Math 2013 Practice Quiz 3 Fall 2011

Name: Last ________________, First ________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question. You must show your work to get credit.

Evaluate the integral.

1) $\int_{-6}^{-4} \int_{-3}^{9} dy\, dx$ 1

A) 1 B) -54 C) 24 D) -18

$\int_{-6}^{-4}\int_{-3}^{9} dy\, dx = \int_{-6}^{-4} y\Big|_{-3}^{9} dx = \int_{-6}^{-4} 9-(-3)\; dx = \int_{-6}^{-4} 12\, dx = 12x\Big|_{-6}^{-4} = -48-(-72) = 24$

C

2) $\int_{0}^{7} \int_{0}^{4} (2x^2y - 6xy)\, dy\, dx$

A) $\frac{280}{3}$ B) $\frac{490}{3}$ C) $\frac{1960}{3}$ D) $\frac{70}{3}$

$= \int_0^7 (x^2y^2 - 3xy^2)\Big|_0^4 dx = \int_0^7 (16x^2 - 48x)\, dx = \left(\frac{16x^3}{3} - 24x^2\right)\Big|_0^7 = \frac{5488}{3} - 1176 = \frac{1960}{3}$

C

Integrate the function f over the given region.

3) $f(x, y) = xy$ over the rectangle $2 \le x \le 7$, $1 \le y \le 6$

A) $\frac{1575}{4}$ B) $\frac{525}{2}$ C) $\frac{1575}{2}$ D) 525

$\int_1^6\int_2^7 xy\, dx\, dy = \int_1^6 \frac{x^2y}{2}\Big|_2^7 dy = \int_1^6 \frac{45y}{2} dy = \frac{45y^2}{4}\Big|_1^6 = \frac{45}{4}(35) = \frac{1575}{4}$ A

4) $f(x, y) = 4x \sin xy$ over the rectangle $0 \le x \le \pi$, $0 \le y \le 1$

A) 4π B) $4\pi - 4$ C) π D) $\frac{\pi}{4}$

$\int_0^\pi\int_0^1 4x\sin xy\, dy\, dx = \int_0^\pi -4\cos xy\Big|_0^1 dx = \int_0^\pi (-4\cos x + 4)\, dx = (-4\sin x + 4x)\Big|_0^\pi = 4\pi$

A

Find the volume under the surface z = f(x,y) and above the rectangle with the given boundaries.

5) $z = 8x + 4y + 7;\ 0 \le x \le 1,\ 1 \le y \le 3$

A) 28 B) 26 C) 38 D) 36

$$V=\int_1^3\int_0^1 (8x+4y+7)\,dx\,dy=\int_1^3 (4x^2+4xy+7x)\Big|_0^1 dy=\int_1^3 (4+4y+7)\,dy=\int_1^3 (11+4y)\,dy=(11y+2y^2)\Big|_1^3$$

$$=33+18-11-2=38 \ \text{C}$$

6) $z=\frac{x}{y};\ 0 \le x \le 1,\ 1 \le y \le e$

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{6}$ D) $\frac{1}{2}$

$$V=\int_1^e\int_0^1 \frac{x}{y}\,dx\,dy=\int_1^e \frac{x^2}{2y}\Big|_0^1 dy=\int_1^e \frac{1}{2y}\,dy=\frac{1}{2}\ln y\Big|_1^e$$

$$=\frac{1}{2}\ln e-\frac{1}{2}\ln 1=\frac{1}{2}$$

Evaluate the integral.

