Exam Math 2013 Practice Test 1

Name________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find an equation for the sphere with the given center and radius.

1) Center (−4, 7, 0), radius = 8

A) $x^2 + y^2 + z^2 + 8x - 14y = -1$

B) $x^2 + y^2 + z^2 + 8x + 14y = -1$

C) $x^2 + y^2 + z^2 - 8x - 14y = -1$

D) $x^2 + y^2 + z^2 - 8x + 14y = -1$

Equation for sphere

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

$$(x-(-4))^2 + (y-7)^2 + z^2 = 64$$

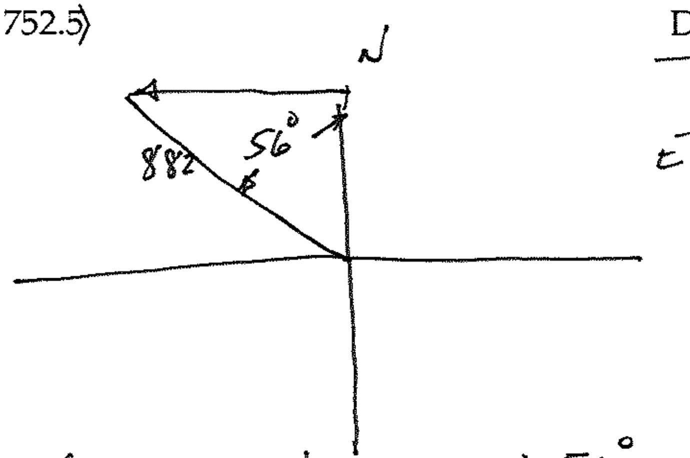
$$x^2 + 8x + 16 + y^2 - 14y + 49 + z^2 = 64$$

$$x^2 + y^2 + 8x - 14y + 65 = 64$$

$$\boxed{x^2 + y^2 + 8x - 14y = -1}$$

2) An airplane is flying in the direction 56° west of north at 882 km/hr. Find the component form of the velocity of the airplane, assuming that the positive x–axis represents due east and the positive y–axis represents due north.

A) $\langle -493.2, 731.2 \rangle$
B) $\langle -0.8290, 0.5592 \rangle$
C) $\langle 460.0, 752.5 \rangle$
D) $\langle -731.2, 493.2 \rangle$



x component: $\frac{|x|}{882} = \sin 56°$ $\quad |x| = 731.2$ neg

y $\quad |y| = 882 \cos 56° = 493.2$

$(-731.2, 493.2)$

or $\quad -731.2\, i + 493.2\, j$

Give a geometric description of the set of points whose coordinates satisfy the given conditions.

3) $6 \leq y \leq 7, 6 \leq z \leq 7$

A) The cube located in the first quadrant and with sides 6 units in length

B) The square with corners at (0, 6, 6), (0, 6, 7), (0, 7, 6), and (0, 7, 7)

C) The line between the points (0, 6, 6) and (0, 7, 7)

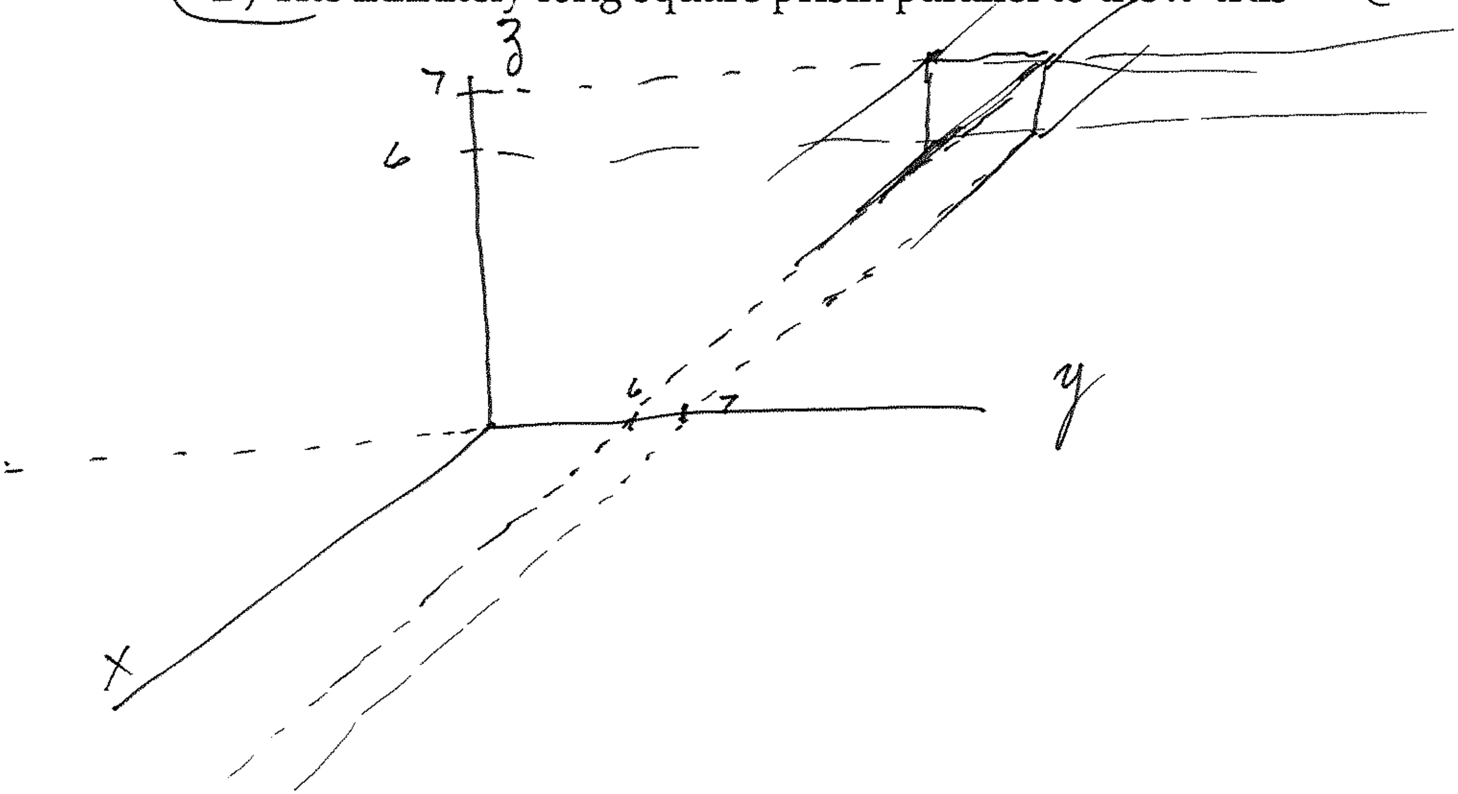
D) The infinitely long square prism parallel to the x-axis

(Need to say

$6 \leq y \leq 7$

$6 \leq z \leq 7$

Defines a square for any value of x



Write one or more inequalities that describe the set of points.

4) The exterior of the sphere of radius 1 centered at the point (2, −5, −3)

A) $(x + 2)^2 + (x - 5)^2 + (x - 3)^2 < 1$

B) $(x - 2)^2 + (x + 5)^2 + (x + 3)^2 < 1$

C) $(x + 2)^2 + (x - 5)^2 + (x - 3)^2 \geq 1$

D) $(x - 2)^2 + (\cancel{x}^{y} + 5)^2 + (\cancel{x}^{z} + 3)^2 > 1$

error

$$(x-2)^2 + (y+5)^2 + (y+3)^2 > 1$$

Find the angle between u and v in radians.

5) $\mathbf{u} = 2\mathbf{j} - 4\mathbf{k}$, $\mathbf{v} = 9\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$

A) 0.44 B) 1.56 C) 1.13 D) 1.44

$$\theta = \cos^{-1}\left(\frac{u_1 v_1 + u_2 v_2 + u_3 v_3}{|\mathbf{u}||\mathbf{v}|}\right) = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$$

$$\theta = \cos^{-1}\left[\frac{(0i + 2j - 4k)\cdot(9i - 4j - 8k)}{\sqrt{4+16}\ \sqrt{81+16+64}}\right]$$

$$\theta = \cos^{-1}\left(\frac{-8 + 32}{\sqrt{20}\ \sqrt{161}}\right) = \cos^{-1}\left[\frac{24}{\sqrt{20}\ \sqrt{161}}\right] \approx 1.13 \text{ rad}$$

or $\approx 64.98^\circ$

Give a geometric description of the set of points whose coordinates satisfy the given conditions.

