Math 2013 Final Study Practice Quiz

Name: Last ______________, First ______________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question. You must show your work to get credit.

Find the angle between u and v in radians.

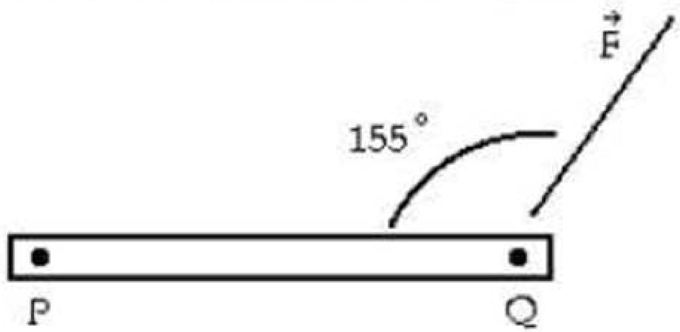
1) $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$, $\mathbf{v} = 10\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$

A) 1.19 B) 1.56 C) 0.38 D) 1.44

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right) = \cos^{-1}\frac{(2i-3j-3k)\cdot(10i-10j-4k)}{\sqrt{4+9+9}\,\sqrt{100+100+16}} = \cos^{-1}\left(\frac{20-12+12}{\sqrt{22}\,\sqrt{132}}\right) = 1.19 \text{ A}$$

Solve the problem.

2) Find the magnitude of the torque in foot-pounds at point P for the following lever:



$|\overrightarrow{PQ}| = 4$ in. and $|\mathbf{F}| = 30$ lb

A) 4.23 ft-lb B) 120 ft-lb C) 14.37 ft-lb D) 9.06 ft-lb

Torque $= |r||F|\sin\theta$

$|\overrightarrow{PQ}| = 4/12$ ft

Torque $= \left(\frac{4}{12}\right)(30)\sin 155°$

$=$ A) 4.23 ft-lb

Find parametric equations for the line described below.

3) The line through the point P(3, –4, 5) parallel to the vector **–8i + 4j – 4k**

A) $x = 8t + 3$, $y = 4t - 4$, $z = -4t + 5$

B) $x = -8t + 3$, $y = 4t - 4$, $z = -4t + 5$

C) $x = 8t - 3$, $y = 4t + 4$, $z = -4t - 5$

D) $x = -8t - 3$, $y = 4t + 4$, $z = -4t - 5$

P(3, –4, 5) parallel to $-8i + 4j - 4k$

start dir start dir

$x = 3 - 8t$, $y = -4 + 4t$, $z = 5 - 4t$ B

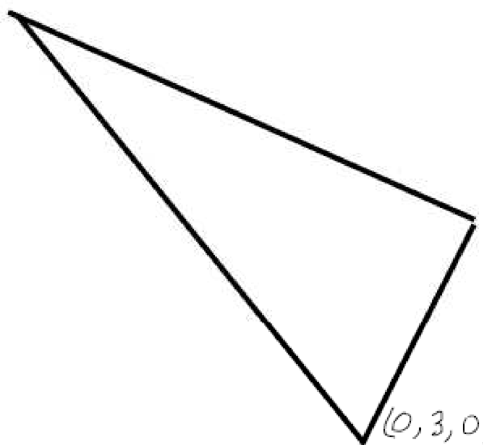
Calculate the requested distance.

4) The distance from the point S(−6, −6, 9) to the plane $2x + 2y + z = 6$

A) $\frac{7}{3}$ B) 7   C) 9   D) 3

$S(-6,6,9)$ to plane $2x + 2y + z = 6$ $\Rightarrow$ normal vector to plane is $2i + 2j + k = n$

$|n| = \sqrt{4+4+1} = 3$



$\overrightarrow{PS} = -6i - 9j + 9k$

$(0,3,0)$ Pick an easy pt. on plane

P

$d = \left| \overrightarrow{PS} \cdot \frac{n}{|n|} \right|$ $\quad$ $d = \left| (-6i - 9j + 9k) \cdot \frac{(2i + 2j + k)}{3} \right| = \left| \frac{-12 - 18 + 9}{3} \right| = \frac{21}{3} = 7$ B

Write the equation for the plane.

5) The plane through the point P(−6, −6, −5) and normal to $\mathbf{n} = -8\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}$.

A) $8x + 7y - 4z = 26$

B) $6x + 6y - 5z = 26$

C) $-6x - 6y + 5z = 26$

D) $-8x - 7y + 4z = 70$

let (x, y, z) be point on the plane, $(x+6)i + (y+6)j + (z+5)k$ is a vector on the plane which must be $\perp$ to $(-8i - 7j + 4k)$, so $[(x+6)i + (y+6)j + (z+5)k] \cdot [-8i - 7j + 4k] = 0$

$-8(x+6) - 7(y+6) + 4(z+5) = 0$, $-8x - 48 - 7y - 42 + 4z + 20 = 0$

$-8x - 7y + 4z = 48 + 42 - 20 = 70$

$\boxed{-8x - 7y + 4z = 70}$ D

Find the intersection of the line and the plane.

6) $x = 7 + 8t$, $y = -9 + 2t$, $z = 3 + 8t$; $2x + 8y + 7z = 7$

A) $(15, -7, 11)$ B) $(-1, -11, -5)$ C) $(11, -8, 7)$ D) $\left(3, -\frac{20}{11}, -3\right)$

Need to find t that causes line to intersect plane

$2(7+8t) + 8(-9+2t) + 7(3+8t) = 7$

$14 + 16t - 72 + 16t + 21 + 56t = 7$

$88t = 7 - 14 + 72 - 21 = 44$

$t = 1/2$

So, pt. of intersection is

$x = 7 + 8(\frac{1}{2}) = 11$, $y = -9 + 2(\frac{1}{2}) = -8$, $z = 3 + 8(\frac{1}{2}) = 7$

the pt. is $(11, -8, 7)$ C

The position vector of a particle is r(t). Find the requested vector.

7) The acceleration at $t = 3$ for $\mathbf{r}(t) = (5t - 2t^4)\mathbf{i} + (10 - t)\mathbf{j} + (8t^2 - 2t)\mathbf{k}$

A) $\mathbf{a}(3) = -216\mathbf{i} + 16\mathbf{k}$

B) $\mathbf{a}(3) = 216\mathbf{i} + 16\mathbf{k}$

C) $\mathbf{a}(3) = -216\mathbf{i} - \mathbf{j} + 16\mathbf{k}$

D) $\mathbf{a}(3) = -54\mathbf{i} + 16\mathbf{k}$

Need 2nd derivative

$v(t) = (5 - 8t^3)i - j + (16t - 2)k$

$a(t) = -24t^2 i + 16k$

$a(3) = -24(3)^2 i + 16k$

$= -216 i + 16k$ A

Write the equation for the plane.

8) The plane through the point A(5, 7, 4) perpendicular to the vector from the origin to A.

