# Math 1540 Test1 Study Guide

1. Calculate the Average Rate of Change (ARC) of a function $y = f(x)$ over an interval.

   ARC = $\frac{f(x_2)-f(x_1)}{x_2-x_1}$ = $\frac{y_2-y_1}{x_2-x_1}$, For example, $y = f(x) = \frac{10}{x^2+1}$ $over\ (1,3) = \frac{\frac{10}{3^2+1}-\frac{10}{1^2+1}}{3-2} = \frac{1-5}{2} = -2$

2. Calculate limits. $\lim_{x \to x_o} f(x) = L$ using various methods.

   a. Plug-In. Example: $\lim_{x\to 2} x^3 - 2x^2 + 3 = 2^3 - 2(2^2) + 3 = 8 - 8 + 3 = 3$

   b. A function with a "hole" Example: $\lim_{x\to -5} \frac{x^2+13x+40}{x+5} = \lim_{x\to -5} \frac{(x+8)\cancel{(x+5)}}{\cancel{x+5}} = \lim_{x\to -5} x + 8 = 3$

   c. A function with a radical. Example: $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x\to 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{(x-1)}}$

   d. A function with $\sin nx. Remember \lim_{anything\to 0} \sin(anything) = anything$

   $$\lim_{x\to 0} \frac{6x + \sin 10x}{x} = \lim_{x\to 0} \frac{6x}{x} + \frac{\sin 10x}{x} = \lim_{x\to 0} 6 + \frac{10x}{x} = 16$$

   $$\lim_{x\to 0} \frac{6x^2 + \sin 5x}{x} = \lim_{x\to 0} \frac{6x^2}{x} + \frac{\sin 5x}{x} = \lim_{x\to 0} 6x + \frac{5x}{x} = 0 + 5 = 5$$

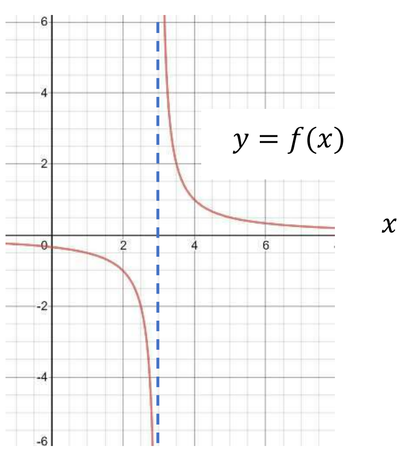
   e. Limits as x→ ∞

   $$\lim_{x\to\infty} \frac{3x^3 + 4x^2 - 2x + 1}{2x^3 - 6x^2 + x - 2} = \lim_{x\to\infty} \frac{3x^3}{2x^3} = \frac{3}{2}$$

   $$\lim_{x\to\infty} \frac{10x^2 - 4x + 3}{5x^3 + 2x + 12} = \lim_{x\to\infty} \frac{10x^2}{5x^3} = \lim_{x\to\infty} \frac{2}{x} = 0$$

   $$\lim_{x\to\infty} \frac{10x^4 + 3x^3 - 2x + 1}{2x^3 - 5x^2 + 6x - 10} = \lim_{x\to\infty} \frac{10x^4}{2x^3} = \lim_{x\to\infty} 5x = \infty$$

   f. Infinite limits (see graph)




$$\lim_{x\to 3^-} f(x) = -\infty$$

$$\lim_{x\to 3^+} f(x) = +\infty$$

3. Calculate the slope M and the equation of the tangent line of $y = f(x)$ at the point $(x_o, y_o)$. You may calculate M for any x by the formula: (You may also calculate for a specific value of $x$ by substituting a specific value for $x$.)

$$M = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example: $y = f(x) = 3x^2 - 2x + 5 \; at \; (2, 13)$

$$M = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - 2(x+h) + 5 - (3x^2 - 2x + 5)}{h}$$

$$M = \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 5 - 3x^2 + 2x - 5}{h}$$