6) $(x - 4)^2 + (y - 5)^2 + (z - 5)^2 < 1, -5 \le z \le 0$

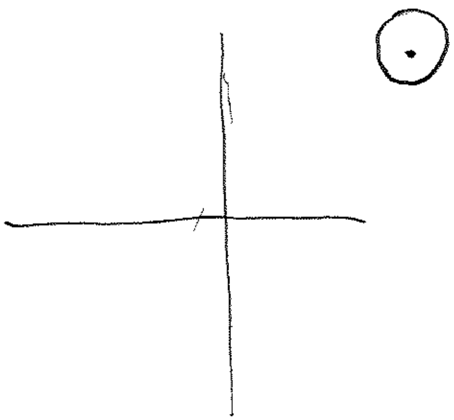
A) All points outside the lower hemisphere centered at (4, 5, 5)

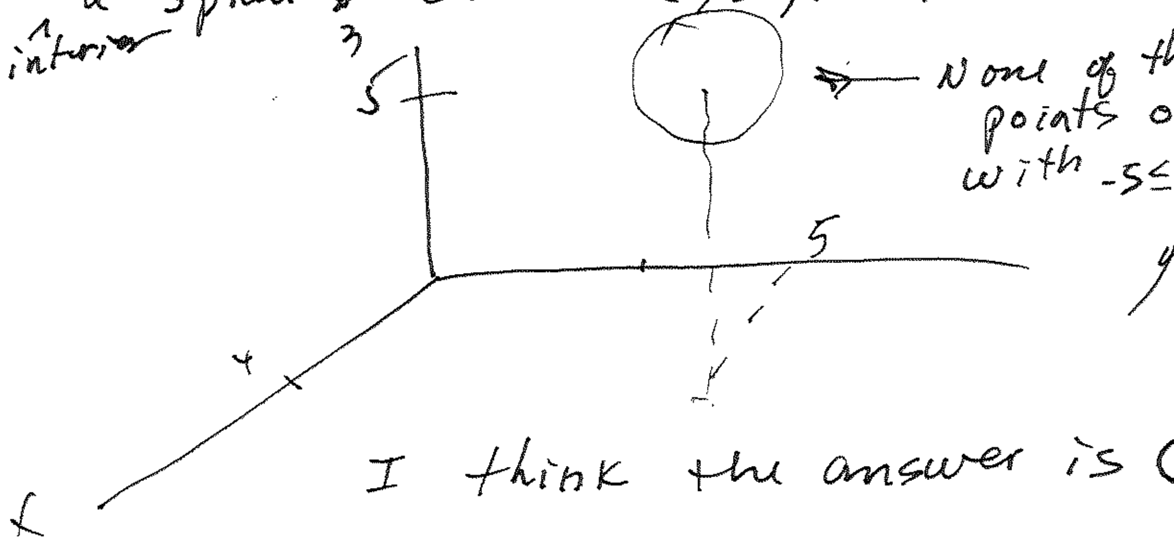
B) All points within the lower hemisphere centered at (4, 5, 5)

C) No set of points satisfy the given relations.

D) All points on the lower hemisphere centered at (4, 5, 5)

1st equation is a interior sphere centered (4, 5, 5) radius 1





None of these points overlap with $-5 \le z \le 0$

I think the answer is C

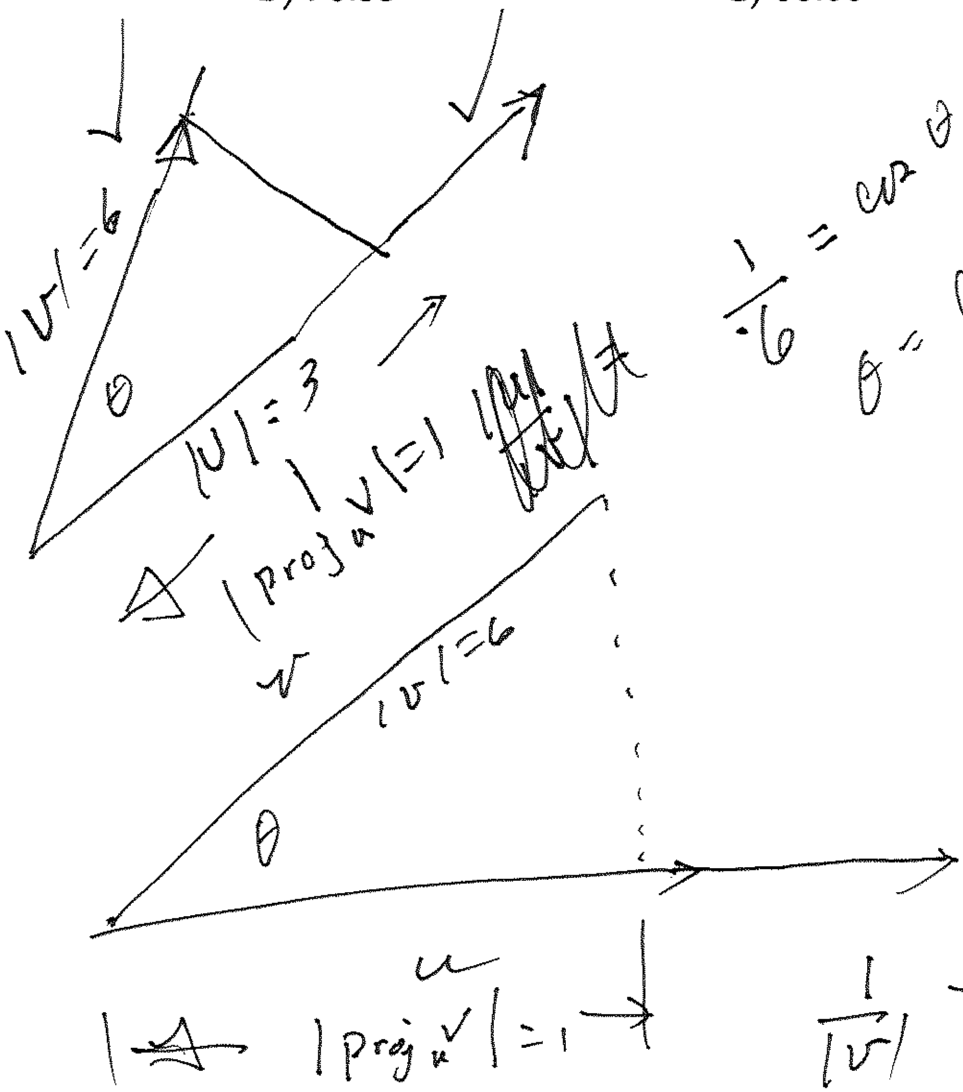
7) For the vectors **u** and **v** with magnitudes $|\mathbf{u}| = 3$ and $|\mathbf{v}| = 6$, find the angle θ between **u** and **v** which makes $|\text{proj}_{\mathbf{u}}\,\mathbf{v}| = 1$

