Quantitative Skills & Reasoning - Math 1001

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Zoom Video 10-31-2019

Question 3. Version 1*/1. Score: **1/1**

How many three-letter "words" can be made from 10 different letters "FGHIJKLMNO" if...

a) Repetition of letters is allowed? 1000 ✓ ⚊

b) Repetition of letters is not allowed? 720 ✓ ⚊

Score: 0.5 /0.5 0.5 /0.5 Full credit all parts Add feedback

Time spent on this version: 4.7 minutes Show all tries

In a lottery daily game, a player picks three numbers from 0, 1, 2, 3 How many different choices does the player have

a) If order matters?

b) If order *does not* matter?

Order Matters - Picking From 0 1 2 3

Use 0 1, and 2	Use 1 2 3	Use 2 3 and 0	Use 3 0 and 1
012	123	230	301
021	132	203	310
102	213	023	031
201	312	320	130
120	231	302	103
210	321	032	013

Question 5.   Version 1*/1. Score: **1/1**

In a lottery daily game, a player picks four numbers from 0 to 9 (without repetition). How many different choices does the player have

a) If order matters? 5040 ✔ ⚷

b) If order *does not* matter? 210 ✔ ⚷

Score: 0.5 /0.5 0.5 /0.5   Full credit all parts   Add feedback

Time spent on this version: 1.4 minutes

12) A company wants to hire a software engineer, an administrative assistant, and a sales representative. There are 4 possible candidates for the position of software engineer, 4 for the position of administrative assistant, and 3 for the position of sales representative. How many ways are there to choose the three people who will be hired?

13) Mr. Larsen's third grade class has 22 students, 12 girls and 10 boys. Two students must be selected at random to be in the fall play. What is the probability that no boys will be chosen? Order is not important.

14) A poker hand consists of 5 cards dealt from an ordinary deck of 52 playing cards. How many different hands are there consisting of four hearts and one spade?

15) How many different 4-topping pizzas can be made if there are 13 individual toppings to choose from? Assume that no topping is used more than once and that the order of the toppings on the pizza is unimportant.

16) There are 5 members on a board of directors. If they must elect a chairperson, a secretary, and a treasurer, how many different slates of candidates are possible?

Question 1.   Version 1*/1. Score: 1/1

A bag contains 2 gold marbles, 8 silver marbles, and 25 black marbles. Someone offers to play this game:
You randomly select one marble from the bag. If it is gold, you win $4. If it is silver, you win $3. If it is
black, you lose $1.

What is your expected value if you play this game?

$ .2 ✓ ⚷

Score: 1 /1 Full credit   Add feedback

Time spent on this version: 2.9 minutes

Find the indicated probability. Round your answer to 6 decimal places when necessary.

18) The Kiwanis Club is selling raffle tickets to raise money for a new playground for the city. A raffle ticket costs $2. There will be 1 winning ticket worth $1000, 3 tickets worth $100, and 5 winning tickets worth $20. 1000 tickets were sold. What is the expected value (to you) of one raffle ticket? *Round your answers to the nearest cent.*

Question 5.   Version 1*/1. Score: **1/1**

In a certain state's lottery, 40 balls numbered 1 through 40 are placed in a machine and eight of them are drawn at random. If the eight numbers drawn match the numbers that a player had chosen, the player wins $1,000,000. In this lottery, the order in which the numbers are drawn does not matter.

Compute the probability that you win the million-dollar prize if you purchase a single lottery ticket. Write your answer as a reduced fraction.

$$P(\text{win}) = \frac{1}{76904685} \quad \checkmark$$

A single lottery ticket costs $2. Compute the Expected Value, to the state, if 10,000 lottery tickets are sold. Round your answer to the nearest dollar.

Answer: $ 19870   ✓

A single lottery ticket costs $2. Compute the Expected Value, to you, if you purchase 10,000 lottery tickets. Round your answer to the nearest dollar.

Answer: $ -19870   ✓

In a certain state's lottery, 40 balls numbered 1 through 40 are placed in a machine and eight of them are drawn at random. If the eight numbers drawn match the numbers that a player had chosen, the player wins $1,000,000. In this lottery, the order in which the numbers are drawn does not matter.