7) $\int_1^3\int_0^y x^2y^2\,dx\,dy$

A) $\frac{350}{3}$ B) $\frac{364}{9}$ C) $\frac{364}{3}$ D) $\frac{350}{9}$

$$=\int_1^3 \frac{y^2x^3}{3}\Big|_0^y dy=\int_1^3 \frac{y^5}{3}\,dy=\frac{y^6}{18}\Big|_1^3$$

$$=\frac{81}{2}-\frac{1}{18}=\frac{729-1}{18}=\frac{728}{18}=\frac{364}{9}=\text{B}$$

8) $\int_0^{\pi/6}\int_0^{\cos 3x} \sin 3x\,dy\,dx$

A) $\frac{\pi}{6}$ B) $\frac{1}{12}$ C) $\frac{1}{6}$ D) $\frac{\pi}{12}$

$$=\int_0^{\frac{\pi}{6}} y\sin 3x\Big|_0^{\cos 3x} dx=\int_0^{\frac{\pi}{6}} \sin 3x\cos 3x\,dx=\int_0^{\frac{\pi}{6}} \frac{\sin 6x}{2}\,dx=-\frac{\cos 6x}{12}\Big|_0^{\frac{\pi}{6}}=$$

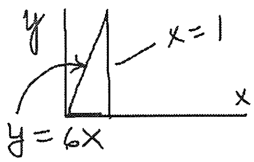
$$=-\frac{\cos\pi}{12}+\frac{\cos 0}{12}=\frac{1}{12}+\frac{1}{12}=\frac{1}{6}\ \text{C}$$

Integrate the function f over the given region.

9) $f(x, y) = y^2 e^{x^4}$ over the triangular region in the first quadrant bounded by the lines $x = y/6$, $x = 1$, $y = 0$

A) $18(e - 1)$ B) $18e$ C) $216(e + 1)$ D) $72(e - 1)$

$$\int_0^1 \int_0^{6x} y^2 e^{x^4}\, dy\, dx = \int_0^1 \frac{y^3}{3} e^{x^4} \Big|_0^{6x} = \int_0^1 \frac{216}{3} x^3 e^{x^4}\, dx$$



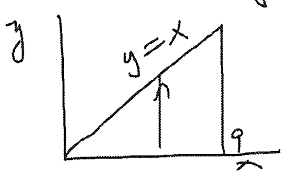
$u = x^4 \quad du = 4x^3 dx, \quad x^3 dx = \frac{du}{4}, \quad \int_0^1 \frac{216}{3} \frac{e^u du}{4}$

$$= \int_0^1 \frac{54}{3} e^u du = \frac{54}{3} e^u \Big|_0^1 = \frac{54}{3}(e-1) = 18(e-1) \quad A$$

Write an equivalent double integral with the order of integration reversed.

10) $\int_0^9 \int_0^x dy\, dx$

need to find region of int.



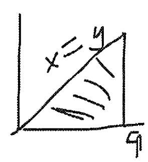
A) $\int_0^9 \int_{-9}^y dx\, dy$

B) $\int_0^x \int_0^9 dx\, dy$

C) $\int_0^9 \int_9^y dx\, dy$

D) $\int_0^9 \int_y^9 dx\, dy$

$\int_0^9 \int_y^9 dx\, dy$ D



11) $\int_0^1 \int_0^{\tan^{-1} x} dy\, dx$

$y = \tan^{-1} x$

or $x = \tan y$

x goes from $\tan y$ to 1

y from 0 to $\frac{\pi}{4}$

A) $\int_0^{\pi/4} \int_{\tan y}^{\pi/2} dx\, dy$

B) $\int_0^{\pi/4} \int_{\tan y}^{1} dx\, dy$

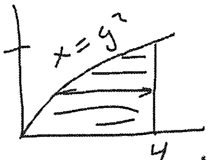
C) $\int_0^{\pi/4} \int_{\tan^{-1} y}^{1} dx\, dy$

D) $\int_0^{\pi/4} \int_{\tan^{-1} y}^{\pi/2} dx\, dy$

12) $\int_0^2 \int_{y^2}^4 6y \, dx \, dy$

y goes from 0 to $\sqrt{x}$

x goes from 0 4



$y = \sqrt{x}$ or $x = y^2$, $x = y^2$, $y = \sqrt{x}$

A) $\int_0^2 \int_0^{\sqrt{x}} 6y \, dy \, dx$

B) $\int_0^4 \int_2^{\sqrt{x}} 6y \, dy \, dx$

C) $\int_0^2 \int_2^{\sqrt{x}} 6y \, dy \, dx$

D) $\int_0^4 \int_0^{\sqrt{x}} 6y \, dy \, dx$

$\int_0^4 \int_0^{\sqrt{x}} y \, dy \, dx$ D

Reverse the order of integration and then evaluate the integral.