A) 19.47

B) 70.53

C) 60.00

D) 80.41



$\frac{1}{6} = \cos\theta$

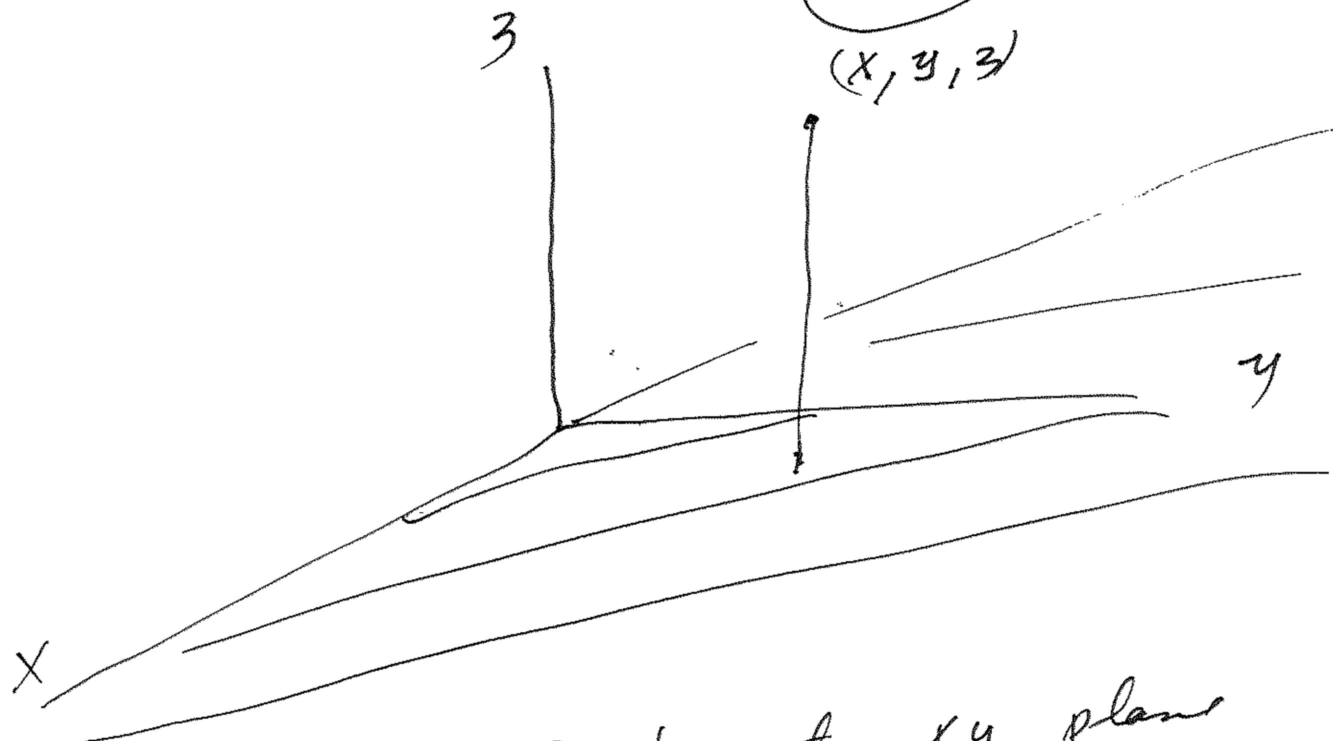
$\theta = \cos^{-1}\frac{1}{6} = 80.41°$

$\frac{1}{|v|} = \cos\theta$

$\theta = \cos^{-1}\frac{1}{6} = 80.41$

8) Find a formula for the distance from the point P(x, y, z) to the xy plane.

A) $\sqrt{x^2 + y^2}$ B) y C) z D) x



Distance to xy plane is just z

Find the indicated vector.

9) Let $\mathbf{u} = \langle -4, -8 \rangle$, $\mathbf{v} = \langle 2, 5 \rangle$. Find $\frac{4}{5}\mathbf{u} + \frac{3}{5}\mathbf{v}$.

A) $\left\langle -2, -\frac{17}{5} \right\rangle$ B) $\left\langle -8, \frac{23}{5} \right\rangle$ C) $\left\langle -\frac{8}{5}, -\frac{9}{5} \right\rangle$ D) $\left\langle -\frac{17}{5}, -2 \right\rangle$

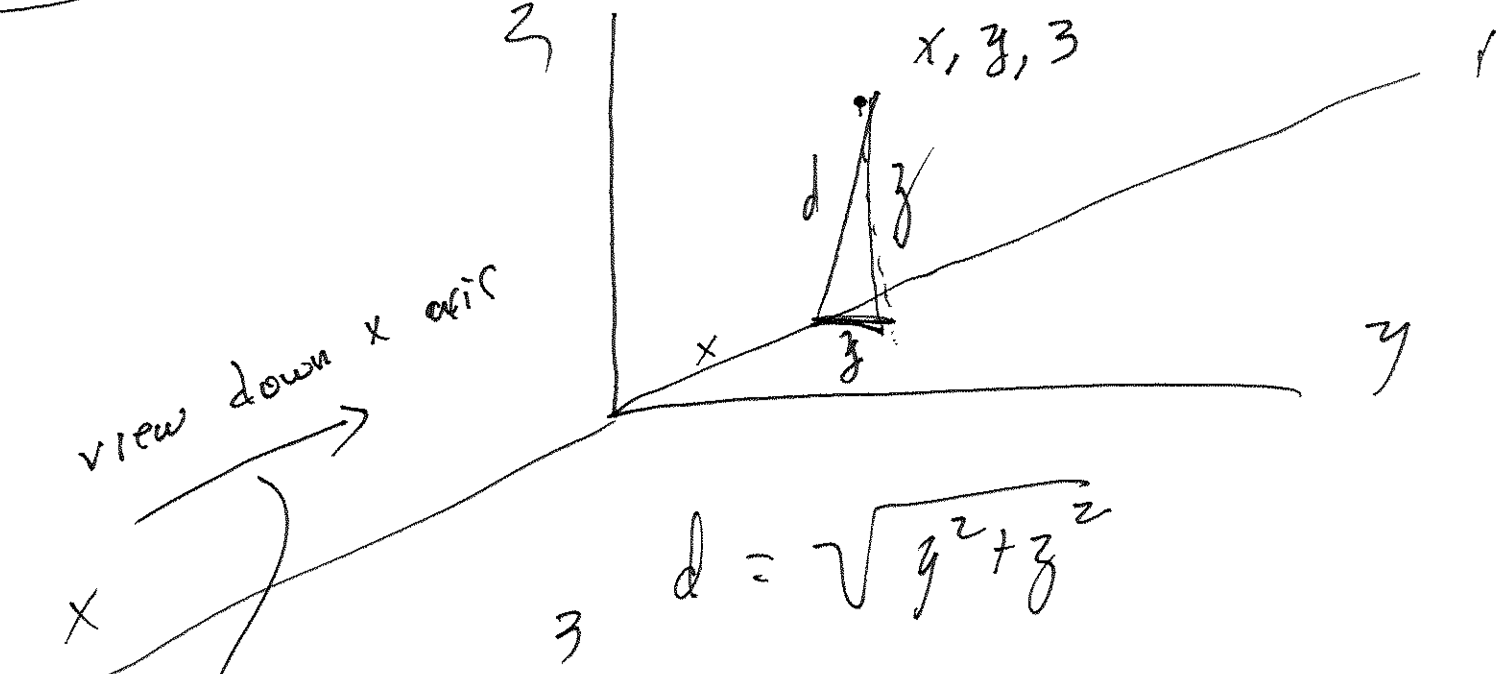
$$\frac{4}{5}u + \frac{3}{5}v = \frac{4}{5}(-4, -8) + \frac{3}{5}(2, 5)$$

$$= \left(\frac{-16}{5}, \frac{-32}{5}\right) + \left(\frac{6}{5}, \frac{15}{5}\right)$$

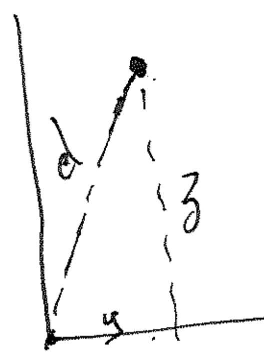
$$= \left(\frac{-10}{5}, \frac{-17}{5}\right)$$

10) Find a formula for the distance from the point P(x, y, z) to the x–axis.

A) $\sqrt{y^2 + z^2}$ B) $\sqrt{x + z}$ C) $\sqrt{y + z}$ D) $\sqrt{x^2 + z^2}$



$d = \sqrt{y^2 + z^2}$



$d^2 = x^2 + z^2$

$d = \sqrt{\cancel{y}^2 + z^2}$

Find the component form of the specified vector.

11) The vector $\overrightarrow{PQ}$, where $P = (6, -9)$ and $Q = (-1, -7)$

A) $\langle 5, -16 \rangle$ B) $\langle -3, -7 \rangle$ C) $\langle 7, -2 \rangle$ D) $\langle -7, 2 \rangle$

$$\overrightarrow{PQ} = (-1-6, -7-(-9))$$

$$= (-7, 2)$$

Express the vector as a product of its length and direction.