$Possible\ Number\ of\ 8\ ball\ combinations\ 76904685,\ 76904685 - 10000 = 76894685\ That\ won't\ win$

Expected Value to State

$$Outcome \qquad Value \qquad Probability\ of\ Outcome$$

$$Nobody\ Wins \qquad \$20{,}000 \qquad \frac{76894685}{76904685}$$

$$Someone\ Wins \qquad -\$980{,}000 \qquad \frac{10000}{76904685}$$

$$Expected\ Value = 20000\left(\frac{76894685}{76904685)}\right) - 980000\left(\frac{10000}{76904685)}\right)$$

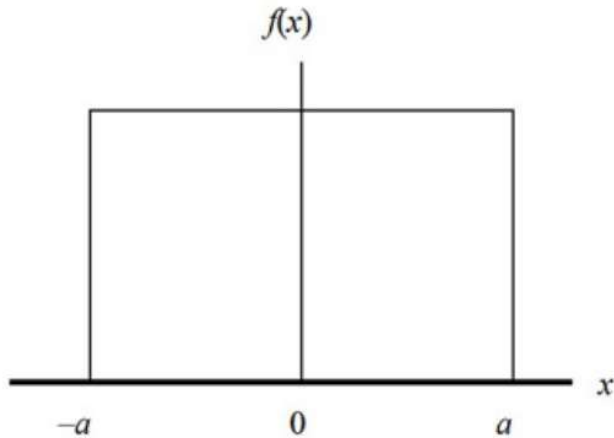$$= \$19{,}869.97\ = \$19{,}870$$

Expected Value to Player

Outcome	Value	Probability of Outcome
He Wins	$ 980000	$\dfrac{10000}{76904685}$
He Loses	-$20,000	$\dfrac{76894685}{76904685}$

$$Expected\ Value = 980000\left(\frac{10000}{76904685)}\right) - 20000\left(\frac{76894685}{76904685)}\right)$$

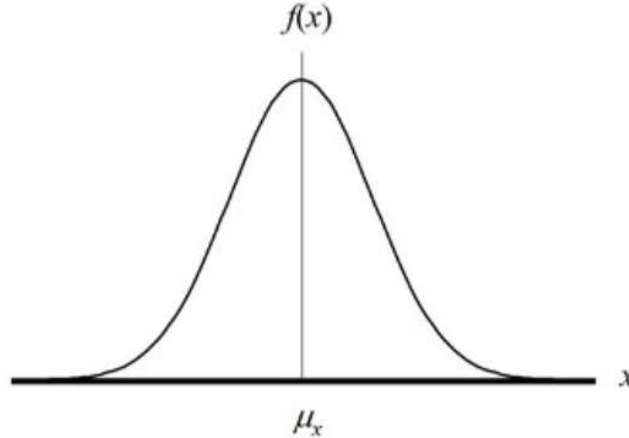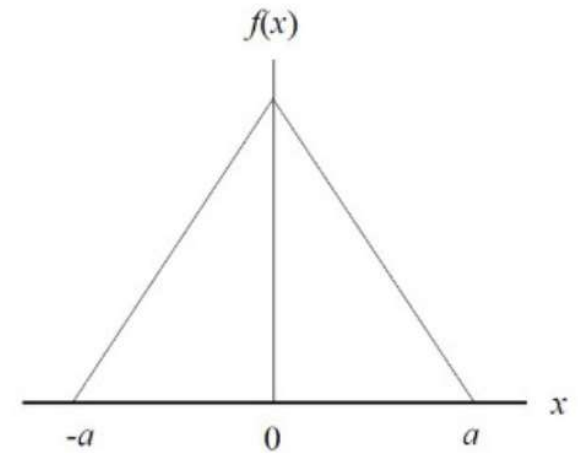$$= -\$19,869.97 = -\$19,870$$