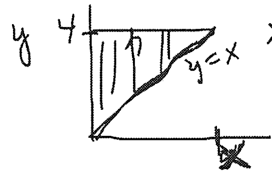
13) $\int_0^4 \int_x^4 \frac{\sin y}{y} \, dy \, dx$

A) cos 4

B) 1 + cos 4

C) 1 − cos 4

D) − cos 4



x goes from 0 to y

y goes from 0 to 4

$\int_0^4 \int_0^y \frac{\sin y}{y} \, dx \, dy = \int_0^4 \frac{x \sin y}{y} \Big|_0^y dy$

$= \int_0^4 \frac{\cancel{y} \sin y}{\cancel{y}} dy = \int_0^4 \sin y \, dy$

$= -\cos y \Big|_0^4$

$= -\cos 4 - (-\cos 0) = 1 - \cos 4$ C

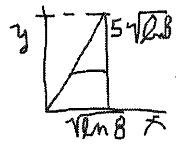
14) $\int_0^{5\sqrt{\ln 8}} \int_{y/5}^{\sqrt{\ln 8}} e^{x^2} \, dx \, dy$

A) $\frac{45}{2}$

B) 16

C) 24

D) $\frac{35}{2}$



$x = y/5$ $y = 5x$

$= \int_0^{\sqrt{\ln 8}} \int_0^{5x} e^{x^2} dy \, dx = \int_0^{\sqrt{\ln 8}} y e^{x^2} \Big|_0^{5x} dx = \int_0^{\sqrt{\ln 8}} 5x e^{x^2} dx$

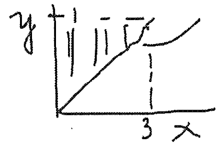
$u = x^2$

$du = 2x \, dx$

$x \, dx = \frac{du}{2}$

$\int_0^{\ln 8} \frac{5}{2} e^u du = \frac{5}{2} e^u \Big|_0^{\ln 8} = \frac{5}{2} e^{\ln 8} - \frac{5}{2} e^0 = \frac{5}{2}(8-1) = \frac{35}{2}$ D

15) $\int_0^3 \int_{x/3}^1 3y^2 \cos xy \, dy \, dx$



$y = x/3 \quad x = 3y$

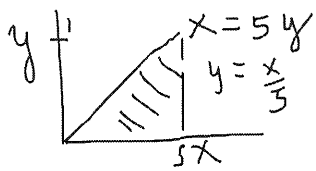
$\int_0^1 \int_0^{3y} 3y^2 \cos xy \, dx \, dy =$

A) $\cos 3 - 1$ B) $3(1 - \cos 3)$ C) $5(1 - \cos 3)$ D) $\frac{1}{2}(1 - \cos 3)$

$\int_0^1 3y \sin xy \Big|_0^{3y} dy = \int_0^1 3y \sin 3y^2 \, dy$ $\quad u = y^2 \quad du = 2y\,dy \quad y\,dy = \frac{du}{2}$ $\rightarrow \int_0^1 \frac{3}{2} \sin 3u \, du$

$= -\frac{3}{2}\left(\frac{1}{3}\right) \cos 3u \Big|_0^1 = -\frac{1}{2}\cos 3 + \frac{1}{2} = \frac{1}{2}(1 - \cos 3)$ D

16) $\int_0^1 \int_{5y}^5 x^4 e^{x^2 y} \, dx \, dy$



$x = 5y$

$y = \frac{x}{5}$

$\int_0^5 \int_0^{x/5} x^4 e^{x^2 y} \, dy \, dx =$

A) $e^{25} - 1$ B) $e^{25} - \frac{130}{3}$ C) $\frac{5e^{25} - 130}{3}$ D) $5e^{25} - 130$