12) $4\mathbf{i} + 8\mathbf{j} + 8\mathbf{k}$

A) $12\left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$

B) $12(\mathbf{i} + \mathbf{j} + \mathbf{k})$

C) $12(4\mathbf{i} + 8\mathbf{j} + 8\mathbf{k})$

D) $12\left(\frac{1}{36}\mathbf{i} + \frac{1}{18}\mathbf{j} + \frac{1}{18}\mathbf{k}\right)$

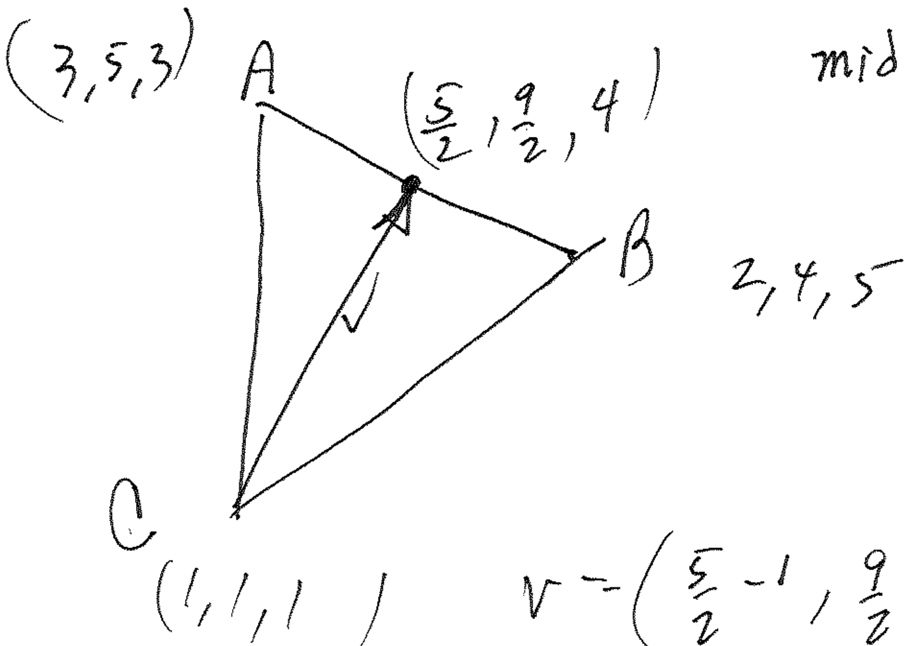
Length $= |u| = \sqrt{16+64+64} = \sqrt{144} = 12$

$\frac{u}{|u|}$ = Direction $\frac{4i+8j+8k}{12} = \left(\frac{1}{3}i + \frac{2}{3}j + \frac{2}{3}k\right)$

Vector $u = 12\left(\frac{1}{3}i + \frac{2}{3}j + \frac{2}{3}k\right)$

13) For the triangle with vertices located at A(3, 5, 3), B(2, 4, 5), and C(1, 1, 1) , find a vector from vertex C to the midpoint of side AB.

A) $\frac{5}{2}\mathbf{i} + \frac{9}{2}\mathbf{j} + 4\mathbf{k}$ B) $\frac{7}{2}\mathbf{i} + \frac{11}{2}\mathbf{j} + 5\mathbf{k}$ C) $\frac{3}{2}\mathbf{i} + \frac{7}{2}\mathbf{j} + 3\mathbf{k}$ D) $\frac{1}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$



midpoint $x_m = \frac{x_1 + x_2}{2}$

$y_m = \frac{y_1 + y_2}{2}$

$z_m = \frac{z_1 + z_2}{2}$

$v = \left(\frac{5}{2} - 1, \frac{9}{2} - 1, 4 - 1\right)$

$= \frac{3}{2} i + \frac{7}{2} j + 3k$

Calculate the direction of $\overrightarrow{P_1P_2}$ and the midpoint of line segment P_1P_2.

14) $P_1(6, -6, -4)$ and $P_2(8, -5, -2)$

A) $\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}; \left(1, \frac{1}{2}, 1\right)$

B) $\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}; \left(4, -\frac{5}{2}, -1\right)$

C) $\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}; \left(7, -\frac{11}{2}, -3\right)$

D) $2\mathbf{i} + 2\mathbf{j} + \frac{4}{3}\mathbf{k}; (3, -3, -2)$

Two independent things Are being asked:

Direction $\dfrac{(8-6)i + (-5-(-6))j + (-2-(-4))k}{|\overrightarrow{P_1P_2}|}$

$= \dfrac{2i + j + 2k}{|2i + j + 2|} = \dfrac{2i + j + 2k}{\sqrt{4+1+4}} = \frac{2}{3}i + \frac{1}{3}j + \frac{2}{3}k$

midpoint $\left(\frac{6+8}{2}, \frac{-6-5}{2}, \frac{-4-2}{2}\right) = \left(7, \frac{-11}{2}, -3\right)$

Find v • u.

15) $\mathbf{v} = 9\mathbf{i} - 2\mathbf{j}$ and $\mathbf{u} = -2\mathbf{i} + 7\mathbf{j}$

A) -32　　B) $7\mathbf{i} + 5\mathbf{j}$　　C) $-18\mathbf{i} - 14\mathbf{j}$　　D) -4

$$= 9(-2) + (-2(7))$$

$$v \cdot u = -18 - 14 = \underline{-32}$$

Find the vector $\text{proj}_{\mathbf{v}}\,\mathbf{u}$.

16) $\mathbf{v} = 3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{u} = 11\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$

A) $\frac{195}{19}\mathbf{i} - \frac{65}{19}\mathbf{j} + \frac{195}{19}\mathbf{k}$

B) $\frac{671}{15}\mathbf{i} + \frac{122}{15}\mathbf{j} + \frac{122}{3}\mathbf{k}$

C) $\frac{671}{225}\mathbf{i} + \frac{122}{225}\mathbf{j} + \frac{122}{45}\mathbf{k}$

D) $\frac{183}{19}\mathbf{i} - \frac{61}{19}\mathbf{j} + \frac{183}{19}\mathbf{k}$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} = \frac{\overset{u\cdot v}{(33 - 2 + 30)}\,\overset{v}{(3i - j + 3k)}}{9 + 1 + 9}$$

$$= \frac{61\,(3i - j + 3k)}{19} = \frac{163i}{19} - \frac{61j}{19} + \frac{183k}{19}$$

Find the angle between u and v in radians.

17) $\mathbf{u} = -2\mathbf{i} - 9\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$

A) 1.80 B) 2.07 C) –0.50 D) 1.58

$$\theta = \cos^{-1}\left(\frac{u_1 v_1 + u_2 v_2 + u_3 v_3}{|\mathbf{u}||\mathbf{v}|}\right) = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$$

$$= \cos^{-1} \frac{-4-27}{\underset{9.2195}{\sqrt{4+81}}\ \underset{(7)}{\sqrt{4+9+36}}} = \cos^{-1}\left[\frac{-31}{\sqrt{85}\sqrt{49}}\right] = 2.07$$

Find the angle between u and v in radians.

18) $\mathbf{u} = 4\mathbf{j} - 6\mathbf{k}$, $\mathbf{v} = 6\mathbf{i} - 9\mathbf{j} - 4\mathbf{k}$

A) 1.72 B) 1.64 C) 1.57 D) −0.14

$$\theta = \cos^{-1}\left(\frac{u_1v_1 + u_2v_2 + u_3v_3}{|\mathbf{u}||\mathbf{v}|}\right) = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$$

$$= \cos^{-1}\left(\frac{0 - 36 + 24}{\sqrt{16+36}\,\sqrt{36+81+16}}\right) = 1.7155 = 1.72$$

Find an equation for the line that passes through the given point and satisfies the given conditions.

19) $P = (-8, 4)$; perpendicular to $\mathbf{v} = -5\mathbf{i} - 3\mathbf{j}$

A) $y - 4 = -\frac{7}{3}(x + 5)$

B) $-3x + 5y = 44$

C) $-5x - 3y = 28$

D) $-5x - 3y = 34$

vector from P to point x, y

$= (x+8,\ y-4)$, To be perpendicular to v their dot product must be zero

$[(x+8)i + (y-4)j] \cdot [-5i - 3j] = 0$

vector from P to x, y

$-5x - 40 - 3y + 12 = 0$

$-5x - 3y - 28 = 0$

$\boxed{-5x - 3y = 28}$

Solve the problem.

20) How much work does it take to slide a box 37 meters along the ground by pulling it with a 242 N force at an angle of 45° from the horizontal?

