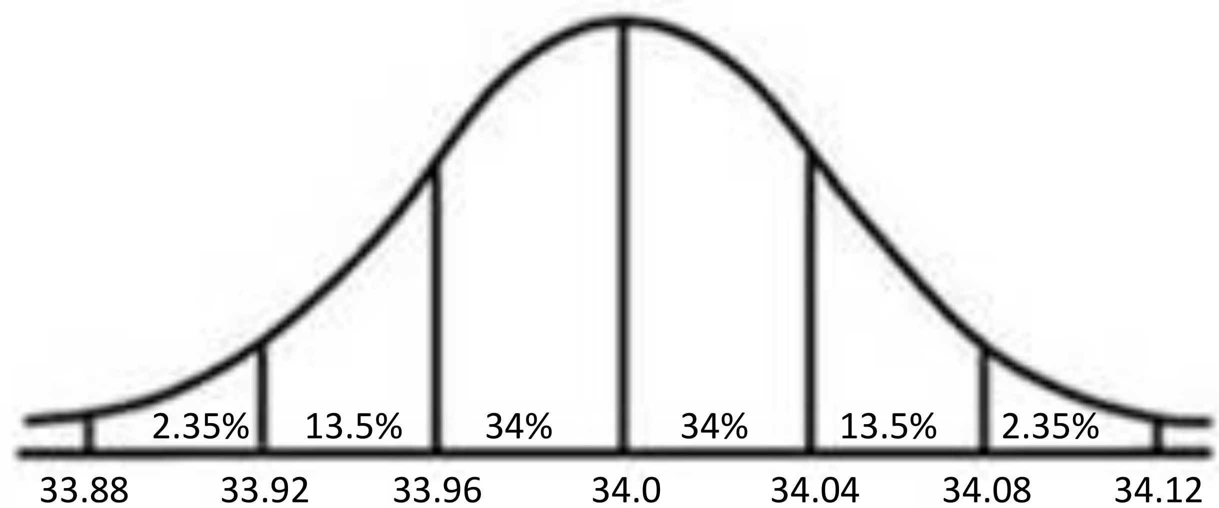
Gear circumferences for a manufactured bicycle part were normally distributed with a mean of 34 inches and a standard deviation of 0.04 inches.

- 68% of the gear circumferences were between 33.96 and 34.04.

- 95% of the gear circumferences were between 33.92 and 34.08.

- 99.7% of the gear circumferences were between 33.88 and 34.12.

Example (cont.)



Standard Scores (z-scores)

- The Empirical Rule only applies when a value is **exactly** 1, 2, or 3 standard deviations away from the mean. This is not usually the case. Therefore, we use a **standard score** (also called "**z-score**")to find the number of standard deviations a data value is from the mean of the distribution.
- We can plot z-scores on a special normal distribution called the **standard normal distribution**. The standard normal distribution is a normal distribution that always has a population *mean of 0* and population *standard deviation of 1*.

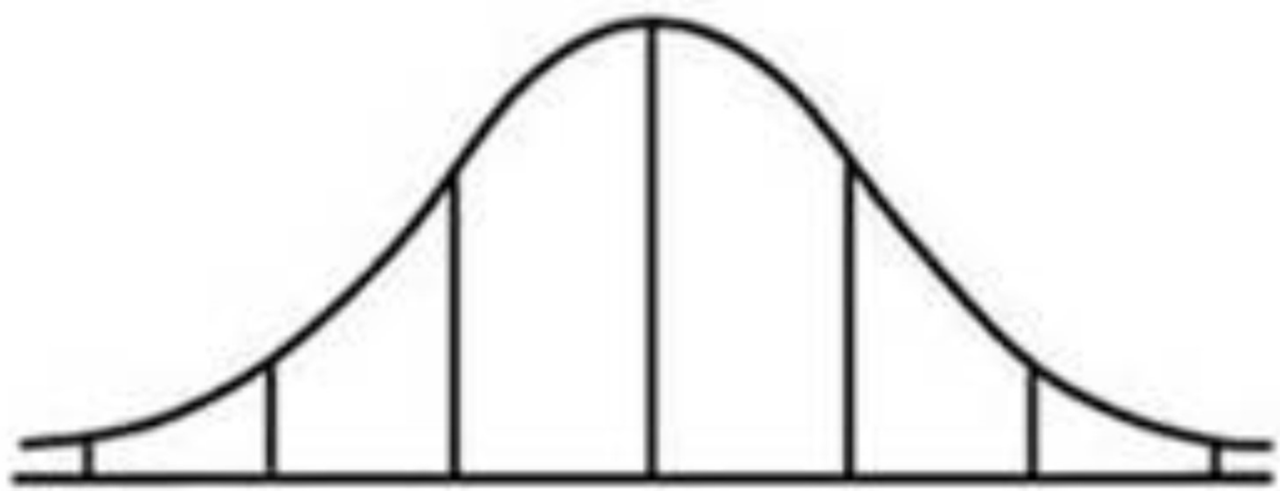
Standard Scores (z-scores) cont.

