Solve the problem.

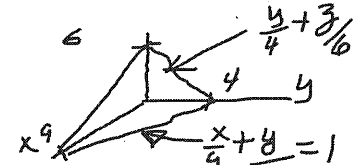
1) Write an iterated triple integral in the order dx dy dz for the volume of the tetrahedron cut from the first octant by the plane $\frac{x}{9} + \frac{y}{4} + \frac{z}{6} = 1$.

A) $\int_0^6 \int_0^{9(1 - y/4)} \int_0^{9(1 - y/4 - z/6)} dx\, dy\, dz$

B) $\int_0^6 \int_0^{4(1 - z/6)} \int_0^{9(1 - y/4 - z/6)} dx\, dy\, dz$

C) $\int_0^6 \int_0^{1 - z/6} \int_0^{1 - y/4 - z/6} dx\, dy\, dz$

D) $\int_0^6 \int_0^{1 - y/4} \int_0^{1 - y/4 - z/6} dx\, dy\, dz$



$\frac{y}{4} + \frac{z}{6} = 1$ $\quad y = 4(1 - z/6)$

$\frac{x}{9} + \frac{y}{4} = 1$

x goes from 0 to ?

Tet: $\frac{x}{9} + \frac{y}{4} + \frac{z}{6} = 1$, $x = 9(1 - \frac{y}{4} - \frac{z}{6})$

y goes from 0 to $4(1 - z/6)$

z goes from 0 to 6

$\int_0^6 \int_0^{4(1-z/6)} \int_0^{4(1 - \frac{y}{4} - \frac{z}{6})} dx\,dy\,dz$

B

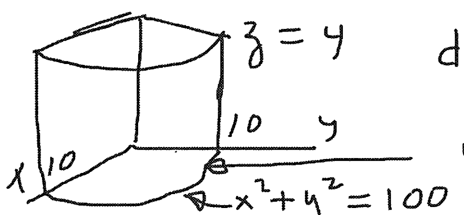
2) Write an iterated triple integral in the order dz dy dx for the volume of the region in the first octant enclosed by the cylinder $x^2 + y^2 = 100$ and the plane $z = 4$.

A) $\int_0^{10} \int_0^{\sqrt{100 - x^2}} \int_0^{y} dz\, dy\, dx$

B) $\int_0^{10} \int_0^{\sqrt{100 - x^2}} \int_0^{4} dz\, dy\, dx$

C) $\int_0^{10} \int_0^{\sqrt{100 - y^2}} \int_0^{4 - y} dz\, dy\, dx$

D) $\int_0^{10} \int_0^{\sqrt{100 - y^2}} \int_0^{4} dz\, dy\, dx$



$dz\, dy\, dx$

$y = \sqrt{100 - x^2}$

$\int_0^{10} \int_0^{\sqrt{100-x^2}} \int_0^{4} 1\, dz\, dy\, dx$

3) Rewrite the integral

$$\int_0^{1/2}\int_0^{(1-2z)/9}\int_0^{(1-9y-2z)/6} dx\, dy\, dz$$

in the order dz dy dx.

A) $\displaystyle\int_0^{1/2}\int_0^{(1-2z)/9}\int_0^{(1-9y-2z)/6} dz\, dy\, dx$

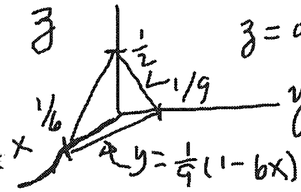
B) $\displaystyle\int_0^{1/6}\int_0^{(1-6x)/9}\int_0^{(1-6x-9y)/2} dz\, dy\, dx$

C) $\displaystyle\int_0^{1/2}\int_0^{(1-6x)/9}\int_0^{(1-2x-9y)/6} dz\, dy\, dx$

D) $\displaystyle\int_0^{1/6}\int_0^{(1-6x)/9}\int_0^{(1-9x-6y)/2} dz\, dy\, dx$

$6x = 1 - 9y - 2z$

$z = \frac{1}{2}(1 - 6x - 9y)$



$z = 0$ to $\frac{1}{2}(1-6x-9y)$

$1 - 9y - 2z = 0$

x — 0 to $1/6$

$$\int_0^{1/6}\int_0^{\frac{(1-6x)}{9}}\int_0^{\frac{(1-6x-9y)}{2}} dz\, dy\, dx$$

Evaluate the integral.

4) $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_0^{3x+6y} dz\,dx\,dy$

A) 9 B) 27 C) 243 D) 81

$$= \int_0^3 \int_0^{\sqrt{9-y^2}} (3x+6y)\,dx\,dy = \int_0^3 \left(\frac{3x^2}{2} + 6yx\right)\Bigg|_0^{\sqrt{9-y^2}} dy = \int_0^3 \frac{3}{2}(9-y^2) + 6y(9-y^2)^{1/2}\,dy$$

$$\left[\frac{3}{2}\left(9y - \frac{y^3}{3}\right) - 2(9-y^2)^{3/2}\right]\Bigg|_0^3 = \frac{3}{2}(27-9) - 0 + 54 = 27 + 54 = 81$$

5) $\int_{-1}^{1}\int_{0}^{5}\int_{0}^{1} (x^2 + y^2 + z^2)\, dx\, dy\, dz$

A) 124 B) 23.2 C) 126 D) 90

$$\int_{-1}^{1}\int_{0}^{5}\left(\frac{x^3}{3} + y^2x + z^2x\right)\Big|_0^1 dy\,dz = \int_{-1}^{1}\int_{0}^{5}\left(\frac{1}{3} + y^2 + z^2\right)dy\,dz = \int_{-1}^{+1}\left(\frac{y}{3} + \frac{y^3}{3} + z^2y\right)\Big|_0^5$$

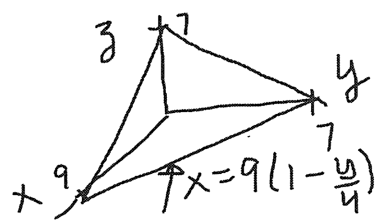
$$\int_{-1}^{+1}\left(\frac{5}{3} + \frac{125}{3} + 5z^2\right)dz = \left(\frac{130z}{3} + \frac{5z^3}{3}\right)\Big|_{-1}^{1} = \frac{130}{3} + \frac{5}{3} \quad -\left(\frac{-130}{3} - \frac{5}{3}\right)$$

$$= \frac{260}{3} + \frac{10}{3} = \frac{270}{3} = 90 \text{ D}$$

Find the volume of the indicated region.