A) $8954\sqrt{2}$ joules B) $\frac{8954}{\sqrt{2}}$ joules C) 8954 joules D) $4477\sqrt{2}$ joules

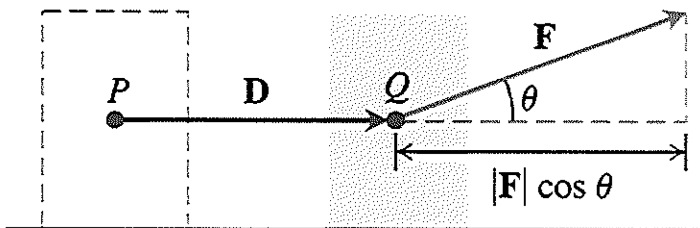


FIGURE 10.25 The work done by a constant force **F** during a displacement **D** is $(|\mathbf{F}|\cos\theta)|\mathbf{D}|$.

$|D| = 37$

$|F| = 242$ N

$\theta = 45^\circ$

$W = 37(\cancel{242}^{121})\frac{\sqrt{2}}{\cancel{2}}$ joules

$W = 4477\sqrt{2}$ joules

Find the length and direction (when defined) of u × v.

21) $\mathbf{u} = 4\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$, $\mathbf{v} = -\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$

A) $6\sqrt{5}; \frac{2\sqrt{5}}{5}\mathbf{i} + \frac{\sqrt{5}}{5}\mathbf{k}$

B) $6\sqrt{5}; \frac{2\sqrt{5}}{5}\mathbf{i} - \frac{\sqrt{5}}{5}\mathbf{k}$

C) $180; \frac{1}{15}\mathbf{i} + \frac{1}{30}\mathbf{k}$

D) $180; \frac{2\sqrt{5}}{15}\mathbf{i} + \frac{\sqrt{15}}{15}\mathbf{j} + \frac{\sqrt{5}}{15}\mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} i & j & k \\ 4 & 2 & 8 \\ -1 & -2 & -2 \end{vmatrix}$$

$$= i(-4+16) - j(-8+8) + k(-8+2)$$

$$= 12i - 6k$$

$$\text{length} = \sqrt{144+36} = \sqrt{180} = \sqrt{36(5)} = 6\sqrt{5}$$

$$\text{Direction} = \frac{12i}{6\sqrt{5}} - \frac{\cancel{6}}{\cancel{6}\sqrt{5}}k$$

$$= \frac{12\sqrt{5}}{30} - \frac{\sqrt{5}}{5}k = \left(\frac{2\sqrt{5}}{5}j - \frac{\sqrt{5}}{5}k\right)$$

Find the length and direction (when defined) of $\mathbf{u} \times \mathbf{v}$.

22) $\mathbf{u} = -\frac{1}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$

A) $2\sqrt{2}; -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \frac{\sqrt{2}}{2}\mathbf{k}$

B) $8; \frac{1}{4}\mathbf{i} - \frac{1}{4}\mathbf{j} + \frac{1}{2}\mathbf{k}$

C) $8; \frac{1}{2}\mathbf{i} - \frac{1}{4}\mathbf{j} - \frac{1}{4}\mathbf{k}$

D) $2\sqrt{3}; \frac{\sqrt{3}}{3}\mathbf{i} + \frac{\sqrt{3}}{3}\mathbf{j} - \frac{\sqrt{3}}{3}\mathbf{k}$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} i & j & k \\ -\frac{1}{2} & \frac{3}{2} & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

4

$$u \times v = i(3-1) - j(-1-1) + k(-2)$$

$$= -2i + 2j - 2k$$

$$|u \times v| = \sqrt{4(3)} = 2\sqrt{3}$$

$$\text{Direction} = \frac{-2}{2\sqrt{3}} + \frac{2}{2\sqrt{3}}j - \frac{2}{2\sqrt{3}}k = -\frac{1}{\sqrt{3}}i + \frac{1}{\sqrt{3}}j - \frac{1}{\sqrt{3}}k$$

$$= \left(-\frac{\sqrt{3}}{3}i + \frac{\sqrt{3}}{3}j - \frac{\sqrt{3}}{3}k\right)$$

Solve the problem.

23) Find the area of the triangle determined by the points P(−3, 6, −4), Q(2, −9, −7), and R(5, −8, −7).

A) $\frac{\sqrt{2590}}{2}$ B) $\sqrt{2590}$ C) $\frac{\sqrt{45,190}}{2}$ D) $\sqrt{45,190}$

Remember Why?

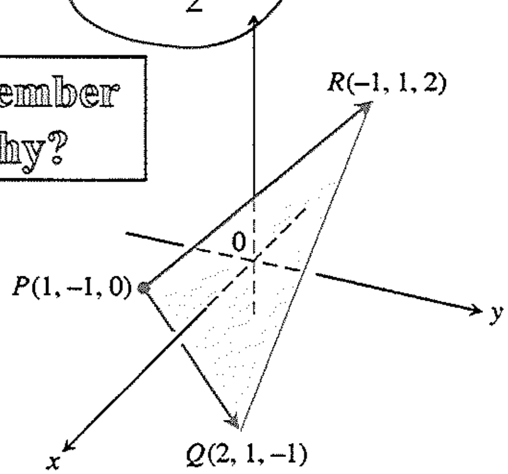


FIGURE 10.30 The area of triangle PQR is half of $|\overrightarrow{PQ} \times \overrightarrow{PR}|$ (Example 2).

$PQ = 5i - 15j - 3k$

$PR = 8i - 14j - 3k$

$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \left\| \begin{matrix} i & j & k \\ 5 & -15 & -3 \\ 8 & -14 & -3 \end{matrix} \right\|$$

$$\begin{array}{r} 15 \\ 8 \\ \hline 120 \end{array}$$

$$= |i(45-42) - j(-15+24) + k(-70+120)|$$

$$= |3i - 9j + 50k|$$

$$\frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2}\sqrt{9+81+2500} = \frac{\sqrt{2590}}{2}$$

Find the triple scalar product (u x v) · w of the given vectors.

24) **u** = 4**i** + 2**j** − **j**; **v** = 7**i** + 6**j** − 6**k**; **w** = 8**i** + 5**j** − 9**k**

A) −197 B) −113 C) −53 D) −22

$$(u \times v) = \begin{vmatrix} i & j & k \\ 4 & 2 & -1 \\ 7 & 6 & -6 \end{vmatrix} = i(-12+6) - j(-24+7) + k(24-14)$$

$$= -6i + 17k + 10k$$

$$(u \times v) \cdot w = (-6i + 17k + 10k) \cdot (8i + 5j - 9k)$$

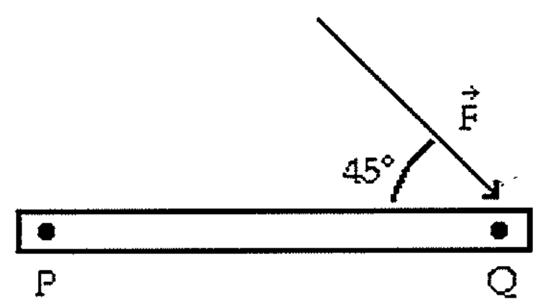
$$= -48 + 85 - 90$$

$$= -53$$

$$\begin{array}{r} -138 \\ 85 \\ \hline 53 \end{array}$$

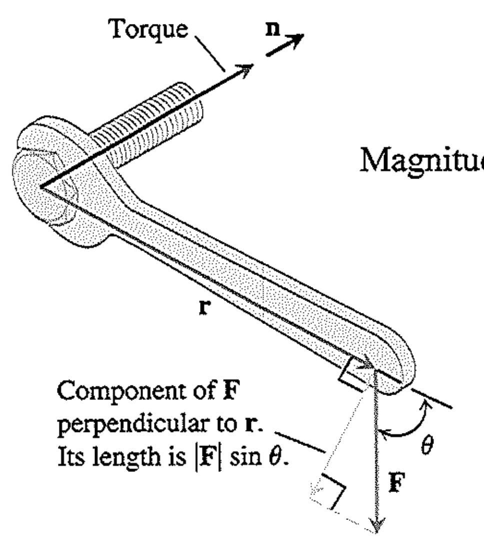
Solve the problem.

25) Find the magnitude of the torque in foot-pounds at point P for the following lever:



$\left|\overrightarrow{PQ}\right| = 8$ in. and $|\mathbf{F}| = 10$ lb

A) 877.56 ft–lb
B) −3900.25 ft–lb
C) 3900.25 ft–lb
D) 80 ft–lb



Magnitude of torque vector $= |\mathbf{r}|\,|\mathbf{F}| \sin \theta$,

$\frac{8}{12}(10) \sin 45°$

$= \frac{\cancel{80}\,40}{12} \frac{\sqrt{2}}{\cancel{2}} = 4{,}714$ ft lbs

$\boxed{?}$

I think this is error

Find parametric equations for the line described below.