- If z is **positive**, then the data value is **above** the mean.
- If z is **negative**, then the data value is **below** the mean.
- It can be helpful to sketch the distribution to verify the z-score.

$$z = \frac{x - \mu}{\sigma} = \frac{x - mean}{standard\ deviation}$$

Example

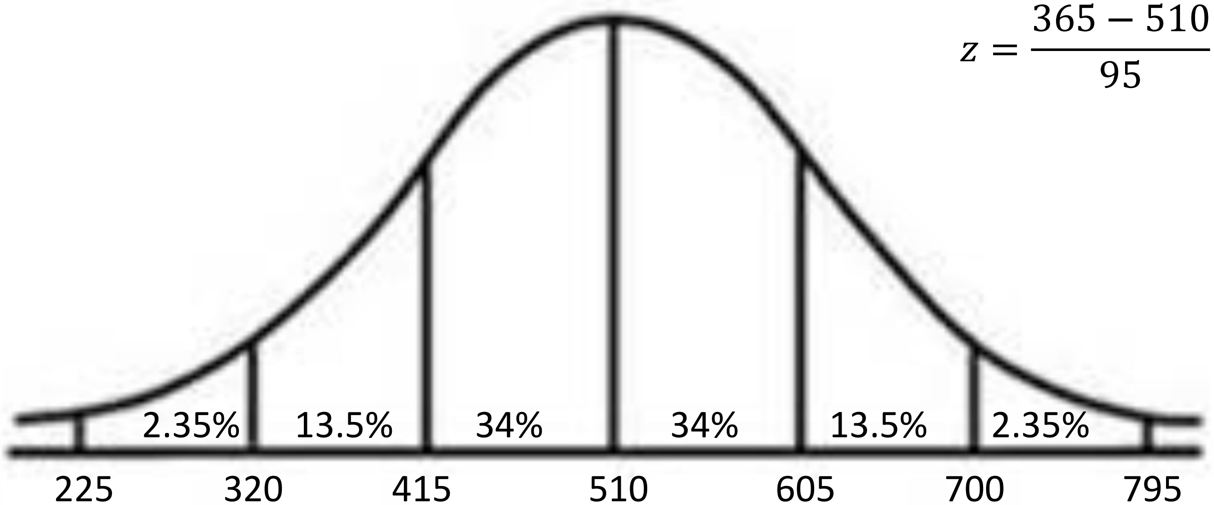
Scores on a standardized test were normally distributed with a mean of 510 and a standard deviation of 95. A student scores 365 points on the test. What is his standard score?



Example (cont.)

$$z = \frac{x - \mu}{\sigma} = \frac{x - mean}{standard\ deviation}$$

$$z = \frac{365 - 510}{95} = -1.525$$



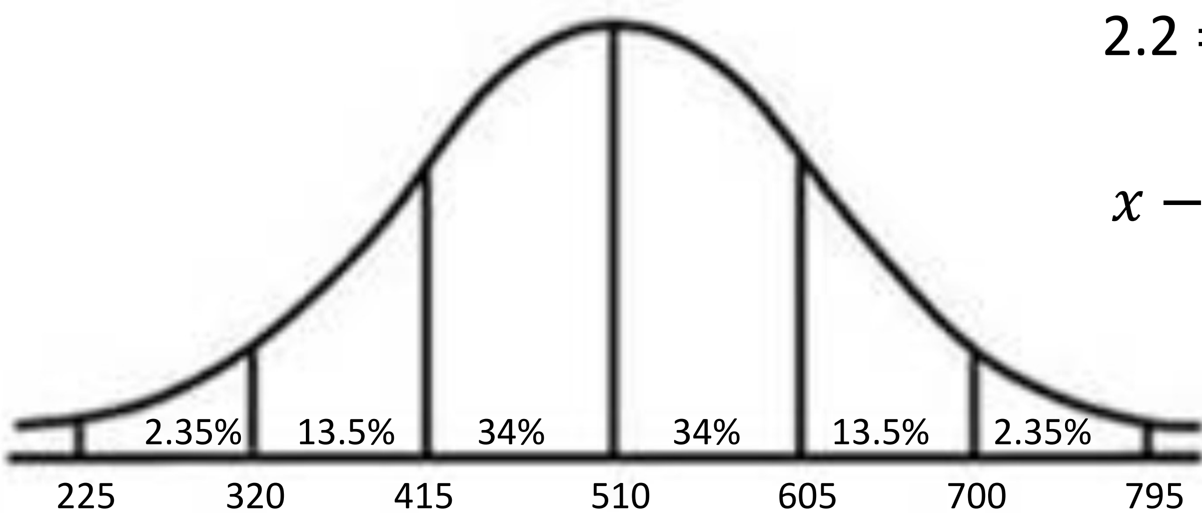
Example (cont.)