A) $5x + 7y + 4z = \sqrt{90}$ B) $5x + 7y + 4z = -90$

C) $5x + 7y + 4z = 90$ D) $5x + 7y + 4z = 16$

Vector from origin to A is $(5i + 7j + 4k)$, vector from pt. on plane to A is $(x-5)i + (y-7)j + (z-4)k$

To be perpendicular the dot products must be zero, so

$(5i + 7j + 4k) \cdot [(x-5)i + (y-7)j + (z-4)k] = 0$

$5(x-5) + 7(y-7) + 4(z-4) = 0$

$5x - 25 + 7y - 49 + 4z - 16 = 0$

$5x + 7y + 4z = 25 + 49 + 16 = 90$ so $\boxed{5x + 7y + 4z = 90}$ C

Solve the problem.

9) Find an equation for the level surface of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ that passes through the point $(4, 3, 12)$.

A) $x^2 + y^2 + z^2 = 13$

B) $x + y + z = \pm 13$

C) $x^2 + y^2 + z^2 = 169$

D) $x + y + z = 13$

$\sqrt{x^2+y^2+z^2} = c$ level curve, $\sqrt{16+9+144} = \sqrt{169} = c = 13$

$\uparrow 4 \quad \uparrow 3 \quad \uparrow 12$

$\sqrt{x^2+y^2+z^2} = 13$

$x^2+y^2+z^2 = 169$

C

Find f_x, f_y, and f_z.

10) $f(x, y, z) = \dfrac{\cos y}{xz^2}$

A) $f_x = \dfrac{\cos y}{z^2}$; $f_y = \dfrac{\sin y}{xz^2}$; $f_z = \dfrac{2 \cos y}{xz}$

B) $f_x = -\dfrac{\cos y}{z^2}$; $f_y = -\dfrac{\sin y}{xz^2}$; $f_z = -\dfrac{2 \cos y}{xz}$

C) $f_x = \dfrac{\cos y}{x^2z^2}$; $f_y = \dfrac{\sin y}{xz^2}$; $f_z = \dfrac{2 \cos y}{xz^3}$

D) $f_x = -\dfrac{\cos y}{x^2z^2}$; $f_y = -\dfrac{\sin y}{xz^2}$; $f_z = -\dfrac{2 \cos y}{xz^3}$

$f_x = \dfrac{\cos y}{z^2} \dfrac{d}{\partial x} x^{-1} = -\dfrac{\cos y}{x^2 z^2}$, $f_y = -\dfrac{\sin y}{xz^2}$, $f_z = \dfrac{\cos y}{x^2} \dfrac{d}{\partial z} z^{-2} = -\dfrac{2 \cos y}{x z^3}$ D

Solve the problem.

11) Evaluate $\frac{dw}{dt}$ at $t = 3\pi$ for the function $w = x^2 - y^2 - 5x$; $x = \cos t$, $y = \sin t$.

A) -9 B) -7 C) 0 D) -3

Chain Rule $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$

$= (2x-5)(-\sin t) - 2y\cos t = (2\cos t - 5)(-\sin t) - 2\sin t \cos t \Big|_{t=3\pi}$

$= (2\cos 3\pi - 5)(-\overset{\nearrow 0}{\sin 3\pi}) - 2\overset{\nearrow 0}{\sin 3\pi}\cos 3\pi = 0$ C

Compute the gradient of the function at the given point.

12) $f(x, y, z) = 2xy^3z^2$, $(2, 8, 4)$

A) $\nabla f = 16{,}384\mathbf{i} + 8192\mathbf{j} + 24{,}576\mathbf{k}$

B) $\nabla f = 8192\mathbf{i} + 8192\mathbf{j} + 6144\mathbf{k}$

C) $\nabla f = 16{,}384\mathbf{i} + 12{,}288\mathbf{j} + 16{,}384\mathbf{k}$

D) $\nabla f = 8192\mathbf{i} + 12{,}288\mathbf{j} + 4096\mathbf{k}$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$$

$$\nabla f = 2y^3z^2 i + 6xy^2z^2 j + 4xy^3z k,$$

$$\nabla f\Big|_{2,8,4}$$

$$= 2(8)^3(4)^2 i + 6(2)(8)^2(4)^2 j + 4(2)(8)^3(4) k$$

$$= 16{,}384 i + 12{,}288 j + 16384 k \quad C$$

Provide an appropriate response.

13) Find the direction in which the function is increasing most rapidly at the point P_0.

$f(x, y, z) = xy - \ln(z)$, $P_0(2, -2, 2)$

A) $\frac{1}{\sqrt{33}}(-4i - 4j + k)$

B) $\frac{1}{\sqrt{33}}(-4i + 4j - k)$

C) $\frac{33}{\sqrt{33}}(-4i + 4j - k)$

D) $\frac{1}{33}(-4i + 4j - k)$

$\nabla f = yi + xj - \frac{1}{z}k$

$\nabla f\big|_{2,-2,2} = -2i + 2j - \frac{1}{2}k$

$|v| = \sqrt{4+4+1/4} = \sqrt{\frac{33}{4}} = \frac{\sqrt{33}}{2}$

so direction $= \dfrac{-2i+2j-\frac{1}{2}k}{\frac{\sqrt{33}}{2}}$

$= \dfrac{-4i+4j-k}{\sqrt{33}}$ B

Find the derivative of the function at P_0 in the direction of u.

14) $f(x, y, z) = 4x + 3y + 9z$, $P_0(2, -8, 6)$, $\mathbf{u} = 3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$

A) $-\frac{24}{7}$ B) $-\frac{33}{7}$ C) $-\frac{12}{7}$ D) $-\frac{15}{7}$

Derivative in direction of u is $\nabla f \cdot \frac{u}{|u|}$, $\nabla f = 4i + 3j + 9k$, $|u| = \sqrt{9+36+4} = 7$

So $\nabla f \cdot \frac{u}{|u|} = \frac{(4i+3j+9k)\cdot(3i-6j-2k)}{7}$

$= \frac{12-18-18}{7} = \frac{-24}{7}$ A

Find the linearization of the function at the given point.

15) $f(x, y, z) = 6xy + 2yz - 9zx$ at $(1, 1, 1)$

A) $L(x, y, z) = -3x + 8y - 7z + 2$

B) $L(x, y, z) = 6x + 2y - 9z + 1$

C) $L(x, y, z) = 6x + 2y - 9z + 2$

D) $L(x, y, z) = -3x + 8y - 7z + 1$

$$L(x,y,z)\Big|_{P_0} = f(x_0,y_0,z_0) + \frac{\partial f}{\partial x}\Big|_{P_0}(x-x_0) + \frac{\partial f}{\partial y}\Big|_{P_0}(y-y_0) + \frac{\partial f}{\partial x}\Big|_{P_0}(z-z_0)$$

$$= (6xy+2yz-9zx)\Big|_{1,1,1} + (6y-9z)(x-1) + (6x+2z)\Big|_{1,1,1}(y-1) + (2y-9x)\Big|_{1,1,1}(z-1)$$

$$= (6+2-9) - 3(x-1) + 8(y-1) - 7(z-1) = -1 - 3x + 3 + 8y - 8 - 7z + 7 = -3x + 8y - 7z + 1$$

D

Find all the local maxima, local minima, and saddle points of the function.