6) the tetrahedron cut off from the first octant by the plane $\frac{x}{9} + \frac{y}{4} + \frac{z}{7} = 1$

A) 84 B) 63 C) 42 D) 126



$$V=\int_0^7\int_0^{9(1-\frac{y}{4})}\int_0^{7(1-\frac{x}{9}-\frac{y}{4})} dz\,dx\,dy$$

$$=\int_0^7\int_0^{9(1-\frac{y}{4})}\left(7-\frac{7x}{9}-\frac{7y}{4}\right)dx\,dy$$

$$=\int_0^7 \left(7x-\frac{7x^2}{18}-\frac{7yx}{4}\right)\Big|_0^{9(1-\frac{y}{4})} dy=\int_0^7 63\left(1-\frac{y}{4}\right)-\frac{7}{18}\left(1-\frac{y}{4}\right)^2-\frac{63y}{4}\left(1-\frac{y}{4}\right)\,dy$$

$$\left[63y-\frac{63y^2}{8}+\frac{28}{54}\left(1-\frac{y}{4}\right)^3-\frac{63y^2}{8}+\frac{63y^3}{48}\right]\Big|_0^7$$

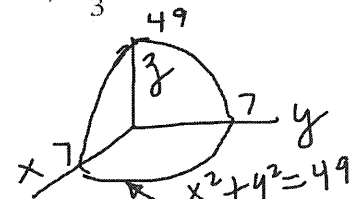
$$63(7)-\frac{63(49)}{8}-\frac{\cancel{28}^{7}}{\cancel{54}_{2}}\left(\frac{\cancel{27}}{\cancel{64}_{16}}\right)-\frac{63(49)}{8}+\frac{\cancel{63}^{21}(7)49}{\cancel{48}_{16}}$$

$$\to 441-\frac{3087}{4}-\frac{7}{32}+\frac{7203}{16}$$

Find the volume of the indicated region.

7) the region bounded by the paraboloid $z = 49 - x^2 - y^2$ and the xy–plane

A) $\frac{343}{3}\pi$ B) $\frac{343}{2}\pi$ C) $\frac{2401}{2}\pi$ D) $\frac{2401}{3}\pi$



$$V = 4\int_0^7 \int_0^{\sqrt{49-x^2}} \int_0^{49-x^2-y^2} dz\,dy\,dx$$

$$= 4\int_0^7 \int_0^{\sqrt{49-x^2}} (49-x^2-y^2)\,dy\,dx = 4\int_0^7 \left[(49-x^2)y - \frac{y^3}{3}\right]\Bigg|_0^{\sqrt{49-x^2}} dx$$

$$= 4\int_0^7 (49-x^2)^{3/2} - \frac{(49-x^2)^{3/2}}{3} \cancel{dx} = \frac{8}{3}\int_0^7 (49-x^2)^{3/2}\,dx = 3771.48$$

$$= \frac{2401\pi}{2}$$

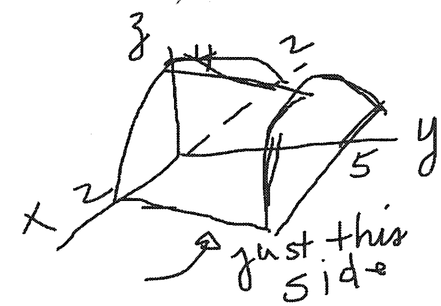
8) the region bounded by the coordinate planes, the parabolic cylinder $z = 4 - x^2$, and the plane $y = 5$

A) $\frac{80}{3}$ B) 80 C) 30 D) 60

$$V = \int_0^2 \int_0^5 \int_0^{4-x^2} dz\,dy\,dx = \int_0^2 \int_0^5 (4-x^2)\,dy\,dx$$

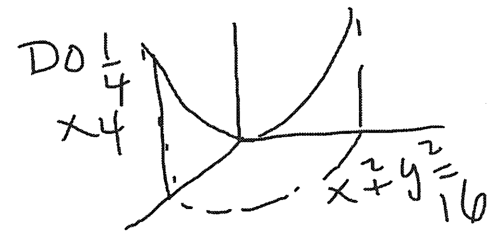
$$= \int_0^2 5(4-x^2)\,dx = 5\left(4x - \frac{x^3}{3}\right)\Big|_0^2$$

$$= 5\left(8 - \tfrac{8}{3}\right) = 5\left(\frac{16}{3}\right) = \frac{80}{3} \quad A$$



9) the region bounded by the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 16$

A) $\frac{1024}{3}\pi$ B) 128π C) 384π D) $\frac{256}{3}\pi$



$$4\int_0^4\int_0^{\sqrt{16-x^2}}\int_0^{x^2+y^2} dz\,dy\,dx$$

$$=4\int_0^4\int_0^{\sqrt{16-x^2}}(x^2+y^2)\,dy\,dz=4\int_0^4\left(x^2y+\frac{y^3}{3}\right)\Big|_0^{\sqrt{16-x^2}}$$

$$=4\int_0^4 x^2\sqrt{16-x^2}+(16-x^2)^{3/2}\,dx$$

$$=4(100.53)=128\pi\ B$$

Find the average value of F(x, y, z) over the given region.

10) $F(x, y, z) = xyz$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 5$, $y = 10$, $z = 7$

A) $\frac{175}{8}$ B) $\frac{175}{4}$ C) $\frac{175}{9}$ D) $\frac{175}{6}$

The volume of the cube $= 5 \cdot 10 \cdot 7 = 350$, Now we integrate $\int_0^7 \int_0^{10} \int_0^5 xyz \, dx\, dy\, dz$

$$= \int_0^7 \int_0^{10} \frac{x^2}{2} yz \Big|_0^5 dy\, dz = \int_0^7 \int_0^{10} \frac{25}{2} yz \, dy\, dz = \int_0^7 \frac{25y^2 z}{4} \Big|_0^{10} dz = \int_0^7 \frac{2500}{4} z \, dz$$

$$= \frac{2500z^2}{8} \Big|_0^7 = \frac{2500(49)}{8}, \text{ AVG VALUE} = \frac{2500(49)}{8(350)} = \frac{175}{4} \quad \text{B}$$

Find the average value of F(x, y, z) over the given region.