26) The line through the point P(5, 1, 5) parallel to the vector $-6\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$

A) $x = 6t - 5,\ y = 4t - 1,\ z = -4t - 5$
B) $x = -6t - 5,\ y = 4t - 1,\ z = -4t - 5$
C) $x = -6t + 5,\ y = 4t + 1,\ z = -4t + 5$
D) $x = 6t + 5,\ y = 4t + 1,\ z = -4t + 5$

Parametric Equations for a Line

The standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ is

$$x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3, \quad -\infty < t < \infty \qquad (3)$$

$x = 5 - 6t,\ y = 1 + 4t\quad z = 5 - 4t$

Find parametric equations for the line described below.

27) The line through the points P(−1, −1, −3) and Q(5, −6, 5)

A) $x = t - 6, y = t + 5, z = -3t - 8$

B) $x = 6t - 1, y = -5t - 1, z = 8t - 3$

C) $x = 6t + 1, y = -5t + 1, z = 8t + 3$

Parametric Equations for a Line

The standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ is

$$x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3, \quad -\infty < t < \infty \qquad (3)$$

vector $\quad 6i - 5j + 8k$

$x = -1 + 6t, \quad y = -1 - 5t, \quad z = -3 + 8t$

Find a parametrization for the line segment beginning at P_1 and ending at P_2.

28) $P_1(7, 3, 1)$ and $P_2(0, 3, -4)$

A) $x = 7t, y = 3t, z = 5t - 4$

B) $x = -7t + 7, y = 3t, z = -5t + 1$

C) $x = -7t + 7, y = 3, z = -5t + 1$

D) $x = 7t, y = 3, z = 5t - 4$

Parametric Equations for a Line

The standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ is

$$x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3, \quad -\infty < t < \infty \qquad (3)$$

$$v = \underline{-7i} \ \underline{-5k} = (0-7)i + (3-3)j + (-4-1)k$$

$$x = 7 - 7t, \quad y = 3, \quad z = 1 - 5t$$

$t = 0$

$\boxed{0 < t < 1}$

Write the equation for the plane.

29) The plane through the point P(3, −3, 7) and normal to **n** = −6**i** − 8**j** + 2**k**.

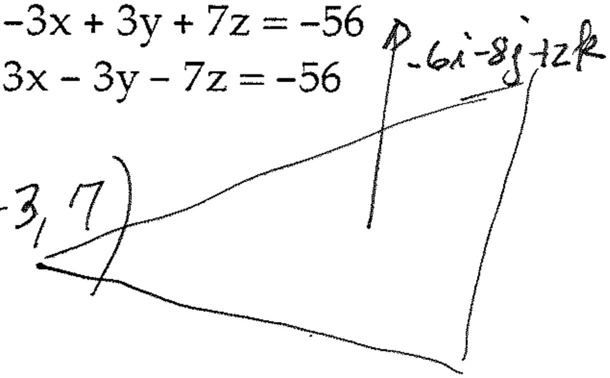
A) $6x + 8y - 2z = -56$
B) $-3x + 3y + 7z = -56$
C) $-6x - 8y + 2z = 20$
D) $3x - 3y - 7z = -56$

Equation for a Plane

The plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$ has

Vector equation:	$\mathbf{n} \cdot \overrightarrow{P_0P} = 0$
Component equation:	$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
Component equation simplified:	$Ax + By + Cz = D$, where $D = Ax_0 + By_0 + Cz_0$

$(3, -3, 7)$



$$-6(x-3) - 8(y+3) + 2(z-7) = 0$$

$$-6x + 18 - 8y - 24 + 2z - 14 = 0$$

$$-6x - 8y + 2z = 20$$

Note: x, y, z is a point on the plane. A vector in the plane containing P is $(x-3)i + (y+3)j + (z-7)k$

if it is normal to n then the dot product must be zero.

$[(x-3)i + (y+3)j + (z-7)k] \cdot [-6i - 8j + 2k] = 0$

$\Rightarrow -6(x-3) - 8(y+3) + 2(z-7) = 0$

Write the equation for the plane.

30) The plane through the point P(5, −7, −7) and parallel to the plane $-8x - 5y + 8z = -67$.

A) $4y = -61$

B) $-8x - 5y + 8z = -51$

C) $-8x - 5y + 8z = -61$

D) $-8x - 5y + 8z = 61$

if Parallel

$$-8x - 5y + 8z = k$$
$$-8(5) - 5(-7) + 8(-7) = k$$
$$-40 + 35 - 56 = k$$
$$-61 = k$$

$$-8x - 5y + 8z = -61$$

Note Any plane parallel to $Ax + By + Cz = k_1$

must be $Ax + By + Cz = k_2$ with a different k

$k_2 = A(x_0) + B(y_0) + Cz_0$

Write the equation for the plane.

31) The plane through the point A(–4, –3, 9) perpendicular to the vector from the origin to A.

A) ~~$4x - 3y - 9z = -2$~~

B) $4x + 3y - 9z = \sqrt{106}$

C) $4x + 3y - 9z = -106$

D) $-4x - 3y + 9z = -106$

Equation for a Plane

The plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$ has

Vector equation:	$\mathbf{n} \cdot \overrightarrow{P_0P} = 0$
Component equation:	$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
Component equation simplified:	$Ax + By + Cz = D$, where
	$D = Ax_0 + By_0 + Cz_0$

from origin

Vector $-4i - 3j + 9k$

$-4(x-(-4)) - 3(y-(-3)) + 9(z-9) = 0$

$-4x - 16 - 3y - 9 + 9z - 81 = 0$

$-4x - 3y + 9z = 106$

$4x + 3y - 9z = -106$

25
81
106

Calculate the requested distance.

32) The distance from the point S(−3, 3, −7) to the line x = 5 + 2t, y = 9 + 6t, z = −10 + 9t

A) $\frac{12564}{121}$ B) $\frac{6\sqrt{349}}{11}$ C) $\frac{6\sqrt{349}}{121}$ D) $\frac{12564}{11}$

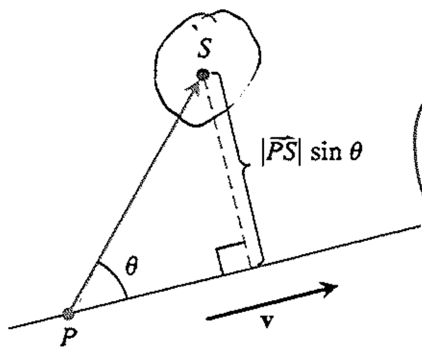


FIGURE 10.37 The distance from S to the line through P parallel to $\mathbf{v}$ is $|\overrightarrow{PS}|\sin\theta$, where θ is the angle between $\overrightarrow{PS}$ and $\mathbf{v}$.

$$d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|}$$

S — point

(−3, 3, −7)

let t=0

P

(5, 9, −10)

Q t = 1

(7, 15, −1)

$\overrightarrow{PS} = -8i - 6j + 3k$

Note: must make two points P and Q on the line (pick t=0, t=1)

$v = 2i + 6j + 9k$.