16) $f(x, y) = x^3 + y^3 - 300x - 75y - 3$

A) $f(10, -5) = -1753$, saddle point; $f(-10, 5) = 1747$, saddle point

B) $f(-10, -5) = 2247$, local maximum; $f(10, 5) = -2253$, local minimum

C) $f(10, 5) = -2253$, local minimum; $f(10, -5) = -1753$, saddle point; $f(-10, 5) = 1747$, saddle point; $f(-10, -5) = 2247$, local maximum

D) $f(-10, -5) = 2247$, local maximum

$\frac{\partial f}{\partial x} = 3x^2 - 300 = 0$, $x^2 = 100$, $x = \pm 10$

$\frac{\partial f}{\partial y} = 3y^2 - 75 = 0$, $y^2 = 25$, $y = \pm 5$

Need to check

$(10,5)$, $(10,-5)$, $(-10,5)$, $(-10,-5)$

$(10,5)$

$f_{xx} = 6x\big|_{10} = 60 > 0$

$f_{xy} = 0$, $f_{yy} = 6y\big|_5 = 30$

$f_{xx}f_{yy} - f_{xy} = (60)(15) - 0 > 0$

$\Rightarrow$ No saddle point

$f_{xx} > 0 \Rightarrow$ concave up

Local MIN $f(10,5) = -2253$

$(10,-5)$

$f_{xx} = 6x\big|_{10} = 60 > 0$

$f_{yy} = 6y\big|_{-3} = -30$

$f_{xy} = 0$

$f_{xx}f_{yy} - f_{xy} = 60(-30) < 0$

$\Rightarrow$ saddle Point

$f(10,-5) = -1733$

$-10, 5$

$f_{xx} = 6x\big|_{-10} = -60 < 0$

$f_{xy} = 0$, $f_{yy} = 6y\big|_5 = 30$

$f_{xx}f_{yy} - f_{xy} = -60(30) < 0$

$\Rightarrow$ Saddle Pt

$f(-10,5) = 1747$

$-10, -5$

$f_{xx} = 6x\big|_{-10} = -60 < 0$

$f_{xy} = 0$ $f_{yy} = 6y\big|_{-5} = -30$

$f_{xx}f_{yy} - f_{xy} = -60(-30) > 0$

$\Rightarrow$ No Saddle pt.

$f_{xx} < 0$ concave down

$\frown$ Local MAX $f(-10,-5) = 2247$

C

17) **Find the extreme values of the function subject to the given constraint.**

$f(x, y) = xy, \quad x^2 + y^2 = 800$

A) Maximum: 400 at (20, 20) and (−20, −20); minimum: −400 at (20, −20) and (−20, 20)

B) Maximum: 400 at (20, 20); minimum: −400 at (−20, −20)

C) Maximum: 400 at (20, 20); minimum: 0 at (0, 0)

D) Maximum: 400 at (20, −20) and (−20, 20); minimum: −400 at (20, 20) and (−20, −20)

$\nabla f = \lambda \nabla g \quad f = xy,\ g = x^2+y^2-800,\ \nabla f = y\mathbf{i} + x\mathbf{j} = \lambda(2x\mathbf{i} + 2y\mathbf{j})$, so $y = \overset{(1)}{2x\lambda}$, $x = \overset{(2)}{2y\lambda}$

$x=0, y=0$ is a solution to last two equations but doesn't satisfy the constraint $x^2+y^2=800$

solving (1)+(2) for λ we get $\lambda = \frac{y}{2x} = \frac{x}{2y}$, so $2y^2 = 2x^2$, this leads to $x = \pm y$, Substitue in $x^2+y^2=800$

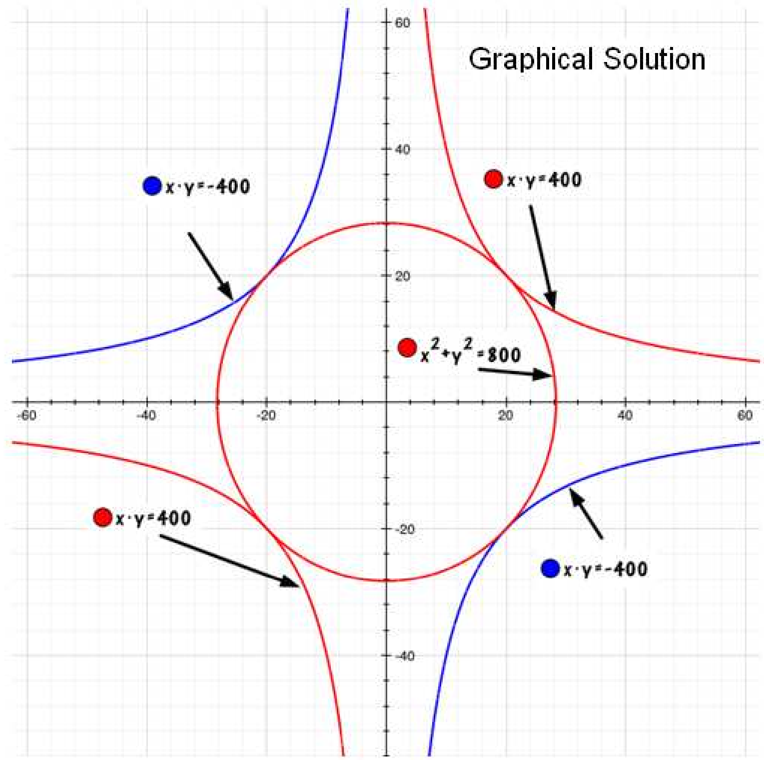
we get $y^2+y^2 = 800$, $y^2 = 400 \Rightarrow y = \pm 20$

Solutions

(20, 20), (−20, −20), (20, −20), (−20, 20)

f = 400 at (20, 20), (20, −20), MAX

f = −400 at (20, −20), (−20, 20) MIN   A



18) $f(x, y) = x^2 + y^2, \quad xy^2 = 686$

A) Maximum: none; minimum: 147 at $(7, \pm 7\sqrt{2})$

B) Maximum: none; minimum: 0 at $(0, 0)$

C) Maximum: 147 at $(7, 7\sqrt{2})$; minimum: -147 at $(7, -7\sqrt{2})$

D) Maximum: 147 at $(7, \pm 7\sqrt{2})$; minimum: 0 at $(0, 0)$

$\nabla f = \lambda \nabla g$, $g = xy^2 - 686$

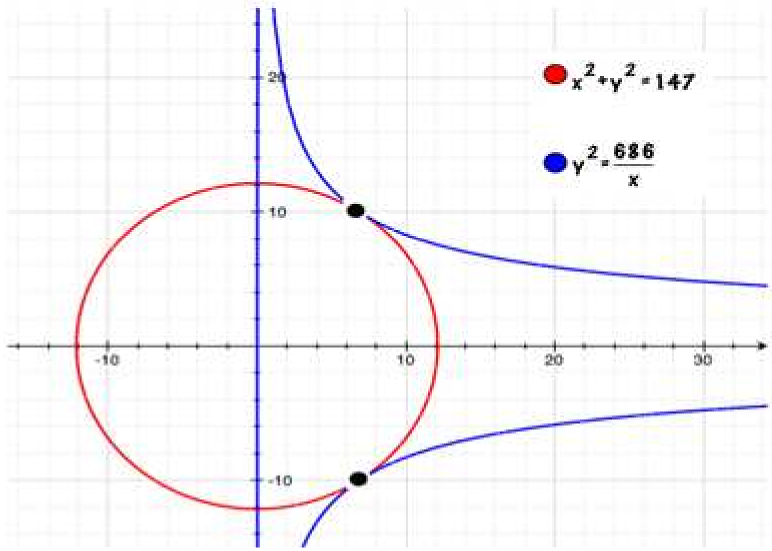
$\nabla f = 2x\,i + 2y\,j = \lambda y^2 i + 2xy\lambda j$, so $2x = y^2\lambda$ (1), $2y = 2xy\lambda$ (2), Note $x = y = 0$ is not a solution (doesn't satisfy $xy^2 = 686$)