11) $F(x, y, z) = x^4 y^3 z^6$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 1, y = 1, z = 1$

A) $\frac{1}{72}$ B) $\frac{1}{126}$ C) $\frac{1}{54}$ D) $\frac{1}{140}$

Volume of cube $= 1 \cdot 1 \cdot 1 = 1$ So we just calculate $\iiint_R f(x,y,z)\,dx\,dy\,dz$

$$= \int_0^1\int_0^1\int_0^1 x^4 y^3 z^6\,dx\,dy\,dz = \int_0^1\int_0^1 \frac{x^5}{5} y^3 z^6 \Big|_0^1 dy\,dz = \int_0^1\int_0^1 \frac{y^3 z^6}{5}\,dy\,dz = \int_0^1 \frac{y^4 z^6}{20}\Big|_0^1 dz = \int_0^1 \frac{z^6}{20}\,dz$$

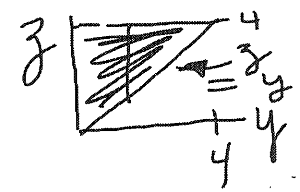
$$= \frac{z^7}{140}\Big|_0^1 = \frac{1}{140}$$ D

Evaluate the integral by changing the order of integration in an appropriate way.

12) $\displaystyle\int_0^1 \int_0^4 \int_y^4 \frac{x \sin z}{z}\, dz\, dy\, dx$

A) $\frac{1 - \cos 4}{2}$ B) $1 + \cos 4$ C) $\frac{1 + \sin 4}{2}$ D) $1 - \sin 4$

$$\int_0^1 \int_0^4 \int_0^z \frac{x \sin z}{z}\, dy\, dz\, dx$$



$$= \int_0^1 \int_0^4 \frac{x y \sin z}{z} \Big|_0^z dz\, dx = \int_0^1 \int_0^4 x \sin z\, dz\, dx$$

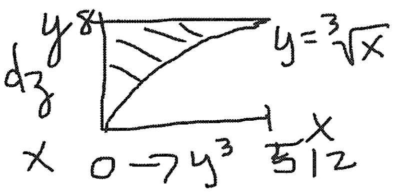
$$= \int_0^1 -x \cos z \Big|_0^4 dx = \int_0^1 (x - x\cos 4)\, dx = \frac{x^2}{2}(1 - \cos 4)\Big|_0^1$$

$$= \frac{1}{2}(1 - \cos 4)$$

13) $\int_0^{512} \int_0^{10} \int_{\sqrt[3]{x}}^{8} \frac{z}{y^4+1}\, dy\, dz\, dx$

A) 25 ln 4097 B) 25 ln 513 C) $\frac{25}{2}$ ln 4097 D) $\frac{25}{2}$ ln 513

$$\int_0^{10}\int_0^{8}\int_0^{y^3} \frac{z}{y^4+1}\, dx\, dy\, dz$$



$$= \int_0^{10}\int_0^{8} \frac{zy^3}{y^4+1}\, dy\, dz$$

$u = y^4+1$
$du = 4y^3\, dy$
$y^3\, dy = \frac{du}{4}$

$$= \frac{25}{2}\ln(4097)$$

$$= \int_0^{10} \frac{z}{4}\ln(y^4+1)\Big|_0^8\, dz = \int_0^{10} \frac{z}{4}\ln(8^4+1)\, dz = \frac{z^2}{8}\ln(8^4+1)\Big|_0^{10} = \frac{100}{8}\ln(8^4+1)$$

Evaluate the line integral along the curve C.

14) $\int_C (y + z)\, ds$, C is the straight–line segment $x = 0$, $y = 5 - t$, $z = t$ from $(0, 5, 0)$ to $(0, 0, 5)$

A) $25\sqrt{2}$ B) $\frac{25}{2}$ C) 0 D) 25

$r(t) = 0i + (5-t)j + tk$, $v(t) = -j + k$, $|v| = \sqrt{2}$

$\int_C (y+z)\, ds = \int_0^5 (5-t+t)\sqrt{2}\, dt = \int_0^5 5\sqrt{2}\, dt = 25\sqrt{2}$

Evaluate the line integral

15) $\int_C \frac{x + y + z}{5}\, ds$, C is the curve $r(t) = 4t\mathbf{i} + (8 \cos \frac{3}{8}t)\mathbf{j} + (8 \sin \frac{3}{8}t)\mathbf{k}$, $0 \le t \le \frac{8}{3}\pi$

A) $\frac{128}{9}\pi^2 + \frac{128}{3}$ B) $\frac{128}{9}\pi^2 + \frac{256}{3}$ C) $\frac{128}{9}\pi$ D) $\frac{128}{9} + \frac{128}{3}$

$$\int_C \frac{x+y+z}{5}\, ds = \int \frac{\left(4t + 8\cos\frac{3}{8}t + 8\sin 3/8\, t\right)}{5} |v(t)|\, dt$$

$$r(t) = 4t\,i + 8\cos 3/8\, t\, j + 8\sin 3/8\, t\, k$$

$$v(t) = 4i - 3\sin\tfrac{3}{8}t\, j + 3\cos\tfrac{3}{8}t\, k$$

$$|v(t)| = \sqrt{16 + 9\sin^2 3/8\, t + 9\cos^2\tfrac{3}{8}t} = \sqrt{25} = 5$$

$$= \int_0^{8/3\pi} \left(4t + 8\cos 3/8\, t + 8\sin 3/8\, t\right) dt =$$

$$\left(2t^2 + \frac{64}{3}\sin\frac{3}{8}t - \frac{64}{3}\cos 3/8\, t\right)\Big|_0^{\frac{8\pi}{3}} = \frac{2(64\pi^2)}{9} + 0 + \frac{64}{3} + \frac{64}{3} = \frac{128\pi^2}{9} + \frac{128}{3}$$