Probability Distributions In General

Uniform Distribution

$f(x)$

$-a$ 0 a x

Normal Distribution

$f(x)$

μ_x x

Triangle Distribution
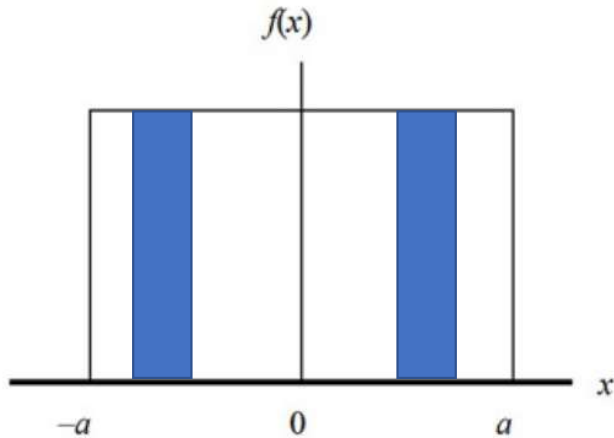
$f(x)$

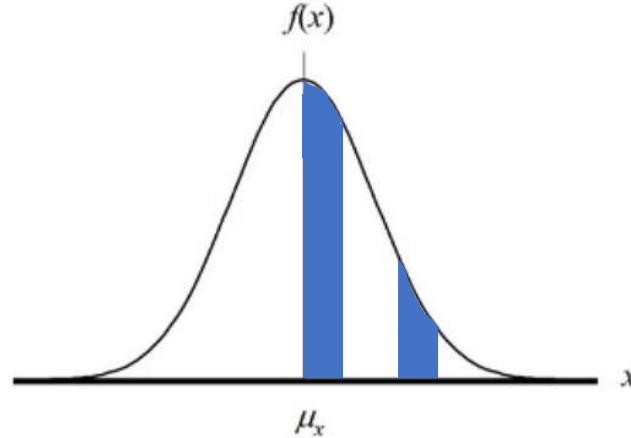$-a$ 0 a x

The Area Under The Curve Is 1.0 For All Distributions (100%)

The Probability Of A Number Being Between a_1 and a_2 = Area Under The Curve
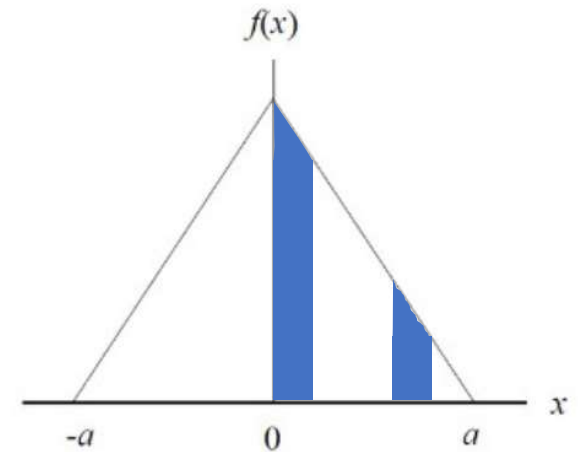
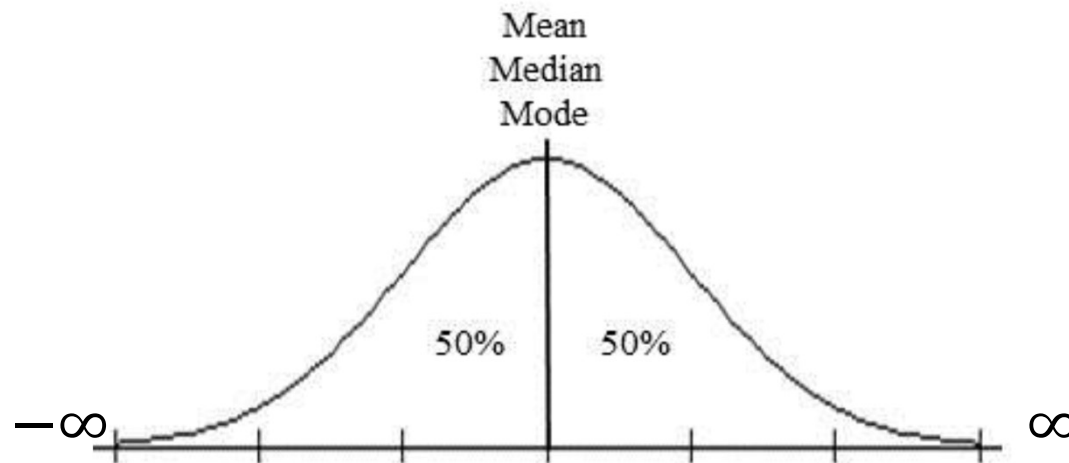# Probability Distributions In General

The Area Under The Curve Is 1.0 For All Distributions (100%)

The Probability Of A Number Being Between a_1 and a_2 = Area Under The Curve

The Normal Distribution

- The Normal Distribution is a symmetric, bell-shaped distribution with a single peak. This peak in the distribution corresponds to the mean, median, and mode.