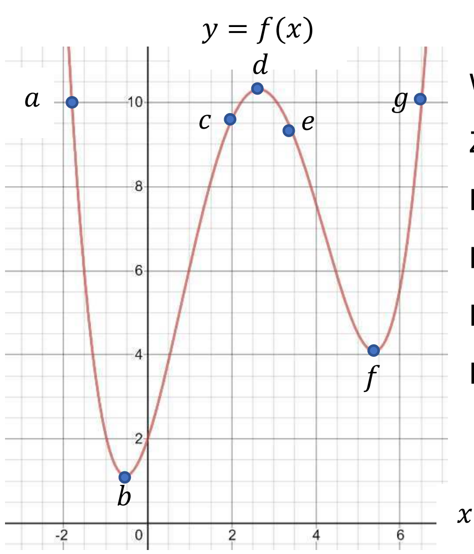
$$M = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 5 - 3x^2 + 2x - 5}{h} = \lim_{h \to 0} \frac{6xh + 3h^2 - 2h}{h}$$

$$M = \lim_{h \to 0} \frac{h(6x + 3h - 2)}{h} = 6x - 2, \qquad at\; x = 2, M = 6(2) - 2 = 10$$

$So, y = Mx + b = 10x + b, \quad must\; go\; through\; (2,13), \quad 13 = 10(2) + b, \quad b = -7$

$y = 10x - 7$ *is the equation of tangent line.*

4. Evaluate the relative values of slope (instantaneous rate of change) at a point on a graph.




Which points are described as?

Zero Slope ____

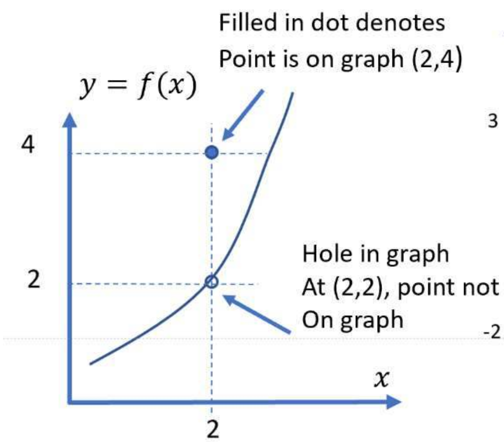
Large Negative Slope ___

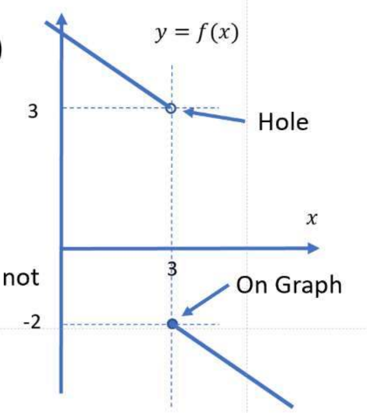
Large Positive Slope ___

Moderate Positive Slope ___

Moderate Negative Slope ___ Try first (See key at end)

5. Analyze graphs and interpret them to evaluate function values and limits.




$f(2) = 4$

$\lim_{x\to 2} f(x) = 2$

$f(3) = -2$

$\lim_{x\to 3^-} f(x) = 3$

$\lim_{x\to 3^+} f(x) = -2$

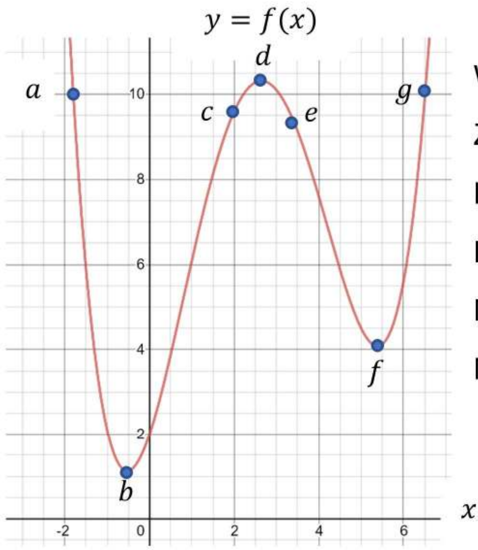
6. More Average and Instantaneous Rate of Change. On earth, an object falls a distance given by $s = 16t^2, s\ in\ feet\ and\ t\ in\ seconds$. What is the average rate of change (ARC) from 1 to 5 seconds? What is the instantaneous rate of change (IRC-velocity) at t = 5?