A

16) $\int_C (y + z)\, ds$, C is the path from (0, 0, 0) to (−3, 3, 1) given by:

$C_1: r(t) = -3t^2 i + 3tj,\ 0 \le t \le 1$

$C_2: r(t) = -3i + 3j + (t-1)k,\ 1 \le t \le 2$

A) $\frac{25}{2}$ B) $\frac{15}{4}\sqrt{5} + \frac{11}{4}$ C) $\frac{15}{4}\sqrt{5} - \frac{11}{4}$ D) $\frac{13}{12}$

C_1 $v(t) = -6t\,i + 3j$ $|v| = \sqrt{36t^2+9}$ C_2 $v(t) = k$ $|v(t)| = 1$

$C_1: \int_C (y+z)\,ds = \int_0^1 (3t+0)\sqrt{36t^2+9}\,dt = \int_0^1 3t\sqrt{36t^2+9}\,dt$ $u = 36t^2+9$, $du = 72t\,dt$

$\int_{C_2} = \int_1^2 (3 + t - 1)(1)\,dt = \int_1^2 2+t\,dt$
$= (2t + \frac{t^2}{2})\Big|_1^2 =$
$= 4 + 2 - 2 - \frac{1}{2} = \frac{7}{2}$

$\int_4^9 \frac{3}{72} u^{1/2}\,du = \frac{3}{36\cancel{72}} \frac{u^{3/2}}{3/\cancel{2}} = \frac{u^{3/2}}{36}\Big|_9^{45} = \frac{(45)^{3/2}}{36} - \frac{27}{36} = \frac{(3\sqrt{5})^3}{36} - \frac{27}{36} = \frac{27(5)\sqrt{5}}{36} - \frac{27}{36}$

$= \frac{15\sqrt{5}}{4} - \frac{3}{4}$ $C_2: \int_C (y+z)\,ds = 6$ from above $\oint_C (y+z)\,ds = \frac{15}{4}\sqrt{5} - \frac{3}{4} + \frac{7}{2} = \frac{15\sqrt{5}}{4} + \frac{11}{4}$ B

17) $\displaystyle\int_C \frac{1}{x^2 + y^2 + z^2}\, ds$, C is the path given by:

C_1: $\mathbf{r}(t) = (5 \cos t)\mathbf{i} + (5 \sin t)\mathbf{j}$ from (5, 0, 0) to (0, 5, 0) $\quad 0 \to \pi/2 \quad z = 0$

C_2: $\mathbf{r}(t) = (5 \sin t)\mathbf{j} + (5 \cos t)\mathbf{k}$ from (0, 5, 0) to (0, 0, 5) $\quad \pi/2 \to 0 \quad x = 0$

C_3: $\mathbf{r}(t) = (5 \sin t)\mathbf{i} + (5 \cos t)\mathbf{k}$ from (0, 0, 5) to (5, 0, 0) $\quad 0 \to \pi/2 \quad y = 0$

A) $\frac{3}{10}\pi$ B) $\frac{\pi}{10}$ C) 0 D) $-\frac{3}{10}\pi$

C_1, $v(t) = -5\sin t\, i + 5\cos t\, j$, $|v| = 5$, $C_2 = 5\sin t\, i + 5\cos t\, j$, $|v| = 5$, C_3 $|v| = 5$ also

C_1 $\displaystyle\int_{C_1} = \int_0^{\frac{\pi}{2}} \frac{1}{25\cos^2 t + 25\sin^2 t}(5)\,dt = \int_0^{\frac{\pi}{2}} \frac{1}{5}\,dt = \frac{\pi}{10}$ $\quad C_2$ $\displaystyle\int_{C_2} = \int_{\frac{\pi}{2}}^{0} \frac{1}{5}\,dt = -\frac{\pi}{10}$

C_3 $\displaystyle\int_{C_2} = \int_0^{\frac{\pi}{2}} \frac{1}{5}\,dt = \frac{\pi}{10}$ $\quad$ so total $\displaystyle\int_C = \frac{\pi}{10} - \frac{\pi}{10} + \frac{\pi}{10} = \frac{\pi}{10}$ B

Evaluate the line integral of f(x,y) along the curve C.

18) $f(x, y) = \frac{x^4}{\sqrt{1 + 4y}}$, C: $y = x^2$, $0 \le x \le 1$

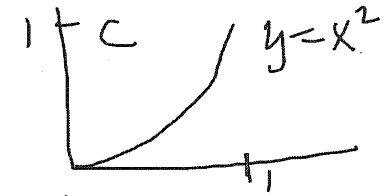
A) $\frac{1}{4}$ B) 1 C) 0 D) $\frac{1}{5}$

What is $r(t)$?

let $x = t$ $y = t^2$

$r(t) = t\,i + t^2 j$

$v(t) = 1\,i + 2t\,j$ $|v(t)| = \sqrt{1+4t^2}$



$$\int_C f(x,y)\,ds = \int_0^1 \frac{t^4}{\cancel{\sqrt{1+4t^2}}} \cancel{\sqrt{1+4t^2}}\,dt = \int_0^1 t^4\,dt = \frac{t^5}{5}\Big|_0^1 = \frac{1}{5}$$ D

Evaluate the line integral of f(x,y) along the curve C.

19) $f(x, y) = \cos x + \sin y$, C: $y = x$, $0 \le x \le \frac{\pi}{2}$

A) 0 B) $\sqrt{2}$ C) $2\sqrt{2}$ D) 2

$r(t) = t\,i + t\,j \quad 0 \le t \le \pi/2$

$(x = y) \quad v(t) = i + j \quad |v| = \sqrt{2}$

$$\int_C f(x,y)\,ds = \int_0^{\frac{\pi}{2}} (\cos t + \sin t)\sqrt{2}\,dt$$

$$= (\sin t - \cos t)\sqrt{2}\,\Big|_0^{\frac{\pi}{2}} = \sqrt{2}\,(1 - 0 - (0 - 1)) = 2\sqrt{2} \quad C$$

Find the center of mass of the wire that lies along the curve r and has density δ.