Solving (1) for λ we get $\lambda = \frac{2x}{y^2}$, Subs. in (2) $2y = 2xy\left(\frac{2x}{y^2}\right) = \frac{4x^2}{y} \Rightarrow 2y^2 = 4x^2$ or $y^2 = 2x^2$, subs into $xy^2 = 686$

we get $x(2x^2) = 686$, $x^3 = 343$, $x = 7$, $y^2 = \frac{686}{x} = \frac{686}{7} = 98 = 2(49)$ so $y = \pm 7\sqrt{2}$

So $f(7, \pm 7\sqrt{2}) = 49 + 49(2) = 147$ is a minimum, The is no MAX because y can be very small and y will apprach ∞ $f \to \infty$

A



Evaluate the integral.

19) $\int_1^3 \int_0^y x^2y^2 \, dx \, dy$

A) $\frac{350}{3}$ B) $\frac{364}{9}$ C) $\frac{350}{9}$ D) $\frac{364}{3}$

$$\int_1^3 \int_0^y x^2y^2 \, dx \, dy = \int_0^3 \frac{x^3y^2}{3} \Big|_0^y dy = \int_1^3 \frac{y^5}{3} dy = \frac{y^6}{18} \Big|_1^3 = \frac{(27)(27)}{18} - \frac{1}{18} = \frac{728}{18} = \frac{364}{9} \; B$$

Write an equivalent double integral with the order of integration reversed.

20) $\int_0^9 \int_0^x dy\,dx$

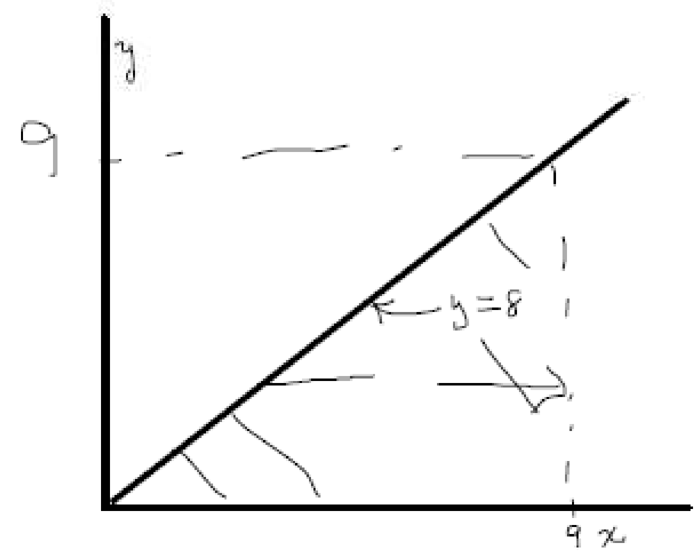
A) $\int_0^9 \int_9^y dx\,dy$

B) $\int_0^x \int_0^9 dx\,dy$

C) $\int_0^9 \int_{-9}^y dx\,dy$

D) $\int_0^9 \int_y^9 dx\,dy$

$\int_0^9 \int_0^x dy\,dx = \int_0^9 \int_y^9 dx\,dy$ D



Find the volume of the indicated region.

21) the region under the surface $z = x^2 + y^4$, and bounded by the planes $x = 0$ and $y = 25$ and the cylinder $y = x^2$

A) $\frac{292,982,500}{33}$ B) $\frac{11,732,500}{33}$ C) $\frac{2,357,500}{33}$ D) $\frac{58,607,500}{33}$

$$z = x^2 + y^2$$

$$V = \int_0^5 \int_{x^2}^{25} \int_0^{x^2+y^4} dz\,dy\,dx = \int_0^5 \int_{x^2}^{25} (x^2+y^4)\,dy\,dx$$

$$= \int_0^5 \left(x^2 y + \frac{y^5}{5}\right)\Big|_{x^2}^{25} dx = \int_0^5 \left(25x^2 + \frac{(25)^5}{5} - x^4 - \frac{x^{10}}{5}\right) dx$$

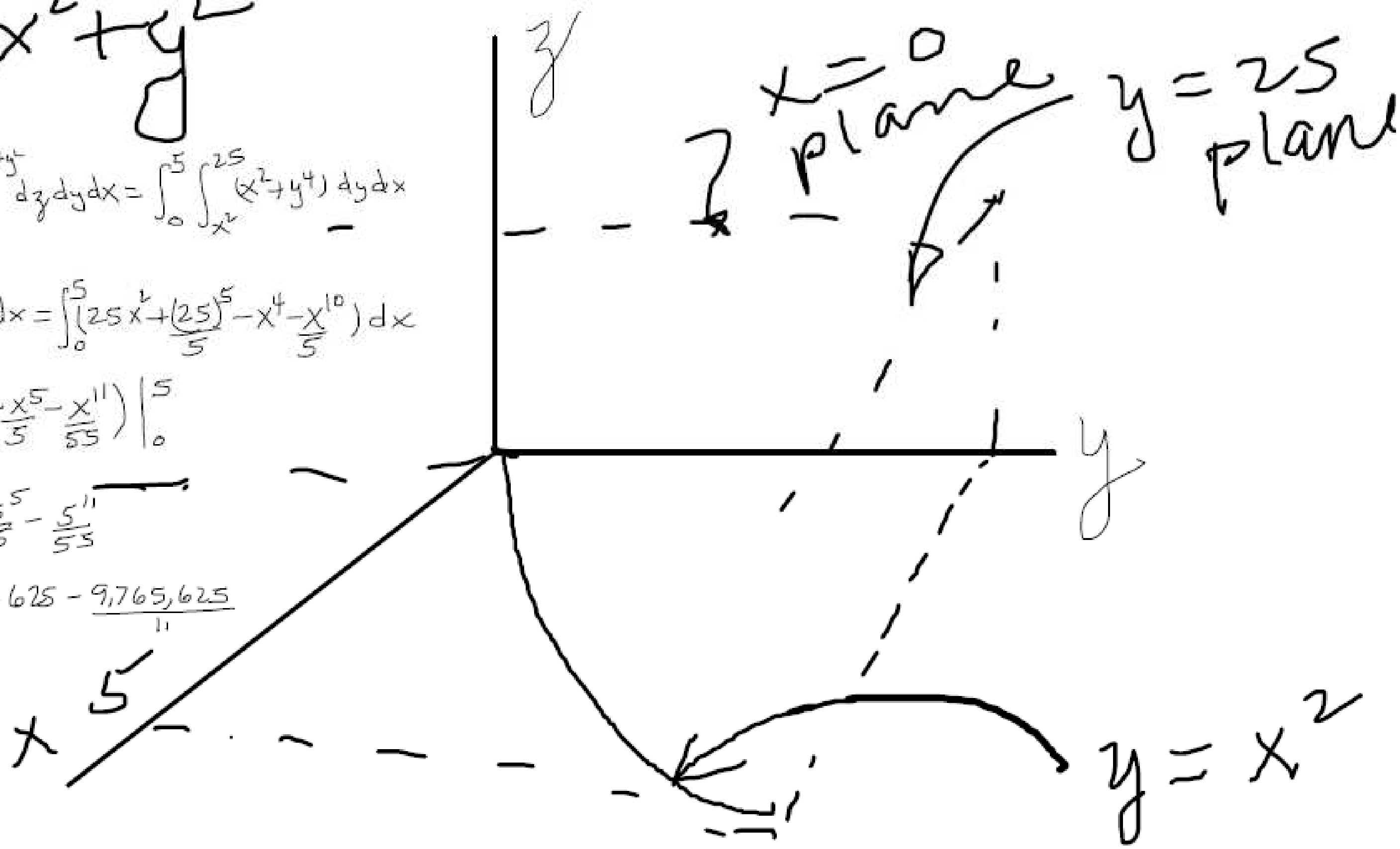
$$= \left(\frac{25x^3}{3} + \frac{(25)^5}{5}x - \frac{x^5}{5} - \frac{x^{11}}{55}\right)\Big|_0^5$$