$|v| = \sqrt{4+36+81} = \sqrt{121} = 11$

$$d = \frac{|\overrightarrow{PS} \times v|}{|v|} = \frac{|\overrightarrow{PS} \times v|}{11}$$

$$= \frac{\left|\begin{vmatrix} i & j & k \\ -8 & -6 & 3 \\ 2 & 6 & 9 \end{vmatrix}\right|}{11} = \frac{|i(-54-18) - j(-72-6) + k(-48+12)|}{11}$$

$$= \frac{|-72i + 78j - 36k|}{11} = \frac{\sqrt{72^2+78^2+36^2}}{11} = \frac{\sqrt{12564}}{11}$$

$$= \frac{\sqrt{36 \times 349}}{11} = \frac{6}{11}\sqrt{349}$$

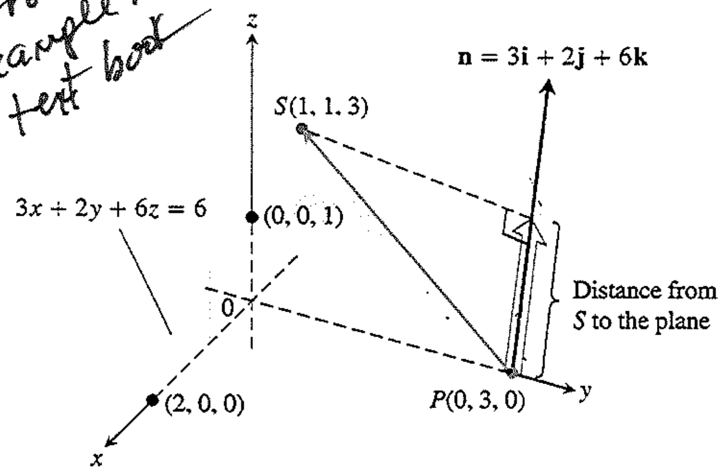
Note: given a plane $Ax + by + cz = k$
A vector normal to plane
is $Ai + Bj + Ck$

Calculate the requested distance.

33) The distance from the point S(−9, 1, −4) to the plane $2x + 2y + z = 1$

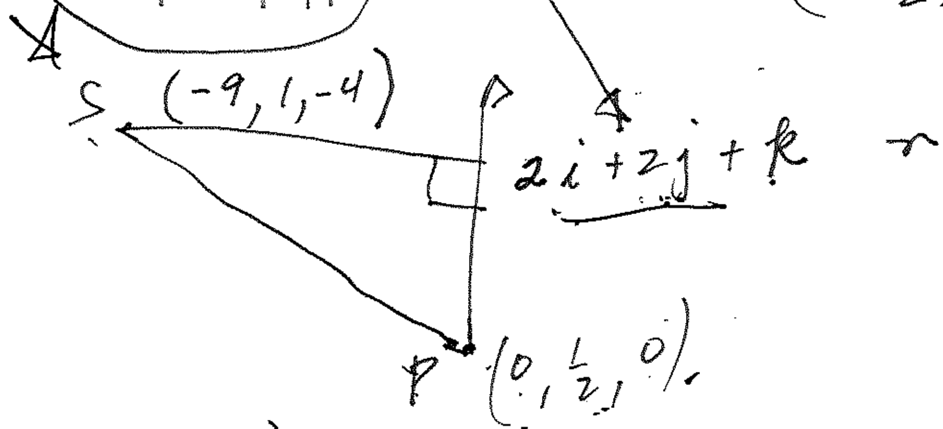
A) $\frac{7}{3}$ B) $\frac{11}{9}$ C) 7 D) $\frac{11}{3}$

From example in text book



$$d = \left| \overline{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$

Pick P on plane
say $(0, \frac{1}{2}, 0)$



$$\overline{PS} = -9i + \frac{1}{2}j - 4k$$

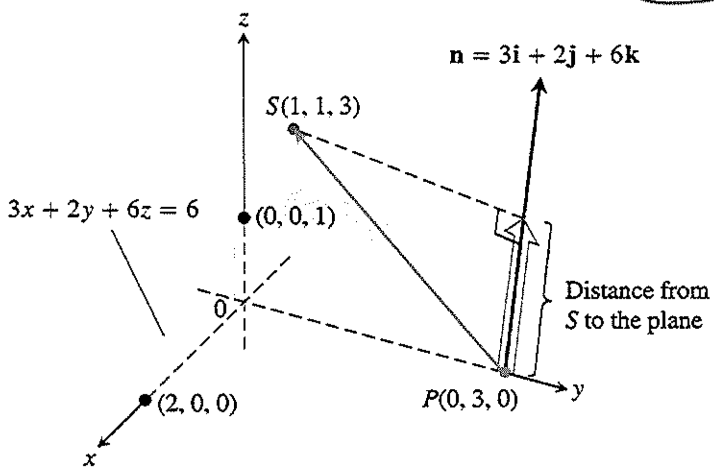
$$d = \frac{(-9i + \frac{1}{2}j - 4k) \cdot (2i + 2j + k)}{\sqrt{4+4+1}} =$$

$$= \left| \frac{-18 + (-4}{3} \right| = \frac{+21}{3} = 7$$

Calculate the requested distance.

34) The distance from the point S(5, −1, 1) to the plane −9x + 6y + 2z = −8

A) $\frac{41}{121}$ B) $\frac{41}{11}$ C) $\frac{61}{11}$ D) $\frac{61}{121}$



$$d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$

S $\cdot (5, -1, 1)$

$n = -9i + 6j + 2k$

P $\cdot \left(0, -\frac{4}{3}, 0\right)$

this point is arbitrary except it must be on the plane.

$\overline{PS} = 5i + \frac{1}{3}j + k$

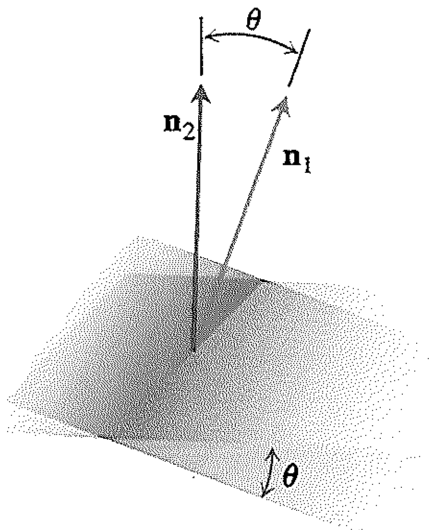
$$d = \frac{\left(5i + \frac{1}{3}j + k\right)\left(-9i + 6j + 2k\right)}{\sqrt{81 + 36 + 4}} = \left| \frac{-45 + 2 + 2}{11} \right|$$

$$= \frac{41}{11}$$

Find the angle between the planes.

35) $9x + 8y + 5z = 7$ and $8x + 7y + 2z = -8$

A) 1.363 B) 1.528 C) 1.414 D) 0.208



$$\theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}\right)$$

normals

$n_1 = 9i + 8j + 5k$

$n_2 = 8i + 7j + 2k$

$$\theta = \cos^{-1}\left[\frac{(9i + 8j + 5k)(8i + 7j + 2k)}{\sqrt{81 + 64 + 25}\sqrt{64 + 49 + 4}}\right]$$

$$= \cos^{-1}\left[\frac{72 + 56 + 10}{\sqrt{170}\sqrt{117}}\right] =$$

$= .208$

128
25
.3

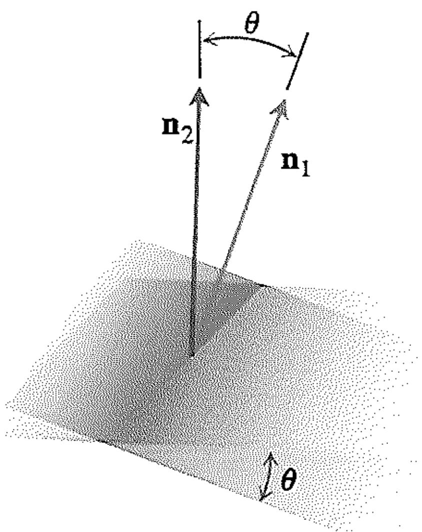
53
64
117

81
64
25
170

Find the angle between the planes.

36) $7x - 6y - 9z = -2$and$- 2x + 9y - 10z = -5$

A) 0.126 B) 1.526 C) 1.282 D) 1.445



$$\theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}\right)$$

$n_1 = 7i - 6j - 9k$

$n_2 = -2i + 9j - 10k$

$$\theta = \cos^{-1} \frac{(7i - 6j - 9k)\cdot(-2i + 9j - 10k)}{\sqrt{49+36+81}\sqrt{4+81+100}}$$

$$= \cos^{-1}\left(\frac{-14 \cancel{+} 54 + 90}{\sqrt{49+36+81}\sqrt{4+81+100}}\right) = 1.4449$$

$$= \underline{1.445}$$

Find the intersection.