20) $\mathbf{r}(t) = (4 + 2t)\mathbf{i} + \mathbf{j} + 3t\mathbf{k},\ 0 \le t \le 1;\ \delta(x, y, z) = x + z^2$

A) $\left(\frac{251}{48}, 0, \frac{59}{32}\right)$ B) $\left(\frac{251}{48}, 1, \frac{59}{32}\right)$ C) $\left(\frac{251}{6}, 8, 177\right)$ D) $(502, 0, 177)$

Note $v(t) = 2i + 3k \quad |v| = \sqrt{13}$

$\bar{x} = \frac{M_{yz}}{M} \quad \bar{y} = \frac{M_{xz}}{M},\ \bar{z} = \frac{M_{xy}}{M} \qquad M = \int_C \delta(x,y,z)\,ds$

$M = \int_0^1 (4+2t+9t^2)\sqrt{13}\,dt = \sqrt{13}(4t + t^2 + 3t^3)\Big|_0^1 = 8\sqrt{13}$

$M_{yz} = \int_C x\,\delta(x,y)\,ds = \int (x^2 + z^2 x)\sqrt{13}\,dt = \sqrt{13}\int_0^1 (4+2t)^2 + 9t^2(4+2t)\,dt$

$M_{yz} = \sqrt{13}\int_0^1 16 + 16t + 4t^2 + 36t^2 + 18t^3\,dt = \sqrt{13}\left[16t + 8t^2 + \frac{40t^3}{3} + \frac{18t^4}{4}\right]\Big|_0^1$

$= \sqrt{13}\left[16 + 8 + \frac{40}{3} + \frac{9}{2}\right] = \frac{\sqrt{13}(251)}{6},\ \bar{x} = \frac{\frac{\sqrt{13}(251)}{6}}{8\sqrt{13}} = \frac{251}{48}$

$$y=1,\ x=4+2t,\ z=3t$$

$$M_{xy} = \int_C y\,\delta(x,y)\,ds = \int (xy + yz^2)\,ds = \int \sqrt{13}(4+2t+9t^2)\,dt$$

$$= \sqrt{13}(4t + t^2 + 3t^3)\Big|_0^1 = \sqrt{13}(8),\ \bar{y} = \frac{\sqrt{13}\ 8}{8\sqrt{13}} = 1$$

$$M_{xy} = \int_C z\,\delta(x,y)\,ds = \int_C (zx + z^3)\,ds = \int_0^1 \sqrt{13}(12t + 6t^2 + 27t^3)\,dt$$

$$= \sqrt{13}\left(6t^2 + 2t^3 + \frac{27t^4}{4}\right)\Big|_0^1 = \sqrt{13}\left(6+2+\frac{27}{4}\right) = \sqrt{13}\left(\frac{59}{4}\right)$$

$$\bar{z} = \frac{\sqrt{13}\left(\frac{59}{4}\right)}{8\sqrt{13}} = \frac{59}{32}$$ (wow, not hard but long)

Find the mass of the wire that lies along the curve r and has density δ.

21) $\mathbf{r}(t) = (8\cos t)\mathbf{i} + (8\sin t)\mathbf{j} + 8t\mathbf{k}$, $0 \le t \le 2\pi$; $\delta = 2$

A) 4π units B) $16\pi\sqrt{2}$ units C) $256\pi\sqrt{2}$ units D) $32\pi\sqrt{2}$ units

$$M = \int_C \delta(x, y), \quad v(t) = -8\sin t\, i + 8\cos t\, j + 8k =$$

$$\sqrt{64\sin^2 t + 64\cos^2 t + 64} = \sqrt{2(64)} = 8\sqrt{2}$$

$$M = \int_0^{2\pi} 2(8\sqrt{2})\, dt = 16\sqrt{2}(2\pi)$$

$$= 32\sqrt{2}\,\pi \quad D$$

Find the work done by F over the curve in the direction of increasing t.

22) $\mathbf{F} = -6y\mathbf{i} + 6x\mathbf{j} + 9z^3\mathbf{k}$; C: $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j}$, $0 \le t \le 7$

A) W = 0 B) W = 84 C) W = 147 D) W = 42

$$W = \int_C \mathbf{F} \cdot \mathbf{T}\, ds = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \frac{d\mathbf{r}}{dt}$$

$$W = \int_0^7 \left[-6\sin t\, i + 6\cos t + 0k\right] \cdot \underbrace{\left[-\sin t\, i + \cos t\, j + 0k\right]}_{\frac{dr}{dt}} dt$$

$$W = \int_0^7 (6\sin^2 t + 6\cos^2 t)\, dt = \int_0^7 6\, dt = 42 \quad D$$

Calculate the circulation of the field F around the closed curve C.

23) $\mathbf{F} = xy\mathbf{i} + 3\mathbf{j}$, curve C is $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$, $0 \le t \le 2\pi$

A) 0 B) 6 C) $\frac{26}{3}$ D) $\frac{10}{3}$

Circulation/Flow $= \int_a^a F \cdot T\, ds = \int_a^b F \cdot \frac{dr}{dt} dt$ $\quad x = 2\cos t$, $y = 2\sin t$

$F = 4\sin t\cos t + 3j$, $\frac{dr}{dt} = -2\sin t\, i + 2\cos t\, j$

Flow/Cir $= \int_0^{2\pi} (4\sin t\cos t(i) + 3j) \cdot (-2\sin t\, i + 2\cos t\, j)\, dt$

$= \int_0^{2\pi} \underbrace{-8\sin^2 t\cos t\, i}_{\text{odd function integral} = 0} + \underbrace{6\cos t}_{\text{integral} = 0}\, dt = 0$!

Calculate the flux of the field F across the closed plane curve C.