$$= \frac{25(125)}{5} + \frac{(25)^5 5}{5} - \frac{5^5}{5} - \frac{5^{11}}{55}$$

$$= \frac{3125}{3} + 9,765,625 - 625 - \frac{9,765,625}{11}$$

$$= 292,982,500$$

A



Find the volume of the indicated region.

22) the region bounded by the paraboloid $z = 100 - x^2 - y^2$ and the xy-plane

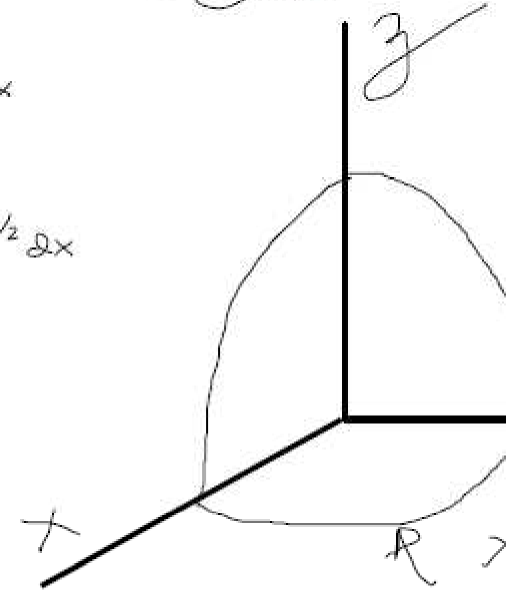
A) $\frac{10000}{3}\pi$ B) $\frac{5000}{3}\pi$ C) 5000π D) 2500π

$$V = 4\int_0^{10}\int_0^{\sqrt{100-x^2}}\int_0^{100-x^2-y^2} 1\,dz\,dy\,dx = 4\int_0^{10}\int_0^{\sqrt{100-x^2}} (100-x^2-y^2)\,dy\,dx$$

$$= 4\int_0^{10}\left[(100-x^2)y - \frac{y^3}{3}\right]\Big|_0^{(100-x^2)^{1/2}} dx = 4\int_0^{10} (100-x^2)^{3/2} - \frac{(100-x^2)^{3/2}}{3}\,dx$$

$$= 4\int_0^{10} \frac{2}{3}(1000-x^2)^{3/2}\,dx$$ do on cal

$= 15{,}707.963 = 5{,}000\pi$ C



we'll do 1/4 and multiply by 4

$x^2 + y^2 = 100$

Solve the problem.

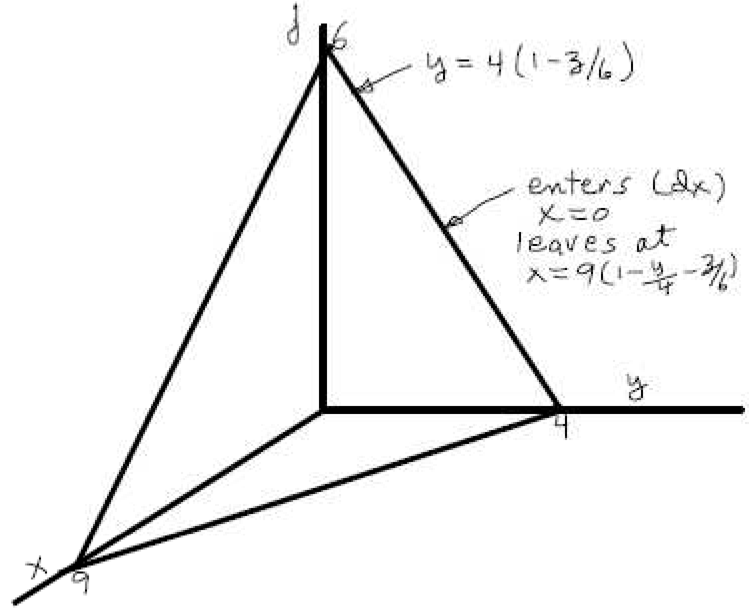
23) Write an iterated triple integral in the order dx dy dz for the volume of the tetrahedron cut from the first octant by the plane $\frac{x}{9} + \frac{y}{4} + \frac{z}{6} = 1$.

A) $\int_0^6 \int_0^{1 - y/4} \int_0^{1 - y/4 - z/6} dx\, dy\, dz$

B) $\int_0^6 \int_0^{9(1 - y/4)} \int_0^{9(1 - y/4 - z/6)} dx\, dy\, dz$

C) $\int_0^6 \int_0^{1 - z/6} \int_0^{1 - y/4 - z/6} dx\, dy\, dz$

D) $\int_0^6 \int_0^{4(1 - z/6)} \int_0^{9(1 - y/4 - z/6)} dx\, dy\, dz$



$$V = \int_0^6 \int_0^{4(1-z/6)} \int_0^{9(1-y/4-z/6)} 1\, dx\, dy\, dz$$

D

Evaluate the integral.

24) $\displaystyle\int_{-1}^{1}\int_{0}^{5}\int_{0}^{1} (x^2 + y^2 + z^2)\,dx\,dy\,dz$

A) 126 B) 90 C) 23.2 D) 124

$= \displaystyle\int_{-1}^{1}\int_{0}^{5}\left(\frac{x^3}{3} + (y^2+z^2)x\right)\Big|_0^1 dy\,dz = \int_{-1}^{1}\int_{0}^{5}\left(\frac{1}{3} + y^2 + z^2\right)dy\,dz = \int_{-1}^{1}\left(\frac{y}{3} + \frac{y^3}{3} + z^2 y\right)\Big|_0^5 dz = \int_{-1}^{1}\left(\frac{5}{3} + \frac{125}{3} + 5z^2\right)dz$

$= \left(\frac{5z}{3} + \frac{125z}{3} + \frac{5z^3}{3}\right)\Big|_{-1}^{+1} = 5/3 + \frac{125}{3} + \frac{5}{3} + \frac{5}{3} + \frac{125}{3} + \frac{5}{3} = \frac{270}{3} = 90$ B

Find the volume of the indicated region.