The Normal Distribution Facts

- Because the distribution is symmetric, 50% of the data values are below the mean, and 50% of the data values are above the mean.

- Data values farther from the mean become increasingly rare.

- The graph of the Normal Distribution is bell-shaped, with tapering tails that approach, but never actually touch the horizontal axis.

- Almost all of the area under a Normal Distribution curve is within three standard deviations of the mean.

- The total area under the curve is 1 (100%)

The Empirical Rule (68-95-99.7 Rule)

Notation
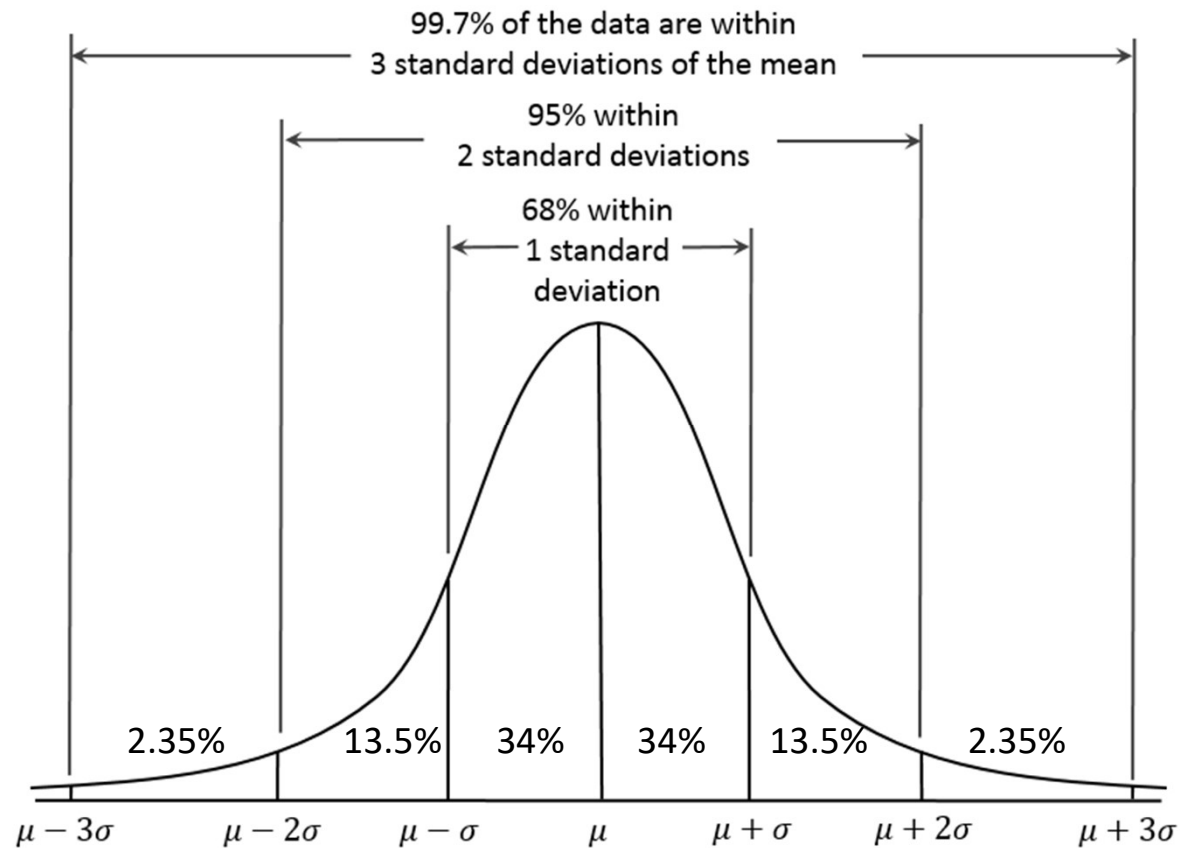
μ = mean

σ = standard deviation

99.7% of the data are within 3 standard deviations of the mean

95% within 2 standard deviations

68% within 1 standard deviation

2.35% 13.5% 34% 34% 13.5% 2.35%

$\mu - 3\sigma$ $\mu - 2\sigma$ $\mu - \sigma$ μ $\mu + \sigma$ $\mu + 2\sigma$ $\mu + 3\sigma$