24) $\mathbf{F} = y^3\mathbf{i} + x^2\mathbf{j}$; the curve C is the closed counterclockwise path formed from the semicircle $\mathbf{r}(t) = 5\cos t\mathbf{i} + 5\sin t\mathbf{j}$, $0 \le t \le \pi$, and the straight line segment from $(-5, 0)$ to $(5, 0)$

A) $-\frac{50}{3}$ B) $\frac{50}{3}$ C) $\frac{100}{3}$ D) 0

$$\text{FLUX} = \oint M\,dy - N\,dx = \int \left(M\frac{dy}{dt} - N\frac{dx}{dt}\right) dt$$

$$C_1 \begin{cases} x = 5\cos t & y = 5\sin t & M = y^3 = 125\sin^3 t \\ \frac{dx}{dt} = -5\sin t & \frac{dy}{dt} = 5\cos t & N = x^2 = 25\cos^2 t \end{cases}$$

$$C_2 \begin{cases} x = 5t & -1 \le t \le 1 & M = y^3 = 0 \\ y = 0 & dx = 5\,dt & N = x^2 \\ & dy = 0 & = 25t^2 \end{cases}$$

$$\oint = \int_{C_1} M\frac{dy}{dt} - N\frac{dx}{dt}\,dt + \int_{C_2}\left(M\frac{dy}{dt} - N\frac{dx}{dt}\right)dt$$

$$= \int_0^{\pi}\left[125\sin^3 t(5\cos t) - 25\cos^2 t(-5\sin t)\right]dt + \int_{-1}^{+1} 0 - 25t^2(5)\,dt$$

$$= \int_0^{\pi}\left(625\sin^3 t\cos t + 125\cos^2 t\sin t\right)dt - \int_{-1}^{+1} 125t^2\,dt$$

do on calculator or use substitution

$$= 83.333 - \left.\frac{125t^3}{3}\right|_{-1}^{+1} = \begin{matrix}83.333\\ -83.333\end{matrix} = 0$$

Calculate the flow in the field F along the path C.

25) $\mathbf{F} = y^2\mathbf{i} + z\mathbf{j} + x\mathbf{k}$; C is the curve $\mathbf{r}(t) = (2 + 2t)\mathbf{i} + 3t\mathbf{j} - 3t\mathbf{k}$, $0 \le t \le 1$

A) $\frac{9}{2}$ B) $-\frac{15}{2}$ C) 39   D) -3

$$\text{Flow} = \oint F \cdot \frac{dr}{dt}\, dt, \quad F = 9t^2 i - 3t j + (2+2t) k$$

$$\frac{dr}{dt} = 2i + 3j - 3k, \quad F \cdot \frac{dr}{dt} = 18t^2 - 9t - 6 - 6t$$

$$\text{FLOW} = \int_0^1 (18t^2 - 15t - 6)\, dt = \left(6t^3 - \frac{15t^2}{2} - 6t\right)\Big|_0^1$$

$$= 6 - 6 - \frac{15}{2} = -\frac{15}{2} \quad B$$

Find the gradient field of the function.

26) $f(x, y, z) = x^7y^8 + \frac{x^3}{z^4}$

A) $\nabla f = (7x^6 + 3x^2)\mathbf{i} + 8y^7\mathbf{j} - \frac{4}{z^5}\mathbf{k}$

B) $\nabla f = \left(7x^6y^8 + \frac{3x^2}{z^4}\right)\mathbf{i} + 8x^7y^7\mathbf{j} - \frac{4x^3}{z^5}\mathbf{k}$

C) $\nabla f = 7x^6y^8\mathbf{i} + 8x^7y^7\mathbf{j} - \frac{4x^7}{z^5}\mathbf{k}$

D) $\nabla f = (7x^6 + 3x^2)\mathbf{i} + 8y^7\mathbf{j} + \frac{4}{z^5}\mathbf{k}$

$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$

$\nabla f = (7x^6y^8 + \frac{3x^2}{z^4}) i + 8x^7y^7 j - \frac{4x^3}{z^5} k$

B

Find the gradient field of the function.

27) $f(x, y, z) = e^{x^6} + y^5 + z^3$

A) $\nabla f = 6x^5e^{x^6}\mathbf{i} + 5y^4e^{y^5}\mathbf{j} + 3z^2e^{z^3}\mathbf{k}$

B) $\nabla f = x^5e^{x^6} + y^5 + z^3\mathbf{i} + y^4e^{x^6} + y^5 + z^3\mathbf{j} + z^2e^{x^6} + y^5 + z^3\mathbf{k}$

C) $\nabla f = 6x^5e^{x^6} + y^5 + z^3\mathbf{i} + 5y^4e^{x^6} + y^5 + z^3\mathbf{j} + 3z^2e^{x^6} + y^5 + z^3\mathbf{k}$

D) $\nabla f = x^6e^{x^6} + y^5 + z^3\mathbf{i} + y^5e^{x^6} + y^5 + z^3\mathbf{j} + z^3e^{x^6} + y^5 + z^3\mathbf{k}$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$$

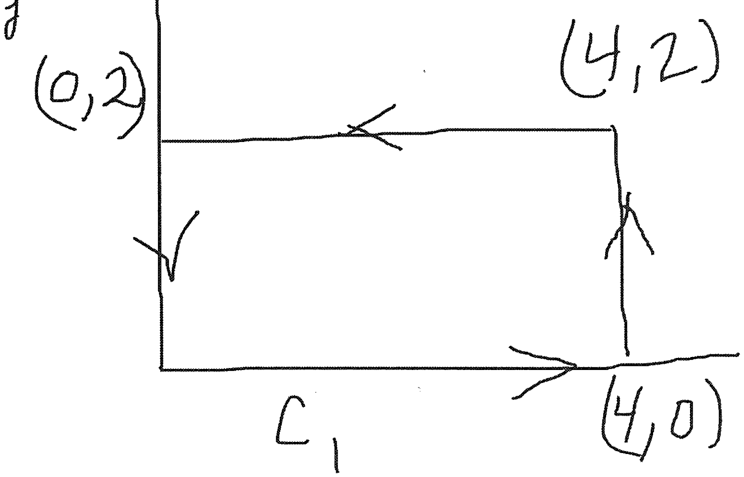
$$= 6x^5 e^{x^6+y^5+z^3} i + 5y^4 e^{x^6+y^5+z^3} j + 3z^2 e^{x^6+y^5+z^3} k \qquad C$$

Calculate the circulation of the field F around the closed curve C.