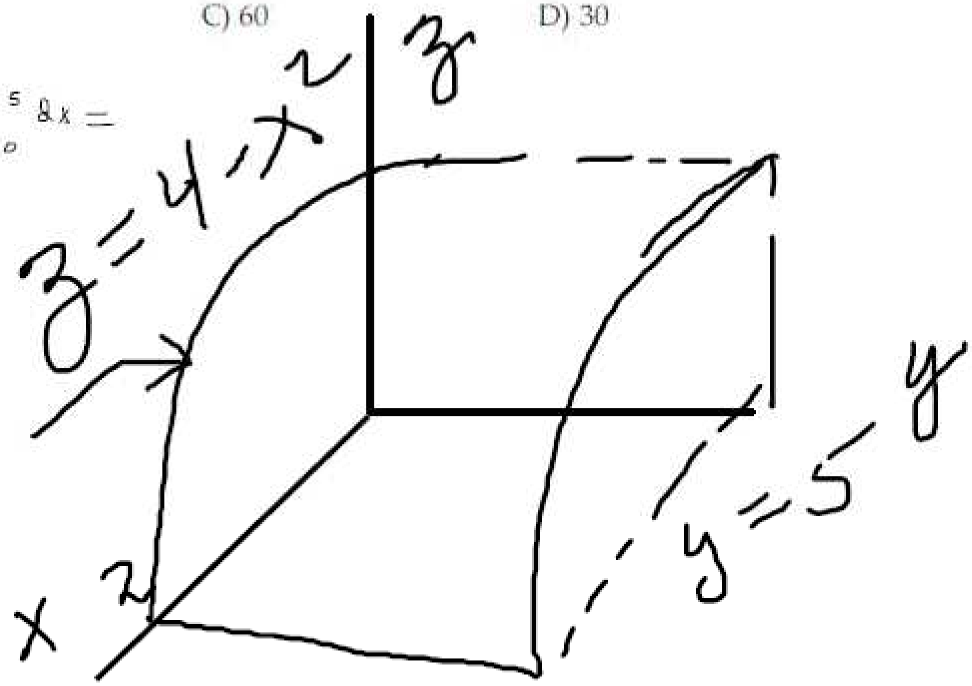
25) the region bounded by the coordinate planes, the parabolic cylinder $z = 4 - x^2$, and the plane $y = 5$

A) 80 B) $\frac{80}{3}$ C) 60 D) 30

$$V = \int_0^2 \int_0^5 \int_0^{4-x^2} dz\,dy\,dx = \int_0^2 \int_0^5 (4-x^2)\,dy\,dx = \int_0^2 (4-x^2)y \Big|_0^5 dx =$$

$$\int_0^2 (20-5x^2)\,dx = \left(20x - \frac{5x^3}{3}\right)\Big|_0^2$$

$$= 40 - \frac{40}{3} = \frac{80}{3} \quad \text{B}$$



Find the volume of the indicated region.

26) the region bounded by the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 16$

A) $\frac{256}{3}\pi$ B) 128π C) $\frac{1024}{3}\pi$ D) 384π

We'll do 1/4 and quadruple it

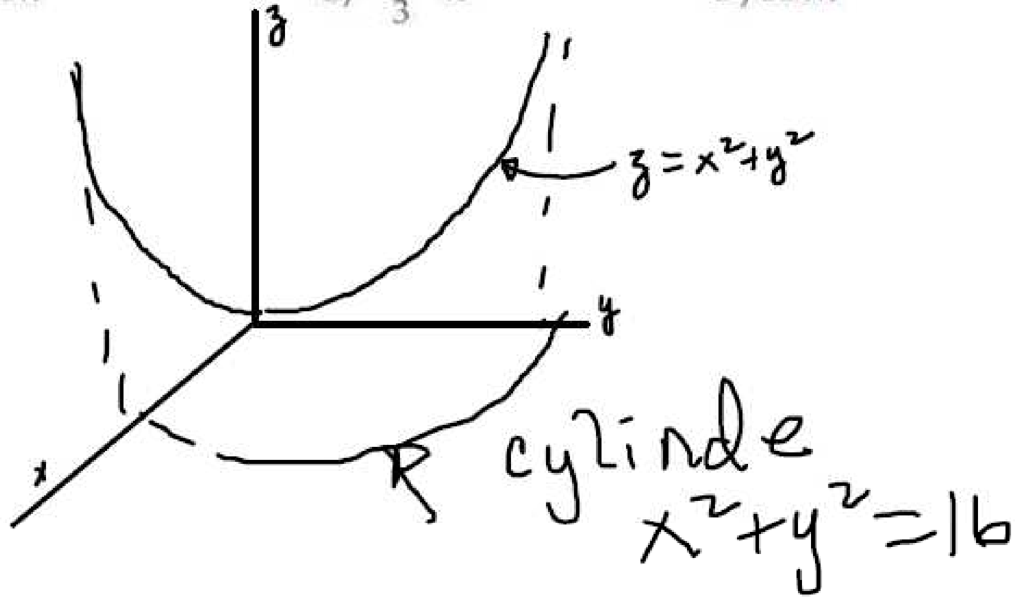
$$V = 4\int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{x^2+y^2} dz\,dy\,dx$$

$$= 4\int_0^4 \int_0^{\sqrt{16-x^2}} (x^2+y^2)\,dy\,dx = 4\int_0^4 x^2 y + \frac{y^3}{3} \Big|_0^{\sqrt{16-x^2}} dx$$

$$= 4\int_0^4 x^2\sqrt{16-x^2} + \frac{(16-x^2)^{3/2}}{3}\,dx$$

$= 402.12396 = 128\pi$

B



Evaluate the line integral along the curve C.

27) $\int_C \frac{x + y + z}{5}\, ds$, C is the curve $\mathbf{r}(t) = 4t\mathbf{i} + (8\cos\frac{3}{8}t)\mathbf{j} + (8\sin\frac{3}{8}t)\mathbf{k}$, $0 \le t \le \frac{8}{3}\pi$

A) $\frac{128}{9}\pi$ B) $\frac{128}{9}\pi^2 + \frac{256}{3}$ C) $\frac{128}{9} + \frac{128}{3}$ D) $\frac{128}{9}\pi^2 + \frac{128}{3}$

$\int_C f(x,y,z)\,ds = \int f(x(t),y(t),z(t))\,|v(t)|\,dt$, $v(t) = \frac{dr}{dt} = 4 - 3\sin\frac{3}{8}t\, j + 3\cos\frac{3t}{8}\, k$, $|v(t)| = \sqrt{16+9} = 5$

$\oint_C f\,ds = \int_0^{\frac{8}{3}\pi} \underbrace{4t + 8\cos\frac{3}{8}t + 8\sin\frac{3}{8}t}_{\cancel{5}}\,(\cancel{5})\,dt = \left(2t^2 + \frac{64}{3}\sin\frac{3}{8}t - \frac{64}{3}\cos\frac{3}{8}t\right)\Big|_0^{\frac{8}{3}\pi} = 2\left(\frac{64}{9}\right)\pi^2 + 0 + \frac{64}{3} + \frac{64}{3}$

$= \frac{128\pi^2}{9} + \frac{128}{3}$ D

Evaluate the line integral along the curve C.