$\int_0^5 x^2 e^{x^2 y} \Big|_0^{\frac{x}{5}} dx = \int_0^5 \left(x^2 e^{\frac{x^3}{5}} - x^2\right) dx = \int_0^5 \left(x^2 e^{\frac{x^3}{5}} - x^2\right) dx = \int_0^5 x^2 e^{\frac{x^3}{5}} dx - \frac{125}{3}$

use sub for 1st

$u = \frac{x^3}{5}, \quad du = \frac{3x^2}{5} dx \quad x^2 dx = \frac{5}{3} du$

$\int_0^{25} \frac{5}{3} e^u du = \frac{5}{3} e^u \Big|_0^{25} = \frac{5}{3} e^{25} - \frac{5}{3}$, Total integral $= \frac{5}{3} e^{25} - \frac{5}{3} - \frac{125}{3} = \frac{5e^{25} - 130}{3}$ C

Find the volume of the indicated region.

17) the region under the surface $z = x^2 + y^4$, and bounded by the planes $x = 0$ and $y = 25$ and the cylinder $y = x^2$

A) $\frac{2,357,500}{33}$ B) $\frac{58,607,500}{33}$ C) $\frac{11,732,500}{33}$ D) $\frac{292,982,500}{33}$

$$V = \int_0^5 \int_{x^2}^{25} \int_0^{x^2+y^4} 1 \, dz\, dy\, dx = \int_0^5 \int_{x^2}^{25} (x^2 + y^4)\, dy\, dx$$

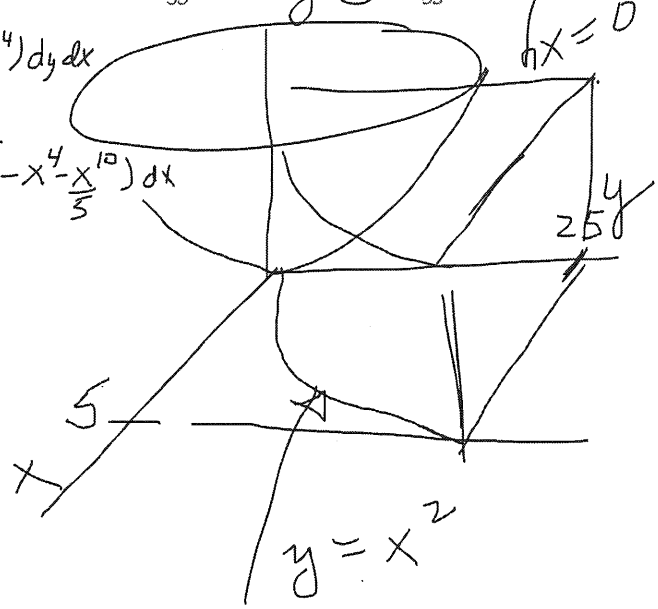
$$= \int_0^5 \left(x^2 y + \frac{y^5}{5}\right)\Big|_{x^2}^{25} dx = \int_0^5 \left(25x^2 + \frac{(25)^5}{5} - x^4 - \frac{x^{10}}{5}\right) dx$$

$$= \left(\frac{25x^3}{3} + \frac{(25)^5}{5} - \frac{x^5}{5} - \frac{x^{11}}{5(11)}\right)\Big|_0^5$$

