## Quantitative Skills \& Reasoning - Math 1001

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## To Be Covered On Test 4

1. General definitions \& principles in introductory probability theory
2. Calculate Probability for various situations
3. Understand how to calculate odds
4. Combinations and Permutations
5. Expected Value
6. Normal Distribution

## Definitions

- Given that all outcomes are equally likely, we can compute the probability of an event E using this formula:

$$
P(E)=\frac{\text { Number of outcomes corresponding to the event } E}{\text { Total number of equal likely outcomes }}
$$

- An impossible event has a probability of 0 .
- A certain event has a probability of 1.

The probability of any event must be $0 \leq P(E) \leq 1$.

- The complement of an event is the event " $E$ does not happen." The notation $\bar{E}$ is used for the complement of event $E$.
- We can compute the probability of the complement using
- $P(\bar{E})=1-P(E)$. Notice also that $P(E)=1-P(\bar{E})$.
- Odds is the ratio of the probability that a particular event will occur to the probability that it will not occur. We always express odds in simplest form.
- The odds for an event $E$ is the ratio of $\mathrm{P}(E)$ to $\mathrm{P}($ not $E)$.
- The odds against an event $E$ is the ratio of $\mathrm{P}($ not $E)$ to $\mathrm{P}(E)$.
- Fundamental Counting Principle

If there are $m$ items in one category and $n$ items in another category, then the total number of available choices is $m^{*} n$.
This principle can be extended to more than two categories as well.

## Definitions

- Independent Events
- Events $A$ and $B$ are independent events if the probability of Event $B$ occurring is the same whether or not Event $A$ occurs.

Formula ( $\mathrm{P}(A$ and $B)$ for independent events)
If events $A$ and $B$ are independent, then the probability of both $A$ and $B$ occurring is

$$
\mathrm{P}(A \text { and } B)=\mathrm{P}(A) \cdot \mathrm{P}(B)
$$

- where $\mathrm{P}(A$ and $B)$ is the probability of events $A$ and $B$ both occurring, $\mathrm{P}(A)$ is the probability of event $A$ occurring, and $P(B)$ is the probability of event $B$ occurring.


## Probability of Either Event

Formula ( $\mathrm{P}(A$ or $B)$ )

- The probability of either $A$ or $B$ occurring (or both) is

$$
P(A \text { or } B)=P(A)+P(B)-P(A \text { and } B) .
$$

Note: Two events are mutually exclusive if they do not share any common outcomes.

## Conditional Probability for Dependent Events

- Independent Events
- Events $A$ and $B$ are dependent events if the probability of Event $B$ is affected by Event $A$ occurring.

Formula ( $\mathrm{P}(A$ and $B)$ for dependent events)
If events $A$ and $B$ are dependent, then the probability of both $A$ and $B$ occurring is

$$
\mathrm{P}(A \text { and } B)=\mathrm{P}(A) \cdot \mathrm{P}(B \mid A)
$$

## Permutations

We say that there are ${ }_{n} P_{r}$ permutations of size $r$ that may be selected from among $n$ choices without replacement when order matters.

$$
{ }_{n} \mathrm{P}_{\mathrm{r}}=\frac{n!}{(n-r)!}
$$

Combinations
We say that there are ${ }_{n} \mathrm{C}_{r}$ combinations of size $r$ that may be selected from among $n$ choices without replacement when order doesn't matter.

$$
{ }_{\mathrm{n}} \mathrm{C}_{\mathrm{r}}=\frac{n!}{(n-r)!r!}
$$

## Do These On Your Calculator!

Expected Value is the average gain or loss of an event if the procedure is repeated many times.

$$
\mu=E(X)=\sum_{i} x_{i} p_{i},
$$

where $x_{i}=$ is an outcome and $p_{i}$ is the probability of that outcome We can compute the expected value by multiplying each outcome by the probability of that outcome, then adding up the products.