28) $\int_C (y + z)\, ds$, C is the path from (0, 0, 0) to (−3, 3, 1) given by:

C_1: $\mathbf{r}(t) = -3t^2\mathbf{i} + 3t\mathbf{j},\ 0 \le t \le 1$

C_2: $\mathbf{r}(t) = -3\mathbf{i} + 3\mathbf{j} + (t-1)\mathbf{k},\ 1 \le t \le 2$

A) $\frac{25}{2}$ B) $\frac{13}{12}$ C) $\frac{15}{4}\sqrt{5} - \frac{11}{4}$ D) $\frac{15}{4}\sqrt{5} + \frac{11}{4}$

C_1: $r(t) = -3t^2 i + 3t j \quad 0 \le t \le 1$

$v(t) = -6t i + 3 j \quad |v| = \sqrt{36t^2+9}$, $x = -3t^2$, $y = 3t$, $z = 0$, C_2: $r(t) = -3i + 3j + (t-1)k$

$v = 1 \quad |v| = 1$

$x = -3,\ y = 3,\ z = t-1$

$$\int_C y + z\, ds = \underbrace{\int_0^1 3t\sqrt{36t^2+9}\, dt}_{C_1} + \int_1^2 (3 + t - 1)(1)\, dt = \int_0^1 9t\sqrt{4t^2+1}\, dt + \int_1^2 (2+t)\, dt$$

$\int_1^2 2+t\, dt = \left(2t + \frac{t^2}{2}\right)\Big|_1^2 = 4 + 2 - 2 - \frac{1}{2} = \frac{7}{2}$ For 1st integral $u = 4t^2+1$, $du = 8t\,dt$, $\int_0^1 9t\sqrt{4t^2+1}\, dt = \int_1^5 \frac{9}{8} u^{1/2}\, du$

$= \frac{9}{8}\left(\frac{2}{3}\right) u^{3/2}\Big|_1^5 = \frac{3}{4} u^{3/2}\Big|_1^5 = \frac{3}{4}\sqrt{125} - 3/4 = \frac{15}{4}\sqrt{5} - 3/4$, So, total $= \frac{15}{4}\sqrt{5} - 3/4 + \frac{14}{4} = \frac{15}{4}\sqrt{5} + \frac{11}{4}$

D

Evaluate the line integral along the curve C.

29) $\int_C \frac{1}{x^2 + y^2 + z^2}\, ds$, C is the path given by:

C_1: $\mathbf{r}(t) = (5\cos t)\mathbf{i} + (5\sin t)\mathbf{j}$ from (5, 0, 0) to (0, 5, 0)

C_2: $\mathbf{r}(t) = (5\sin t)\mathbf{j} + (5\cos t)\mathbf{k}$ from (0, 5, 0) to (0, 0, 5)

C_3: $\mathbf{r}(t) = (5\sin t)\mathbf{i} + (5\cos t)\mathbf{k}$ from (0, 0, 5) to (5, 0, 0)

A) $\frac{3}{10}\pi$ B) $\frac{\pi}{10}$ C) $-\frac{3}{10}\pi$ D) 0

C_1: $r(t) = 5\cos t\, i + 5\sin t\, j$ $|v| = 5$ $x = 5\cos t$ $y = 5\sin t$ $z = 0$

$v(t) = -5\sin t\, i + 5\cos t\, j$

$(5,0,0) \to (0,5,0) \Rightarrow 0 \le t \le \frac{\pi}{2}$

C_2: $r(t) = 5\sin t\, j + 5\cos t\, k$ $|v| = 5$

$v(t) = 5\cos t\, j - 5\sin t\, k$

$(0,5,0) \to (0,0,5) = \frac{\pi}{2} \le t \le 0$

C_3: $r(t) = 5\sin t\, i + 5\cos t\, k$

$v(t) = 5\cos t\, i - 5\sin t\, k$

$(0,0,5)$ to $(5,0,0) \Rightarrow 0 \le t \le \frac{\pi}{2}$

Notice that in all cases $\frac{1}{x^2+y^2+z^2} = \frac{1}{25}$

So, $\int_C \frac{1}{x^2+y^2+z^2}\, ds = \int_0^{\frac{\pi}{2}} \frac{1}{25}(5)\, dt + \int_{\frac{\pi}{2}}^{0} \frac{1}{25}(5)\, dt + \int_0^{\frac{\pi}{2}} \frac{1}{25}(5)\, dt = \frac{\pi}{10} - \frac{\pi}{10} + \frac{\pi}{10} = \frac{\pi}{10}$ B

Find the work done by F over the curve in the direction of increasing t.

30) $\mathbf{F} = -6y\mathbf{i} + 6x\mathbf{j} + 9z^3\mathbf{k}$; C: $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j}$, $0 \le t \le 7$

A) $W = 42$ B) $W = 0$ C) $W = 147$ D) $W = 84$

$W = \int_C F \cdot \frac{dr}{dt}\,dt$ $x = \cos t$, $y = \sin t$ $\frac{dr}{dt} = -\sin t\, i + \cos t\, j$, $W = \int_0^7 (-6\sin t\, i + 6\cos t\, j) \cdot (-\sin t\, i + \cos t\, j)\,dt$

$= \int_0^7 (6\sin^2 t + 6\cos^2 t)\,dt = \int_0^7 6\,dt = 42$ A

Calculate the flux of the field F across the closed plane curve C.

31) $\mathbf{F} = y^3\mathbf{i} + x^2\mathbf{j}$; the curve C is the closed counterclockwise path formed from the semicircle $\mathbf{r}(t) = 5\cos t\mathbf{i} + 5\sin t\mathbf{j}$, $0 \le t \le \pi$, and the straight line segment from $(-5, 0)$ to $(5, 0)$

A) $-\frac{50}{3}$ B) $\frac{50}{3}$ C) 0 D) $\frac{100}{3}$

$\text{FLUX} = \oint_C M\,dy - N\,dx$ for C1: $x = 5\cos t$ $dx = -5\sin t\,dt$, $y = 5\sin t$, $dy = 5\cos t\,dt$
$0 \le t \le \pi$

For C2 $x = 5t$ $-1 \le t \le 1$
$y = 0$

$\text{FLUX} = \oint_C M\,dy - N\,dx = \oint y^3\,dy - x^2\,dx$

$= \int_0^{\pi} 125\sin^3 t\, 5\cos t\,dt - 25\cos^2 t(-5\sin t)\,dt + \int_{-1}^{1} 0^3\,dy - 25t^2(5)\,dt$

C1 C2

$= \int_0^{\pi} 625\sin^3 t\cos t + \int_0^{\pi} 125\cos^2 t\sin t\,dt - 125\int_{-1}^{+1} t^2\,dt = 83.333 - 125\frac{t^3}{3}\Big|_{-1}^{+1}$ C

$= 0$ $\qquad$ 83.333

$= 83.333 - 83.333 = 0$

Calculate the circulation of the field F around the closed curve C.