$$= \frac{25(125)}{3} + (25)^5 - 5^4 - \frac{5^{10}}{11}$$

$$= \frac{292,982,500}{33}$$

D



18) the region bounded by the paraboloid $z = 100 - x^2 - y^2$ and the xy-plane

A) 2500π B) 5000π C) $\frac{10000}{3}\pi$ D) $\frac{5000}{3}\pi$

$$V = 4\int_0^{10}\int_0^{\sqrt{100-y^2}}\int_0^{100-x^2-y^2} 1\, dz\, dx\, dy$$

4 x one quarter of volume

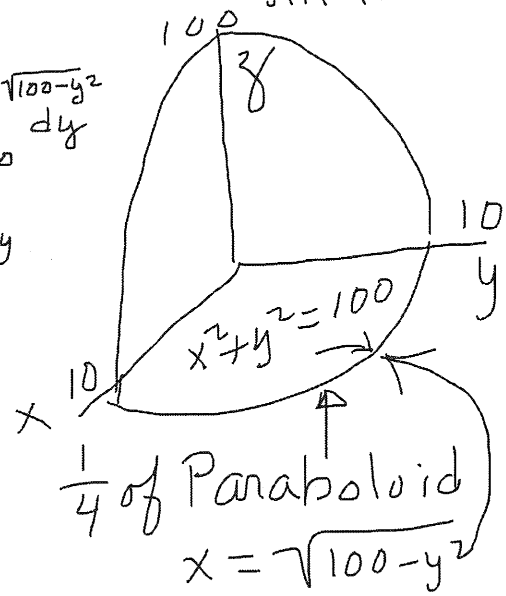
$$4\int_0^{10}\int_0^{\sqrt{100-y^2}} (100-x^2-y^2)\,dx\,dy = 4\int_0^{10}\left[(100-y^2)x - \frac{x^3}{3}\right]_0^{\sqrt{100-y^2}} dy$$

$$= 4\int_0^{10} (100-y^2)^{3/2} - \frac{(100-y^2)^{3/2}}{3}\,dy = \frac{8}{3}\int_0^{10} (100-y^2)^{3/2}\,dy$$

$= 15{,}708$ (on your calculator)

$= 5000\pi$ using Trig Substitution

stitution



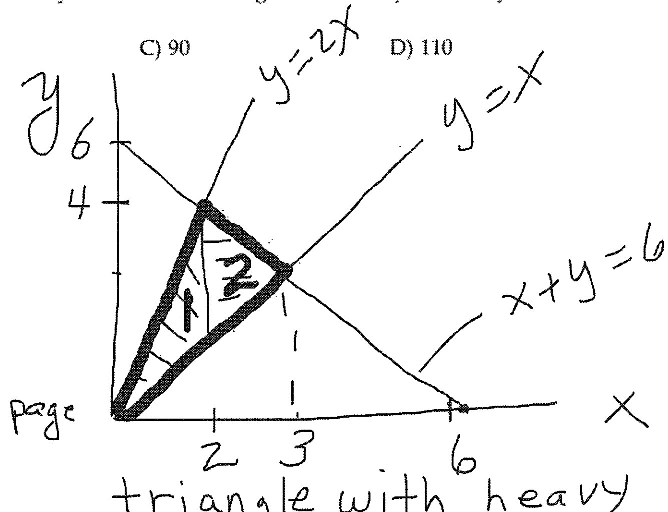
19) the region that lies under the plane $z = 10x + 8y$ and over the triangle bounded by the lines $y = x$, $y = 2x$, and $x + y = 6$

A) 106 B) 126 C) 90 D) 110

We must do the integration over the two sub-areas

$$V = \int_0^2\int_x^{2x}\int_0^{10x+8y} 1\, dz\,dy\,dx + \int_2^3\int_x^{6-x}\int_0^{10x+8y} 1\, dz\,dy\,dx$$

See Integration on next page



triangle with heavy border is triangle in Question

1st integral

$$\int_0^2\int_x^{2x} 10x+8y\,dy\,dx = \int_0^2 (10xy+4y^2)\Big|_x^{2x} dx$$

$$= \int_0^2 20x^2+16x^2-10x^2-4x^2\,dx = \int_0^2 22x^2 dx = \frac{22x^3}{3}\Big|_0^2 = \frac{176}{3}$$

2nd Integral

$$\int_2^3\int_x^{6-x}(10x+8y)\,dy\,dx = \int_2^3 (10xy+4y^2)\Big|_x^{6-x} dx$$

$$= \int_2^3 10x(6-x)+4(6-x)^2-10x^2-4x^2\,dx$$

$$= \int_2^3 (144+12x-20x^2)dx = \left(144x+6x^2-\frac{20x^3}{3}\right)\Big|_2^3$$

$$= \frac{142}{3}, \text{ so total} = \frac{142}{3}+\frac{176}{3} = \frac{318}{3} = 106 \text{ A}$$