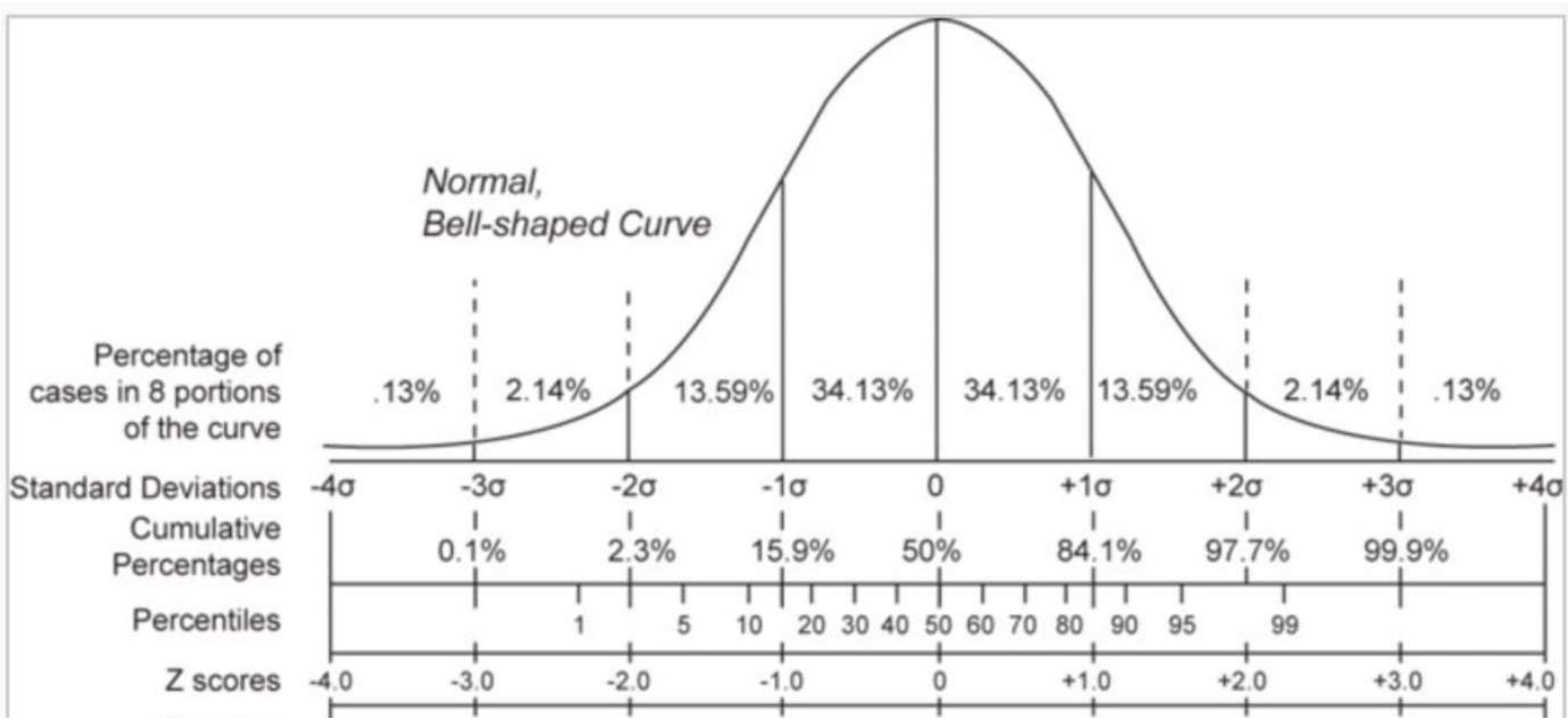
Scores on a standardized test were normally distributed with a mean of 510 and a standard deviation of 95. Suppose a student's z-score is 2.2, what did the student score on the test?

$$2.2 = \frac{x-\mu}{\sigma} = \frac{x-510}{95}$$

$$x - 510 = 2.2(95)$$

$$x = 719$$





Normal,
Bell-shaped Curve
Percentage of cases in 8 portions of the curve
.13%
2.14%
13.59%
34.13%
34.13%
13.59%
2.14%
.13%
Standard Deviations
-4σ
-3σ
-2σ
-1σ
0
+1σ
+2σ
+3σ
+4σ
Cumulative Percentages
0.1%
2.3%
15.9%
50%
84.1%
97.7%
99.9%
Percentiles
1
5
10
20 30 40 50 60 70 80
90
95
99
Z scores
-4.0
-3.0
-2.0
-1.0
0
+1.0
+2.0
+3.0
+4.0