Example

The test scores on a math exam are approximately normally distributed with mean 72 and standard deviation 8. Draw the associated normal distribution curve, and label the axis appropriately.
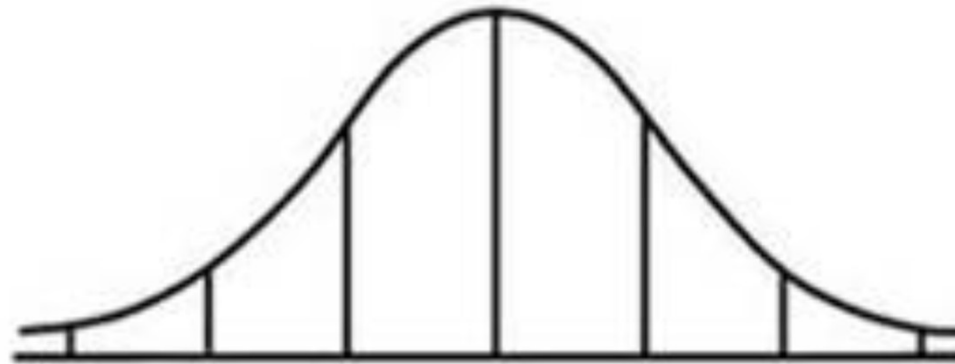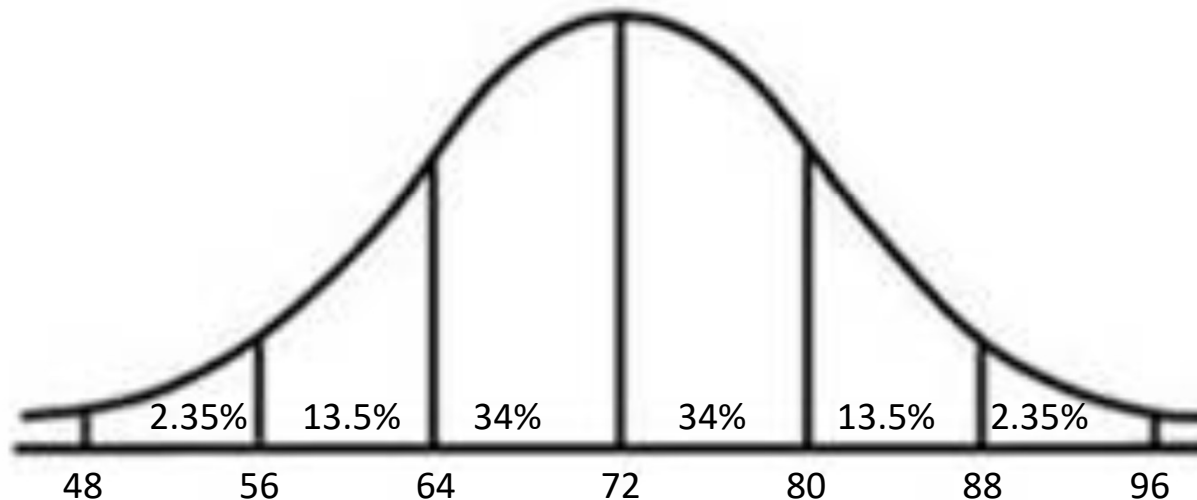
Example

The test scores on a math exam are approximately normally distributed with mean 72 and standard deviation 8. Draw the associated normal distribution curve, and label the axis appropriately.

19) In a normal distribution, what percentage of the data is between $-\infty$ and $+\infty$? (See Normal Curve Attached)