28) $\mathbf{F} = x^2y^3\mathbf{i} + x^2y^3\mathbf{j}$; curve C is the counterclockwise path around the rectangle with vertices at (0, 0), (4, 0), (4, 2), and (0, 2)

A) $\frac{320}{3}$ B) $\frac{704}{3}$ C) -256 D) 0

circulation $= \int F \cdot \frac{dr}{dt}\, dt$



C_1 $x = t, y = 0$ $r = t\,i + 0\,j$

$F = t^2(0)^3 i + t^2 0^3 j = 0$ $\frac{dr}{dt} = i$

C_2 $x = 4,\ y = t$ $0 \le t \le 2$ $F = 16t^3 i + 16t^3 j$

$r = 4i + tj$ $\frac{dr}{dt} = j$

C_3 $x = t, y = 2$ $4 \le t \le 0$

$F = 8t^2 i + 8t^2 j$ $r = ti + 2j$

$\frac{dr}{dt} = i + 0j = i$

C_4 $x = 0$, so $F = 0$

$\text{FLOW}_{C1} = \int_0^4 0 \cdot \frac{dr}{dt} = 0$ FLOW C2 $\int_0^2 (16t^3 i + 16t^3 j) \cdot j \, dt = \int_0^2 16t^3 dt = 4t^4 \Big|_0^2 = 64$

$\text{Flow}_{C3} \int_4^0 (8t^2 i + 8t^2 j) \cdot i \, dt = \int_4^0 8t^2 dt = \frac{8t^3}{3} \Big|_4^0 = -\frac{512}{3}$

$\text{Flow}_{C4} \int \overset{0}{F} \cdot \frac{dr}{dt} dt = 0$

So Flow/circulation

$= 0 + 64 - \frac{512}{3} + 0 = -\frac{320}{3}$ (A)

Calculate the circulation of the field F around the closed curve C.

29) $\mathbf{F} = (-x - y)\mathbf{i} + (x + y)\mathbf{j}$, curve C is the counterclockwise path around the circle with radius 4 centered at (10, 3)

A) 64π B) $32(1 + \pi) + 208$ C) $32(1 + \pi)$ D) 32π

$x = 10 + 4\cos t$ $\quad r = (10 + 4\cos t)i + (3 + 4\sin t)j$

$y = 3 + 4\sin t$ $\quad \frac{dr}{dt} = -4\sin t\, i + 4\cos t\, j$

$0 \le t \le 2\pi$

$F = (-x-y)i + (x+y)j = (-13 - 4\cos t - 4\sin t)i + (13 + 4\cos t + 4\sin t)j$

$\int F \cdot \frac{dr}{dt}\, dt = \int_0^{2\pi} (52\sin t + 16\sin t\cos t + 16\sin^2 t + 52\cos t + 16\cos^2 t + 16\sin t\cos t)\, dt$

Note that $\sin t, \cos t$, $\sin t\cos t$ integrate to 0 from $(0 - 2\pi)$

All we have left is $16\sin^2 t + 16\cos^2 t = 16$

FLOW/CIR. $= \int_0^{2\pi} 16\, dt = 32\pi$ D

Find the potential function f for the field F.

30) $\mathbf{F} = \frac{1}{z}\mathbf{i} - 6\mathbf{j} - \frac{x}{z^2}\mathbf{k}$

A) $f(x, y, z) = \frac{x}{z} + C$

B) $f(x, y, z) = \frac{x}{z} - 6 + C$

C) $f(x, y, z) = \frac{x}{z} - 6y + C$

D) $f(x, y, z) = \frac{2x}{z} - 6y + C$

$F = \nabla f \quad \frac{\partial f}{\partial x} = \frac{1}{z}, \frac{\partial f}{\partial y} = -6, \frac{\partial f}{\partial z} = \frac{-x}{z^2}, \quad f = \frac{x}{z} + g(y,z)$

$\frac{\partial f}{\partial y} = 0 + \frac{\partial g}{\partial y} = -6 \Rightarrow g = -6y + k(z)$

$f = \frac{x}{z} - 6y + k z, \frac{\partial f}{\partial z} = \frac{-x}{z^2} + \frac{\partial k}{\partial z} = \frac{-x}{z^2}$

So $\frac{\partial k}{\partial z} = 0 \Rightarrow K = C$

so $f = \frac{x}{y} - 6y + C$ C

Find the potential function f for the field F.

31) $\mathbf{F} = (y - z)\mathbf{i} + (x + 2y - z)\mathbf{j} - (x + y)\mathbf{k}$

A) $f(x, y, z) = x(y + y^2) - xz - yz + C$

B) $f(x, y, z) = xy + y^2 - x - y + C$

C) $f(x, y, z) = x + y^2 - xz - yz + C$

D) $f(x, y, z) = xy + y^2 - xz - yz + C$

$\frac{\partial f}{\partial x} = y - z, \quad \frac{\partial f}{\partial y} = x + 2y - z, \quad \frac{\partial f}{\partial z} = -x - y$

$f = yx - zx + g(y, z)$

$\frac{\partial f}{\partial y} = \cancel{x} + \frac{\partial g}{\partial y} = \cancel{x} + 2y - z$

$g(y, z) = y^2 - zy + k(z), \quad f = yx - zx + y^2 - zy + k(z)$

$\frac{\partial f}{\partial z} = \cancel{-x} \cancel{-y} + \frac{dk}{dz} = \cancel{-x} \cancel{-y}$, so $k = C$

$f = yx - zx + y^2 - zy + C$ D

Evaluate the work done between point 1 and point 2 for the conservative field F.