## The Normal Distribution

- The Normal Distribution is a symmetric, bell-shaped distribution with a single peak. This peak in the distribution corresponds to the mean, median, and mode.



## The Normal Distribution Facts

- Because the distribution is symmetric, $50 \%$ of the data values are below the mean, and $50 \%$ of the data values are above the mean.
- Data values farther from the mean become increasingly rare.
- The graph of the Normal Distribution is bell-shaped, with tapering tails that approach, but never actually touch the horizontal axis.
- Almost all of the area under a Normal Distribution curve is within three standard deviations of the mean.
- The total area under the curve is 1 (100\%)


## The Empirical Rule (68-95-99.7 Rule)

Notation
$\mu=$ mean
$\sigma=$ standard deviation


## Standard Scores (z-scores) cont.

- If $z$ is positive, then the data value is above the mean.
- If $z$ is negative, then the data value is below the mean.
- It can be helpful to sketch the distribution to verify the $z$-score.

$$
z=\frac{x-\mu}{\sigma}=\frac{x-\text { mean }}{\text { standard deviation }}
$$

## Example

The test scores on a math exam are approximately normally distributed with mean 72 and standard deviation 8 . Draw the associated normal distribution curve, and label the axis appropriately.


## Question 5. Version 1*/1. Score: 1/1

In a certain state's lottery, 40 balls numbered 1 through 40 are placed in a machine and eight of them are drawn at random. If the eight numbers drawn match the numbers that a player had chosen, the player wins $\$ 1,000,000$. In this lottery, the order in which the numbers are drawn does not matter.

Compute the probability that you win the million-dollar prize if you purchase a single lottery ticket. Write your answer as a reduced fraction.


A single lottery ticket costs \$2. Compute the Expected Value, to the state, if 10,000 lottery tickets are sold. Round your answer to the nearest dollar.


A single lottery ticket costs $\$ 2$. Compute the Expected Value, to you, if you purchase 10,000 lottery tickets. Round your answer to the nearest dollar.


In a certain state's lottery, 40 balls numbered 1 through 40 are placed in a machine and eight of them are drawn at random. If the eight numbers drawn match the numbers that a player had chosen, the player wins $\$ 1,000,000$. In this lottery, the order in which the numbers are drawn does not matter.

Possible Number of 8 ball combinations 76904685, 76904685-10000 $=76894685$ That won't win
Expected Value to State

$$
\begin{array}{ccc}
\text { Outcome } & \text { Value } & \text { Probability of Outcome } \\
\text { Nobody Wins } & \$ 20,000 & \frac{76894685}{76904685} \\
\text { Someone Wins } & -\$ 980,000 & \frac{10000}{76904685} \\
\text { Expected Value }=20000\left(\frac{76894685}{76904685)}\right)-980000\left(\frac{10000}{76904685)}\right) \\
=\$ 19,869.97=\$ 19,870
\end{array}
$$

## Expected Value to Player

$$
\left.\begin{array}{ccc}
\begin{array}{c}
\text { Outcome } \\
\text { He Wins }
\end{array} & \begin{array}{c}
\text { Value } \\
\$ 980000
\end{array} & \begin{array}{c}
\text { Probability of Outcome } \\
700004685
\end{array} \\
\text { He Loses } & -\$ 20,000 & \frac{76894685}{76904685}
\end{array}\right] \begin{array}{r}
\text { Expected Value }=980000\left(\frac{10000}{76904685)}\right)-20000\left(\frac{76894685}{76904685)}\right) \\
=-\$ 19,869.97=-\$ 19,870
\end{array}
$$

If the probability of the Atlanta Braves winning the World Series is $3 / 20$

What are the odds of the Braves winning the World Series?

A poll showed that 53\% of Americans say they believe that UFOs exist. What is the probability of randomly selecting someone who does not believe that UFOs exist? (Express answer as a decimal)

A group of people were asked if they owned a TV. 234 responded "yes", and 4 responded "no".

Find the probability that if a person is chosen at random, they own a TV?