99.7% of the data are within 3 standard deviations of the mean

95% within 2 standard deviations

68% within 1 standard deviation

| 2.35% | 13.5% | 34% | 34% | 13.5% | 2.35% |

$\mu - 3\sigma$ $\mu - 2\sigma$ $\mu - \sigma$ μ $\mu + \sigma$ $\mu + 2\sigma$ $\mu + 3\sigma$
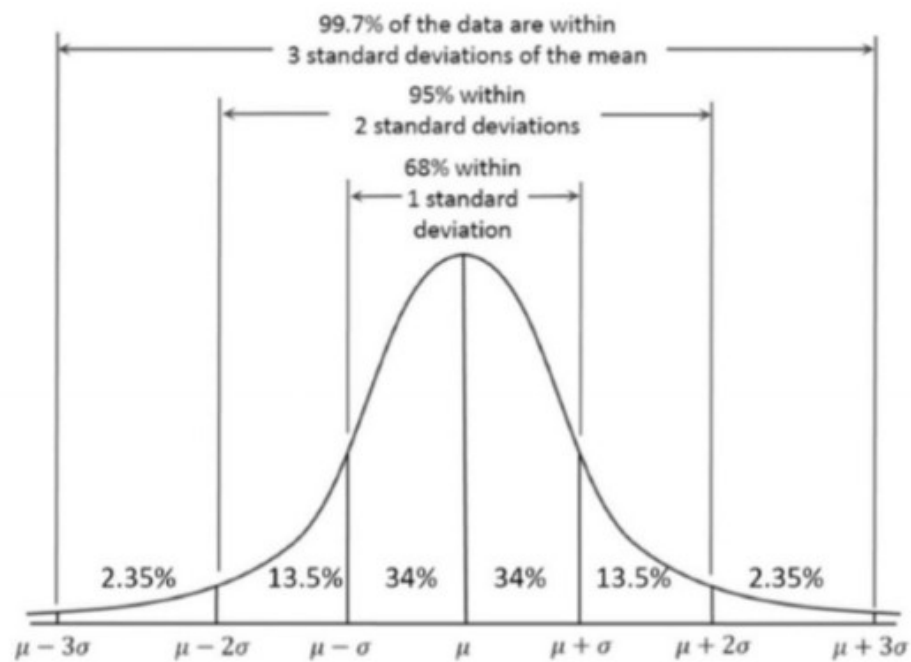
20) What is the probability that a data point will be greater than 2 standard deviations above the mean?
(See Normal Curve Attached)
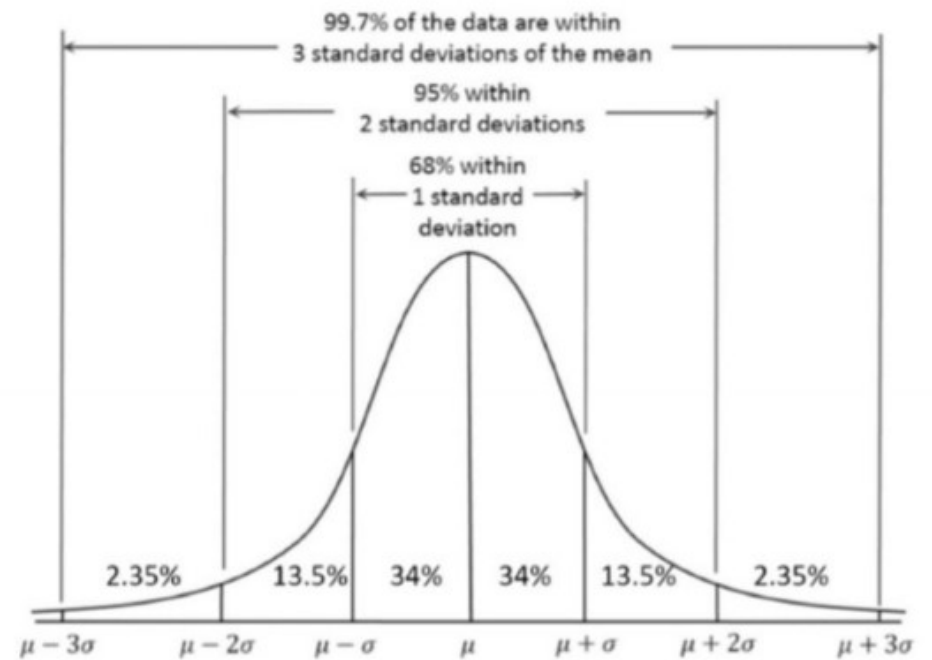
99.7% of the data are within
3 standard deviations of the mean

95% within
2 standard deviations

68% within
1 standard
deviation

| 2.35% | 13.5% | 34% | 34% | 13.5% | 2.35% |

$\mu - 3\sigma$ $\mu - 2\sigma$ $\mu - \sigma$ μ $\mu + \sigma$ $\mu + 2\sigma$ $\mu + 3\sigma$

21) The mean weight of the fish in a pond is 10 pounds with a standard deviation of 3 pounds. What is the probability that a random fish caught is between 13 and 16 pounds? (**See Normal Curve Attached**).