37) $x = -6 + 9t,\ y = -9 + 3t,\ z = 2 + 2t;\ 4x + 9y + 8z = -10$

Plane

A) $(-15, -12, 0)$

B) $\left(3, -\frac{44}{79}, 3\right)$

C) $\left(-15, -\frac{518}{79}, -1\right)$

D) $(3, -6, 4)$

we need to find the t that causes the line to intersect with the plane

$\cancel{-6 + 9t = t}$

$\cancel{9t = 10}$

$\cancel{t = \frac{10}{9}}$

plug in x → plug in y → plug in z

$$4(-6 + 9t) + 9(-9 + 3t) + 8(2 + 2t) = -10$$

$$-24 + 36t - 81 + 27t + 16 + 16t = -10$$

52

$$79t \cancel{52t} = 81 + 24 - 26$$

$$t = \frac{29}{52}$$

$$x = -6 + 9\left(\frac{79}{52}\right)$$

$$79t \cancel{} = 79$$

$$t = 1$$

$$x = -6 + 9 = 3$$

$$y = -9 + 3 = -6$$

$$z = 2 + 2 = 4$$

$(3, -6, 4)$

Find the line of intersection of two planes.

38) $-5x - 7y - 4z = -2$, $6x + 2y + 8z = 5$

A) $x = -48t + \frac{31}{32}$, $y = 16t - \frac{13}{32}$, $z = 32t$

B) $x = -1536t - 31$, $y = 512t + 13$, $z = -32t$

C) $x = -48t - 31$, $y = 16t - 13$, $z = -32t$

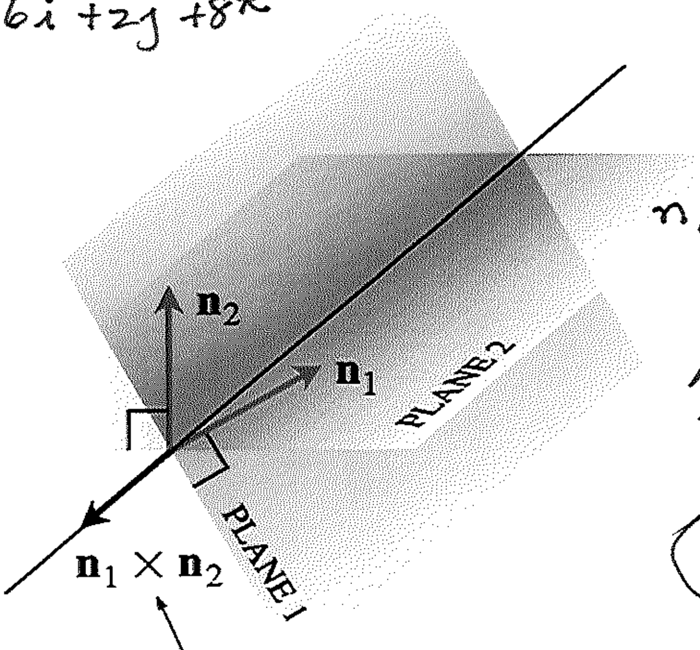
D) $x = 48t + 31$, $y = -16t + -13$, $z = -32t$

Two planes (unless parallel) intersect in a line. The cross product of the two normals n_1 & n_2, $n_1 \times n_2$, is parallel to line of intersection of two planes.

$n_1 = -5i - 7j - 4k$
$n_2 = 6i + 2j + 8k$

Procedure ① take $n_1 \times n_2$ gives vector that is parallel to line of intersection of two planes

② Find just one point of intersection of the two planes [could pick $z = 0$ and find x & y]



Parallell to line of intersection of two planes

$$n_1 \times n_2 = \begin{vmatrix} i & j & k \\ -5 & -7 & -4 \\ 6 & 2 & 8 \end{vmatrix} = i(-56+8) - j(-40+24) + k(-10+42)$$
$$= -48i + 16j + 32k \quad \text{[vector Parallel to line]}$$

Now find a point on line of intersection pick $z = 0$ and substitute into planes

$$-5x - 7y = -2 \qquad -30x - 42y = -12$$
$$6x + 2y = 5 \quad \Rightarrow \quad 30x + 10y = 25$$
$$-32y = 13, \quad y = -\frac{13}{32}$$

$$5x = 2 - 7y = 2 + \frac{91}{32} = \frac{155}{32}$$
$$x = \frac{31}{32}$$

So point on line is $\left(\frac{31}{32}, -\frac{13}{32}, 0\right)$

vector parallel is $-48i + 16j + 32k$

So the line is $x = \frac{31}{32} - 48t$, $y = -\frac{13}{32} + 16t$, $z = 0 + 32t$

If r(t) is the position vector of a particle in the plane at time t, find the indicated vector.

39) $\mathbf{r}(t) = (5t^2 + 3)\mathbf{i} + \left(\frac{1}{12}t^3\right)\mathbf{j}$.

Find the velocity vector of the particle.

A) $\mathbf{v}(t) = (10t)\mathbf{i} - \left(\frac{1}{4}t^3\right)\mathbf{j}$

B) $\mathbf{v}(t) = (10t)\mathbf{i} + \left(\frac{1}{4}t^3\right)\mathbf{j}$

C) $\mathbf{v}(t) = \left(\frac{1}{4}t^2\right)\mathbf{i} + (10t)\mathbf{j}$

D) $\mathbf{v}(t) = (10)\mathbf{i} + \left(\frac{1}{2}t\right)\mathbf{j}$

error

$v(t) = (10t + 0)i + \frac{1}{4}t^2 j$

$= 10t(i) + \frac{1}{4}t^2(j)$

If r(t) is the position vector of a particle in the plane at time t, find the indicated vector.

40) $\mathbf{r}(t) = (\cos 2t)\mathbf{i} + (5 \sin t)\mathbf{j}$

Find the acceleration vector.

A) $\mathbf{a}(t) = (-4 \cos 2t)\mathbf{i} + (-25 \sin t)\mathbf{j}$

B) $\mathbf{a}(t) = (-2 \cos 2t)\mathbf{i} + (5 \sin t)\mathbf{j}$

C) $\mathbf{a}(t) = (-4 \cos 2t)\mathbf{i} + (-5 \sin t)\mathbf{j}$

D) $\mathbf{a}(t) = (4 \cos 2t)\mathbf{i} + (-5 \sin t)\mathbf{j}$

$v = -2\sin 2t\, i + 5\cos t\, j$

$a = -4\cos 2t\, i - 5\sin t\, j$

The position vector of a particle is r(t). Find the requested vector.

41) The velocity at $t = 4$ for $\mathbf{r}(t) = (5t^2 + 3t + 3)\mathbf{i} - 5t^3\mathbf{j} + (5 - t^2)\mathbf{k}$

A) $\mathbf{v}(4) = 23\mathbf{i} - 80\mathbf{j} - 4\mathbf{k}$

B) $\mathbf{v}(4) = 43\mathbf{i} + 240\mathbf{j} + 8\mathbf{k}$

C) $\mathbf{v}(4) = 37\mathbf{i} - 240\mathbf{j} - 8\mathbf{k}$

D) $\mathbf{v}(4) = 43\mathbf{i} - 240\mathbf{j} - 8\mathbf{k}$

$$v(t) = (10t + 3)i - 15t^2 j - 2t k$$

$$v(4) = 43i - 240j - 8k$$

$$\begin{array}{r} 16 \\ 15 \\ \hline 80 \\ +16 \\ \hline \end{array}$$

The position vector of a particle is r(t). Find the requested vector.

42) The acceleration at $t = 2$ for $\mathbf{r}(t) = (4t - 2t^4)\mathbf{i} + (4 - t)\mathbf{j} + (6t^2 - 9t)\mathbf{k}$

A) $\mathbf{a}(2) = 96\mathbf{i} + 12\mathbf{k}$

B) $\mathbf{a}(2) = -24\mathbf{i} + 12\mathbf{k}$

C) $\mathbf{a}(2) = -96\mathbf{i} + 12\mathbf{k}$

D) $\mathbf{a}(2) = -96\mathbf{i} - \mathbf{j} + 12\mathbf{k}$

$V(t) = (4 - 8t^3)i - j + (12t - 9)k$

$a(t) = -24t^2 i + 12k$

$a(2) = -96i + 12k$

The vector r(t) is the position vector of a particle at time t. Find the angle between the velocity and the acceleration vectors at time t = 0.

43) $r(t) = (2t^2 + 4)i + (5t^3 - 4t)k$

A) $\frac{\pi}{2}$ B) 0 C) π D) $\frac{\pi}{4}$

$v(t) = 4t\, i + (15t^2 - 4)k \qquad v(0) = -4k$

$a(t) = 4i + 30t\, k \qquad a(0) = 4i$