ARC = $\frac{16(5^2)-16(1^2)}{5-1} = \frac{400-16}{4} = 96\ ft/sec$

$$IRC = \lim_{h\to 0} \frac{s(t+h)-s(t)}{h} = \lim_{h\to 0} \frac{16(t+h)^2-16t^2}{h} = \lim_{h\to 0} \frac{16\ ^2+32th+16h^2-16t^2}{h}$$

$$= \lim_{h\to 0} \frac{h(32t+16h)}{h} = 32t, at\ t = 5, IRC = 160\ ft/sec$$

Key to slope question:




Which points are described as?

Zero Slope b, d, f

Large Negative Slope a

Large Positive Slope g

Moderate Positive Slope c

Moderate Negative Slope e Try first (See key at end)

Concepts & Skills to Know for Calculus I Test 2

1. The Derivative is:
   a) The rate of change of the function f(x) at a given value of x (or t, if it is a function of time)
   b) Also called the instantaneous rate of change.
   c) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
   d) It is also the slope of the tangent line at a given point.
   e) **<u>It is not the average rate of change</u>**
2. The speed of an object is $|v(t)|$. It is always positive.
3. Know how to calculate the derivative of a variety of functions. You may use the attached sheet which is a differentiation table. It will be attached to test 1. **Print a copy and take it with you if you are testing with ProctorU**
4. When calculating the derivative of a power of x, you should write x in the numerator before applying the formula for the derivative. $\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{d}{dx}x^{-3} = -3x^{-4} = \frac{-3}{x^4}$
5. Know how to use the product, quotient, and chain rule.
6. When you take a derivative of a composite function, be sure that you have considered every function involved and use the chain rule. You must include the derivative of every function and use the Chain Rule! $y = (x^3 + \ln x + sin^2 x)^5$

$$y' = 5(x^3 + \ln x + sin^2 x)^4 \frac{d}{dx}(x^3 + \ln x + sin^2 x)$$

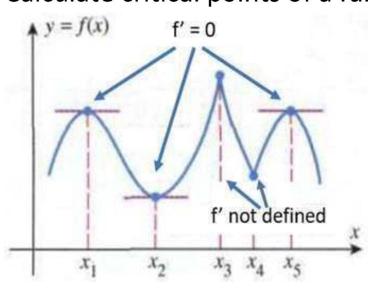
$$y' = 5(x^3 + \ln x + sin^2 x)^4 (3x^2 + \frac{1}{x} + 2\sin x \cos x)$$

7. If $s(t)$ *is an object's position*, $s'(t)$ *is the velocity* $v(t)$, *and the acceleration is* $a(t) = v'(t) = s''(t)$.
8. An object is moving forward (or upward) when v > 0, and is moving backward (or downward) when v < 0.
9. Understand implicit differentiation.
10. Understand how to calculate related rates.
11. Be able to interpret the derivative/slope of a graph at a point (+, -, 0)

Calculus I Students,

Test 3 will be April 1 and April 2 (Augusta students) and April 8 and April 9 (Swainsboro/Statesboro students). To be prepared, you should have done or should do the following:

1. Watched all the videos on Chapter 4.
   **Augusta:**
   https://www.telstarbob.net/bbrown/Math1540_daily_sch_Spring_2021AUG.htm
   **Swainsboro/Statesboro:**
   https://www.telstarbob.net/bbrown/Math1540_daily_sch_Spring_2021SWST.htm
   Completed all the homework for Chapter 4.
2. Work the practice test by yourself.
   http://telstarbob.net/bbrown/Math1540OL/CalculusITest3Practice.pdf
3. Watch Practice test 3 video if you need help on any problem.
   https://www.youtube.com/watch?v=cy92PtEIsYU&feature=youtu.be
4. Ask me any questions or even request a Zoom Video Tutoring Session if you need additional help.
5. Calculate critical points of a function (f' = 0, or f' is not defined)




6. For a function $y = f(x)$, determine intervals in which the function is increasing (y' > 0) or decreasing (y' < 0). Solve y' = 0 to determine the intervals to consider.
7. Be able to calculate any absolute max/min or relative max/min of a function. **<u>Understand that, if the function is defined over a closed interval [a, b], you must evaluate the function at the end points as well as the critical points</u>**.
8. Calculate x = c in the Mean Value Theorem that satisfies $f'(c) = \frac{f(b)-f(a)}{b-a}$
9. Define intervals on which a function is concave up (f" > 0) or concave down (f" < 0).
10. Use l'Hopital's Rule to find limits. You may have to rewrite so that you have $\frac{0}{0}$ $or$ $\frac{\infty}{\infty}$.
11. Solve max or min applications problems.
12. Calculate antiderivatives.
13. Solve initial value problems given y' or y" or position s' or s" and determine y or s as required. You solve by finding the antiderivative, adding a constant each time you integrate, and solve for the constants by using the initial values or values given at a point.

All of these ideas are illustrated in the Practice Test.

On the test, you may have this study guide as well as the derivative and anti-derivative tables at the end of this note.

Good luck,

Calculus I Students,

Test 4 will be Thursday and Friday April 30/May 1. You will take it on-line using MyMathLab. You must complete it in one session where you will have a total of two hours. To be prepared, you should have done or should do the following:

1. Watched all the videos on Chapter 5.
   http://telstarbob.net/bbrown/Math1540_daily_sch_Spring_2020-Revised.htm
2. Completed all the homework for Chapter 4.
3. Work the practice test by yourself.
   http://telstarbob.net/bbrown/Math1540OL/CalculusITest4Practice2.pdf
4. Watch Practice test 4 video if you need help on any problem.
   https://www.youtube.com/watch?v=WZ2B1bEKVIg&feature=youtu.be
5. Ask me any questions or even request a Zoom Video Tutoring Session if you need additional help.
6. Understand summation notation such as
   $\sum_{k=1}^{4} 2\cos\frac{\pi}{k}$ , $\sum_{k=0}^{4} 2(k+1)$
7. Calculate the area under a curve using approximations (see practice test)
8. Understand and use the two versions of the Fundamental Theorem of Calculus:

**The First Fundamental Theorem of Calculus**

*If f is a continuous function at every point on the interval* $[a, b]$ *and F* Is any antiderivative of f on [a, b], then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

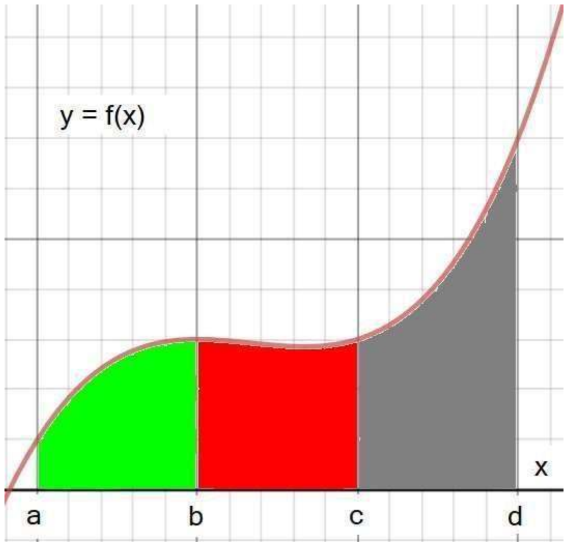
**The Second Fundamental Theorem of Calculus**