99.7% of the data are within
3 standard deviations of the mean

95% within
2 standard deviations

68% within
1 standard
deviation

| 2.35% | 13.5% | 34% | 34% | 13.5% | 2.35% |

$\mu - 3\sigma$ $\mu - 2\sigma$ $\mu - \sigma$ μ $\mu + \sigma$ $\mu + 2\sigma$ $\mu + 3\sigma$
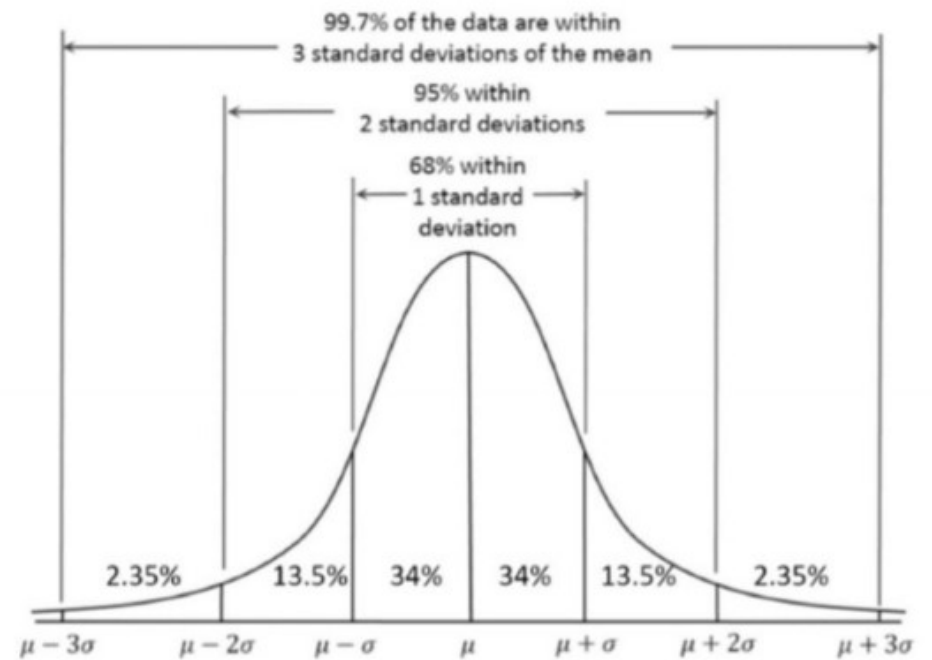
22) A normal distribution has a mean of 500 and a standard deviation of 50. Find the z-score for a data value of 400.

23 The mean diameter (specification) of a roller bearing for a large machine is 2.2 inches. The standard deviation of the bearings coming off the assembly line is .02 inches. Complete the distribution chart below.
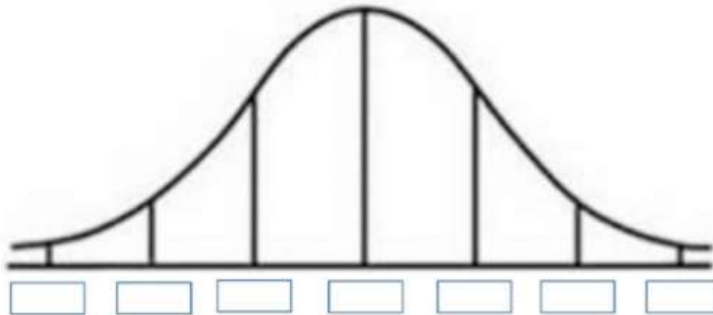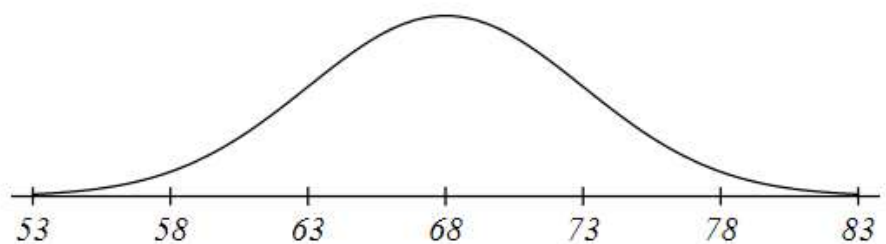
Question 1. Version 1*/1. Score: **2/2**

Which of the following statements are TRUE about the Normal Distribution? *Check all that apply.*

- ☑ The graph of the Normal Distribution is bell-shaped, with tapering tails that never actually touch the horizontal axis.
- ☑ 50% of the data values lie at or above the mean.
- ☑ The distribution is symmetric with a single peak.
- ☑ The mean, median and mode are all equal and occur at the center of the distribution.
- ☑ Data values are spread evenly around the mean.
- ☑ Data values farther from the mean are less common than data values closer to the mean.
- ☐ About 95% of all data values lie within 1 standard deviation of the mean.