32) $\mathbf{F} = xy\mathbf{i} + 3\mathbf{j}$, curve C is $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$, $0 \le t \le 2\pi$

A) $\frac{10}{3}$ B) 6 C) $\frac{26}{3}$ D) 0

Circulation = Flow around closed loop $= \int F \cdot \frac{dr}{dt}\, dt$

$x = 2\cos t$ $\quad r = 2\cos t\, i + 2\sin t\, j$

$y = 2\sin t$ $\quad \frac{dr}{dt} = -2\sin t\, i + 2\cos t\, j$

$$cir = \int_0^{2\pi} (4\sin t\cos t\, i + 3j)\cdot(-2\sin t\, i + 2\cos t\, j)\, dt = \int_0^{2\pi} (\underbrace{-8\cos t\sin^2 t}_{\to 0} + \underbrace{6\cos t}_{\to 0})\, dt = 0 \quad D$$

Calculate the flow in the field F along the path C.

33) $\mathbf{F} = y^2\mathbf{i} + z\mathbf{j} + x\mathbf{k}$; C is the curve $\mathbf{r}(t) = (2 + 2t)\mathbf{i} + 3t\mathbf{j} - 3t\mathbf{k}$, $0 \le t \le 1$

A) 39 B) $\frac{9}{2}$ C) $-\frac{15}{2}$ D) -3

$\text{Flow} = \int_C F \cdot \frac{dr}{dt}\,dt$, $r = (2+2t)i + 3tj - 3tk$, $\frac{dr}{dt} = (2i + 3j - 3k)$ $x = 2+2t$, $y = 3t$, $z = -3t$

$\text{FLOW} = \int_0^1 (9t^2 i - 3tj + 2+2tk)\cdot(2i + 3j - 3k)\,dt = \int_0^1 18t^2 - 9t - 6 - 6t\ dt = \int_0^1 (18t^2 - 15t - 6)\,dt$

$= \left(6t^3 - \frac{15}{2}t^2 - 6t\right)\Big|_0^1$

$= 6 - \frac{15}{2} - 6 = -\frac{15}{2}$ (C)

Find the gradient field of the function.

34) $f(x, y, z) = x^7y^8 + \dfrac{x^3}{z^4}$

A) $\nabla f = (7x^6 + 3x^2)\mathbf{i} + 8y^7\mathbf{j} - \dfrac{4}{z^5}\mathbf{k}$

B) $\nabla f = 7x^6y^8\mathbf{i} + 8x^7y^7\mathbf{j} - \dfrac{4x^7}{z^5}\mathbf{k}$

C) $\nabla f = \left(7x^6y^8 + \dfrac{3x^2}{z^4}\right)\mathbf{i} + 8x^7y^7\mathbf{j} - \dfrac{4x^3}{z^5}\mathbf{k}$

D) $\nabla f = (7x^6 + 3x^2)\mathbf{i} + 8y^7\mathbf{j} + \dfrac{4}{z^5}\mathbf{k}$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} i + \frac{\partial f}{\partial z} k$$

$$= \left(7x^6y^8 + \frac{3x^2}{z^4}\right) i + 8x^7y^7 j - \frac{4x^3}{z^5} k \quad C$$

Find the potential function f for the field F.

35) $\mathbf{F} = \frac{1}{z}\mathbf{i} - 6\mathbf{j} - \frac{x}{z^2}\mathbf{k}$

A) $f(x, y, z) = \frac{x}{z} - 6y + C$

B) $f(x, y, z) = \frac{x}{z} + C$

C) $f(x, y, z) = \frac{2x}{z} - 6y + C$

D) $f(x, y, z) = \frac{x}{z} - 6 + C$

$\frac{\partial f}{\partial x} = \frac{1}{z}$ $\frac{\partial f}{\partial y} = -6$, $\frac{\partial f}{\partial z} = -\frac{x}{z^2}$, $f = \frac{x}{z} + g(y,z)$, $\frac{\partial f}{\partial y} = 0 + \frac{\partial g}{\partial y} = -6 \Rightarrow g = -6y + h(z)$

so far $f = \frac{x}{z} - 6y + h(z)$

$\frac{\partial f}{\partial z} = -\frac{x}{z^2} + \frac{dh}{dz} = -\frac{x}{z^2} \Rightarrow$ $h = c$, so $f = \frac{x}{z} - 6y + c$

A

Evaluate the work done between point 1 and point 2 for the conservative field F.

36) $\mathbf{F} = 6 \sin 6x \cos 4y \cos 6z\mathbf{i} + 4 \cos 6x \sin 4y \cos 6z\mathbf{j} + 6 \cos 6x \cos 4y \sin 6z\mathbf{k}$; $P_1(0, 0, 0)$, P_2 $\left(\frac{1}{3}\pi, \frac{1}{2}\pi, \frac{\pi}{6}\right)$

A) W = 1 B) W = 0 C) W = -2 D) W = 2

Need to fint potential function $\frac{\partial f}{\partial x} = 6 \sin 6x \cos 4y \cos 6z \Rightarrow f = -\cos 6x \cos 4y \cos 6z + g(y,z)$

$\frac{\partial f}{\partial y} = 4 \cos 6x \sin 4y \cos 6z + \frac{\partial g}{\partial y} = 4 \cos 6x \sin 4y \cos 6z \Rightarrow \frac{\partial g}{\partial y} = 0 \quad g = h(z)$

so far $f = -6 \cos 6x \cos 4y \cos 6z + h(z)$, $\frac{\partial f}{\partial z} = 6 \cos 6x \cos 4y \sin 6z + \frac{dh}{dz} = 6 \cos 6x \cos 4y \sin 6z$

$\Rightarrow \frac{dh}{dz} = 0, \; h = C$ so $f = -\cos 6x \cos 4y \cos 6z + C$, work $= f(\frac{\pi}{3}, \frac{\pi}{2}, \frac{\pi}{6}) - f(0,0,0)$

$= -\overset{1}{\cos 2\pi}\; \overset{1}{\cos 2\pi}\; \overset{-1}{\cos \pi} + \cos 0 \cos 0 \cos 0 = 1 + 1 = 2$

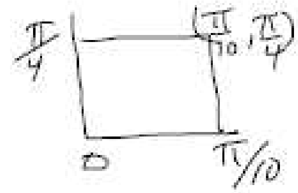
D

Using Green's Theorem, find the outward flux of F across the closed curve C.

37) $\mathbf{F} = \sin 10y\mathbf{i} + \cos 4x\mathbf{j}$; C is the rectangle with vertices at $(0, 0)$, $\left(\frac{\pi}{10}, 0\right)$, $\left(\frac{\pi}{10}, \frac{\pi}{4}\right)$, and $\left(0, \frac{\pi}{4}\right)$

A) $-\frac{2}{5}\pi$ B) 0 C) $-\frac{1}{5}\pi$ D) $\frac{1}{5}\pi$

outward flux across closed curve C



Use Green's Theorem

$$\oint F \cdot n\,ds = \iint_R \left(\overset{0}{\frac{\partial M}{\partial x}} + \overset{0}{\frac{\partial N}{\partial y}}\right) dx\,dy = 0$$

(B)