*If f is continuous function on* $[a, b]$ *then* $F(x) = \int_a^x f(t)\,dt$ *is continuous on* $[a, b]$ *and differentiable on* $(a, b)$ *and its derivative is* $f(x)$

$$F'(x) = \int_a^x f(t)\,dt = f(x)$$

**In the second theorem, if the upper limit is not x, but g(x), then you must use the chain rule and multiply by g'(x).**

9. Understand $\int_a^b f(x)dx = -\int_b^a f(x)dx$
10. Given the function pictured below, understand the properties of definite integrals:




$$If\ a < b < c < d, then \int_a^d f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx + \int_c^d f(x)dx$$

11. Understand how to calculate indefinite integrals using substitutions.

**Steps To Integrate $\int g'(x)f(g(x))dx$**

1. *Choose Substitution* $u = g(x)$
2. *Calculate* $\frac{du}{dx} = g'(x), du = g'(x)dx$
3. *Substitute* $du = g'(x)dx$ *in integral above*
4. *Integrate with respect to* $u$
5. *Substitute* $u = g(x)$

Note that a wise substitution is u = g(x) if g'(x) appears in the integral. See practice test and practice test video.

12. If you are calculating a definite integral and using substitutions, you must change the lower and upper limits using the substitution.

**Definite Integrals & Substitution**
**Change Limits of Integration Using $u = g(x)$**

$$\int_a^b g'(x)f(g(x))dx = \int_{g(a)}^{g(b)} f(u)du$$

All of these ideas are illustrated in the Practice Test.

On the test, you may have this study guide as well as the derivative and anti-derivative tables at the end of this note.

Good luck,

Dr. Brown

# Differentiation Rules

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -\csc x\ \cot x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = (\ln a)a^x$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\csc^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$$

Note: **Remember the Chain Rule! If the variable in the function is not just plain x but is u(x), you must multiply the results by** $\frac{du}{dx}$.