⚷ ✔

Question 3.   Version 1*/1. Score: **2/2**

Based on the graph of this normal distribution,

a. The mean is [68 ✓] [⚲] .

b. The median is [68 ✓] [⚲] .

c. The mode is [68 ✓] [⚲] .

d. The standard deviation is [5 ✓] [⚲] .

The time to complete an exam is approximately Normal with a mean μ = 53 minutes and a standard deviation σ = 2 minutes.

The bell curve below represents the distribution for testing times. The scale on the horizontal axis is equal to the standard deviation. Fill in the indicated boxes.
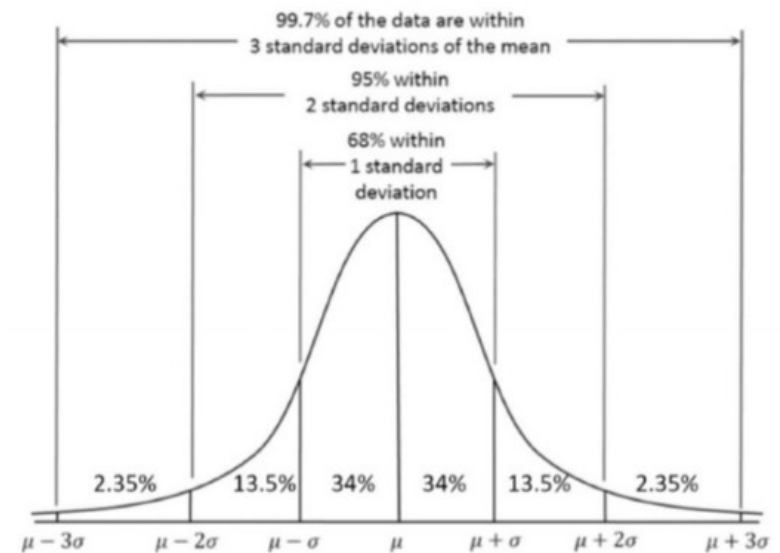
$\mu = 53$
$\sigma = 2$
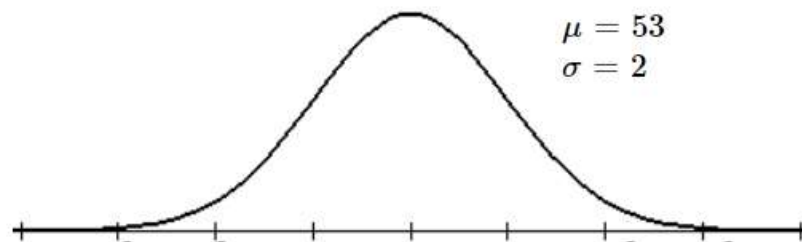
Question 6. Version 1*/1. Score: 6/6

A set of exam scores is normally distributed with a mean = 72 and standard deviation = 8.
Use the Empirical Rule to complete the following sentences.

68% of the scores are between 64 ✓ σ^6 and 80 ✓ σ^6 .

95% of the scores are between 56 ✓ σ^6 and 88 ✓ σ^6 .

99.7% of the scores are between 48 ✓ σ^6 and 96 ✓ σ^6 .

99.7% of the data are within
3 standard deviations of the mean

95% within
2 standard deviations

68% within
1 standard
deviation

2.35% 13.5% 34% 34% 13.5% 2.35%

$\mu - 3\sigma$ $\mu - 2\sigma$ $\mu - \sigma$ μ $\mu + \sigma$ $\mu + 2\sigma$ $\mu + 3\sigma$

21) The mean weight of the fish in a pond is 10 pounds with a standard deviation of 3 pounds. What is the probability that a random fish caught is between 13 and 16 pounds? (See Normal Curve Attached).

99.7% of the data are within 3 standard deviations of the mean

95% within 2 standard deviations

68% within 1 standard deviation

2.35% 13.5% 34% 34% 13.5% 2.35%

$\mu - 3\sigma$ $\mu - 2\sigma$ $\mu - \sigma$ μ $\mu + \sigma$ $\mu + 2\sigma$ $\mu + 3\sigma$