32) $\mathbf{F} = 6 \sin 6x \cos 4y \cos 6z\mathbf{i} + 4 \cos 6x \sin 4y \cos 6z\mathbf{j} + 6 \cos 6x \cos 4y \sin 6z\mathbf{k}$; $P_1(0, 0, 0)$, $P_2 \left(\frac{1}{3}\pi, \frac{1}{2}\pi, \frac{\pi}{6}\right)$ To find the work we must get potential function

A) $W = -2$ B) $W = 2$ C) $W = 0$ D) $W = 1$

$\frac{\partial f}{\partial x} = 6 \sin 6x \cos 4y \cos 6z$, $f = -\cos 6x \cos 4y \cos 6z + g(y,z)$

$\frac{\partial f}{\partial y} = 4 \cos 6x \sin 4y \cos 6z + \frac{\partial g}{\partial y} = 4 \cos 6x \sin 4y \cos 6z \Rightarrow \frac{\partial g}{\partial y} = 0 \Rightarrow g = h(z)$

$f = -\cos 6x \cos 4y \cos 6z + h(z)$, $\frac{\partial f}{\partial z} = 6 \cos 6x \cos 4y \sin 6z + \frac{dh}{dz} = 6 \cos 6x \sin 4y \cos 6z$

$\frac{dh}{dz} = 0 \Rightarrow h(z) = \text{constant}$ So $f = -\cos 6x \cos 4y \cos 6z$

so work $= -\cos 6x \cos 4y \cos 6z \Big|_{0,0,0}^{\frac{\pi}{3}, \frac{\pi}{2}, \frac{\pi}{6}}$

$W = -\overset{+1}{\cos 2\pi}\, \overset{+1}{\cos 2\pi}\, \overset{-1}{\cos \pi} - \overset{+1}{(-\overset{+1}{\cos 0}\, \overset{+1}{\cos 0}\, \overset{+1}{\cos 0})} = 1 + 1 = 2$ B

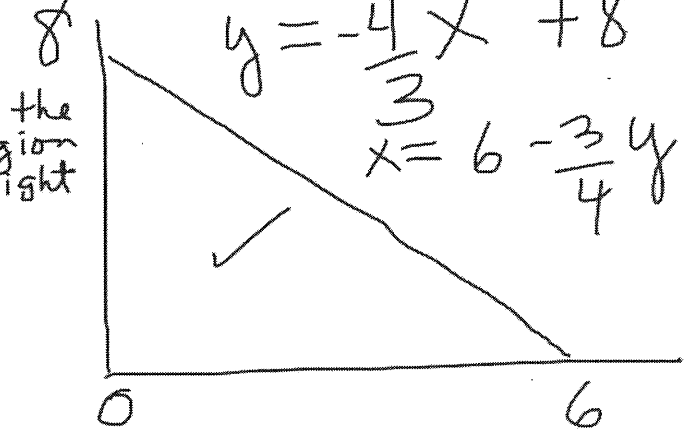
Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

33 F = xyi + xj; C is the triangle with vertices at (0, 0), (6, 0), and (0, 8)

A) 0 B) 24 C) 88 D) 64

2nd Green's Theorem - CC Circulation

$= \iint \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dx\,dy, \ \frac{\partial N}{\partial x} = 1, \ \frac{\partial M}{\partial y} = x$, see the region to right



$y = -\frac{4}{3}x + 8$

$x = 6 - \frac{3}{4}y$

$\text{cc cir} = \int_0^8 \int_0^{6-3/4y} (1-x)\,dx\,dy$

$= \int_0^8 \left(x - \frac{x^2}{2}\right)\Big|_0^{6-3/4y} dy = \int_0^8 \left[6 - 3/4y - \frac{(6-3/4y)^2}{2}\right] dy$

$= \left[6y - \frac{3y^2}{8} + \frac{4}{3}\frac{(6-3/4y)^3}{6}\right]\Big|_0^8 = \left[6y - \frac{3y^2}{8} + \frac{2}{9}(6-3/4y)^3\right]\Big|_0^8$

$= 48 - 24 + 2/9(0) - 48 = -24$ B

Using Green's Theorem, find the outward flux of F across the closed curve C.

34) $\mathbf{F} = \sin 10y\mathbf{i} + \cos 4x\mathbf{j}$; C is the rectangle with vertices at $(0, 0)$, $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{\pi}{10}, \frac{\pi}{4}\right)$, and $\left(0, \frac{\pi}{4}\right)$

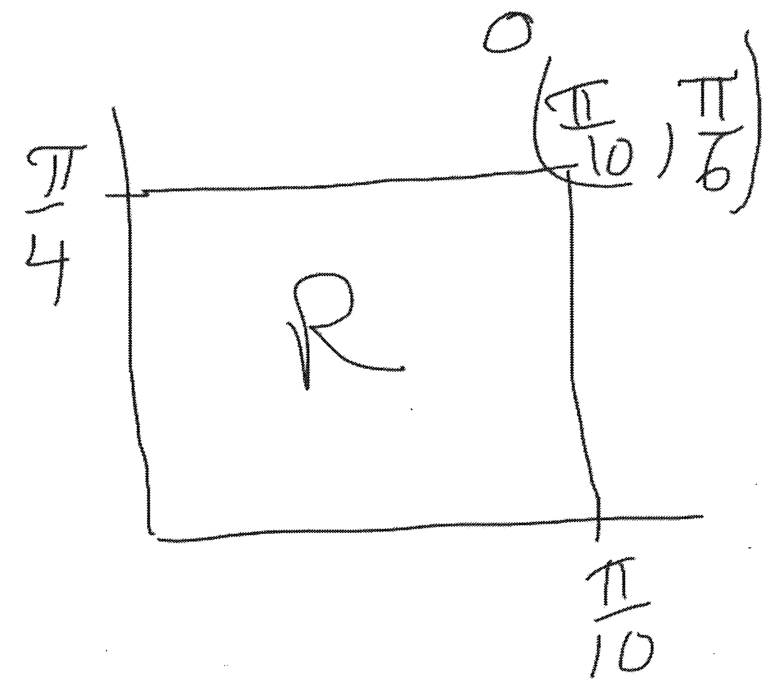
A) $\frac{1}{5}\pi$ B) $-\frac{2}{5}\pi$ C) $-\frac{1}{5}\pi$ D) 0

0

$\text{FLUX} = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy$

$\frac{\partial M}{\partial x} = 0$ $\frac{\partial N}{\partial y} = 0$ so FLUX = 0

D

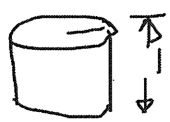


Calculate the area of the surface S.

35 [redacted] S is the portion of the cylinder $x^2 + y^2 = 36$ that lies between $z = 1$ and $z = 2$.

A) 18π B) 12π C) 36π D) 6π

$r = 6$ $2\pi r = 12\pi$



$r = 6$

surface $= 1 \times 2\pi r$

$= 2\pi(6) = 12\pi$