Students In History Class - Test Grades

If one student is chosen at random, Find the probability that the student was male

|  | $A$ | $B$ | $C$ | Total |
| :--- | :--- | :--- | :--- | :--- |
| Male | 15 | 10 | 18 | 43 |
| Female | 8 | 20 | 19 | 47 |
| Total | 23 | 30 | 37 | 90 |

If one student is chosen at random, Find the probability that the student was male AND got a "B"

If one student is chosen at random, Find the probability that the student was male OR got an "B"

If one student is chosen at random, find the probability that the student was male GIVEN they got a 'A'

An Emanuel County Planning Committee is to consist of 4 persons from Swainsboro, 3 from ECI, and 2 from Oak Park. Swainsboro has 10 qualified persons, ECI has 8 qualified, and Oak Park has 7 that are qualified. In how many ways can the committee be chosen?

At an exclusive car show only ten cars are to be displayed. The person planning the show has 15 snazzy cars from which to choose. Order is not important. How many different 10 car lineups are possible?

A bakery has lots of birthday cakes. They offer 4 cake sizes, 8 types of batter, 15 flavors of icing, and 5 colors of candles.

How many different cakes can you get?

A bag contains 3 gold marbles, 6 silver marbles, and 11 black marbles. Someone offers to play this game: You randomly select one marble from the bag. If it is gold, you win $\$ 3$. If it is silver, you win \$2. If it is black, you lose \$1.

What is your expected value if you play this game?

A chef must supply 7 different flavors of salad dressing for a salad party. He/she finds 20 flavors at Publix. How many different selections can he/she make?

20 Students Are Eligible to lead a committee for the senior prom. How many ways can you put together a Chair, Vice-Chair, and a Secretary?

One thousand raffle tickets are sold for $\$ 5.00$ each.
One grand prize of \$800 and two consolation prizes of \$100 each will be awarded. Jeremy purchases one ticket. Find his expected value

Question 3. Version 2*/2. Score: 1/1 *

A 40 -year-old man in the U.S. has a $0.246 \%$ risk of dying during the next year. An insurance company charges $\$ 280$ per year for a life-insurance policy that pays a $\$ 100,000$ death benefit. What is the expected value for the person buying the insurance? Round your answer to the nearest dollar.

Expected Value: $\$-34 \quad \checkmark \quad$ for the year

You draw three cards from a 52 card deck without replacement. What is the probability that you will draw 3 ACEs in a row?

Choice Between Humanities and Science
10 Students from Swainsboro, 8 from Statesboro, and 12 from Augusta were asked if they preferred humanities or science. The data is given below:

| Swainsboro | Statesboro | Augusta |
| :---: | :---: | :---: |
| Humanities 7 | Humanities 3 | Humanities 8 |
| Science 3 | Science 5 | Science 4 |

If a student was picked at random, what is probability he/she preffered science?

A student was picked at random and you were he/she preferred humanities. What is the probability that the student is from Augusta?

Fill in the blanks.

In a normal distribution, $\qquad$ percent of the data are above the mean, and $\qquad$ percent of the data are below the mean.

Similarly,___ percent of all data points are within 1 standard deviation of the mean, $\qquad$ percent of all data points are within 2 standard deviations of the mean, and $\qquad$ percent are within 3 standard deviations of the mean.

Suppose that we know about a population of a particular breed of cats having weights that are normally distributed. Furthermore, suppose we know that the mean of the distribution is 10 pounds and the standard deviation is 2 pounds.

1. What is the $z$-score for 13 pounds?

$$
z=\frac{x-\mu}{\sigma}
$$

2.What is the $z$-score for 6 pounds?
3.How many pounds corresponds to a $z$-score of 1.25 ?
21) The mean weight of the fish in a pond is 10 pounds with a standard deviation of 3 pounds. What is the probability that a random fish caught is between 13 and 16 pounds? (See Normal Curve Attached).

What is the probability that the fish weighs less than 7 pounds?