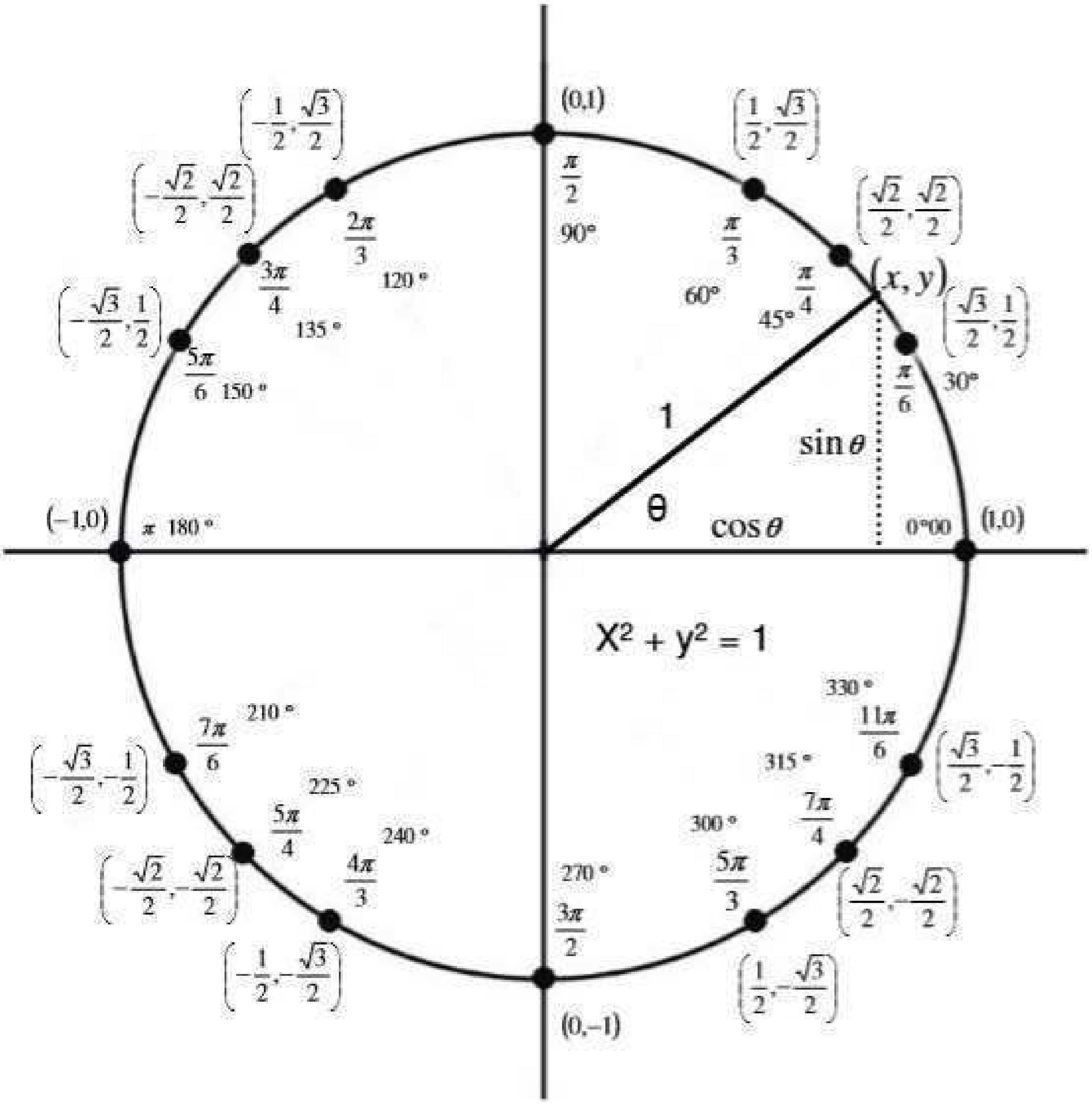
Product Rule: $(uv)' = uv' + vu'$

Quotient Rule: $\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

**TABLE 4.2** Antiderivative formulas, $k$ a nonzero constant

| | Function | General antiderivative |
|---|---|---|
| **1.** | $x^n$ | $\frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$ |
| **2.** | $\sin kx$ | $-\frac{1}{k}\cos kx + C$ |
| **3.** | $\cos kx$ | $\frac{1}{k}\sin kx + C$ |
| **4.** | $\sec^2 kx$ | $\frac{1}{k}\tan kx + C$ |
| **5.** | $\csc^2 kx$ | $-\frac{1}{k}\cot kx + C$ |
| **6.** | $\sec kx \tan kx$ | $\frac{1}{k}\sec kx + C$ |
| **7.** | $\csc kx \cot kx$ | $-\frac{1}{k}\csc kx + C$ |

| | Function | General antiderivative |
|---|---|---|
| **8.** | $e^{kx}$ | $\frac{1}{k}e^{kx} + C$ |
| **9.** | $\frac{1}{x}$ | $\ln\lvert x\rvert + C, \quad x \neq 0$ |
| **10.** | $\frac{1}{\sqrt{1-k^2x^2}}$ | $\frac{1}{k}\sin^{-1} kx + C$ |
| **11.** | $\frac{1}{1+k^2x^2}$ | $\frac{1}{k}\tan^{-1} kx + C$ |
| **12.** | $\frac{1}{x\sqrt{k^2x^2-1}}$ | $\sec^{-1} kx + C, \quad kx > 1$ |
| **13.** | $a^{kx}$ | $\left(\frac{1}{k\ln a}\right)a^{kx} + C, \quad a > 0, \ a \neq 1$ |




# The Unit Circle