Math 2013 Practice Quiz 2 Modified Fall 2011

Name: Last _______________, First _______________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question. You must show your work to get credit.

Find the domain and range and describe the level curves for the function f(x,y).

1) $f(x, y) = \sqrt{64 - x^2 - y^2}$

A) Domain: all points in the x–y plane; range: all real numbers; level curves: circles with centers at (0, 0)

B) Domain: all points in the x–y plane satisfying $x^2 + y^2 = 64$; range: real numbers $0 \le z \le 8$; level curves: circles with centers at (0, 0) and radii r, $0 < r \le 8$

C) Domain: all points in the x–y plane; range: real numbers $0 \le z \le 8$; level curves: circles with centers at(0, 0) and radii r, $0 < r \le 8$

D) Domain: all points in the x–y plane satisfying $x^2 + y^2 \le 64$; range: real numbers $0 \le z \le 8$; level curves: circles with centers at (0, 0) and radii r, $0 < r \le 8$

$64 - x^2 - y^2 \geq 0$
$x^2 + y^2 \leq 64$

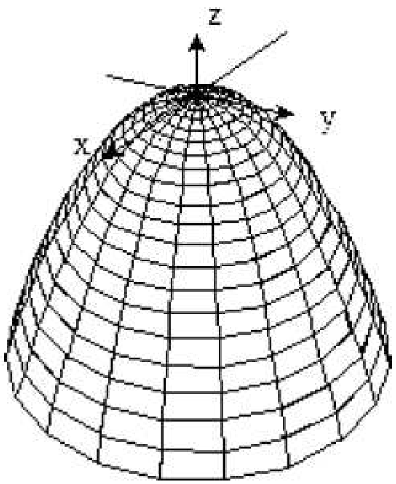
$\sqrt{64 - x^2 - y^2} = c$
$64 - x^2 - y^2 = c$
$x^2 + y^2 = 64 - c$

$0 < r \leq 8$

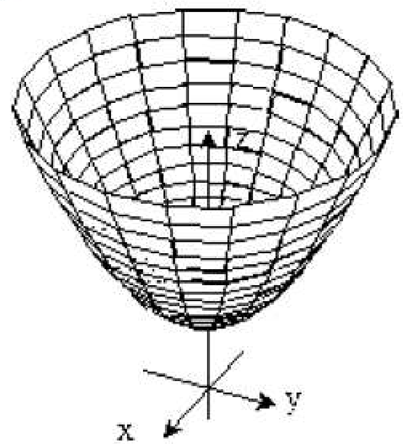
Sketch the surface z = f(x,y).

2) $f(x, y) = -x^2 - y^2$

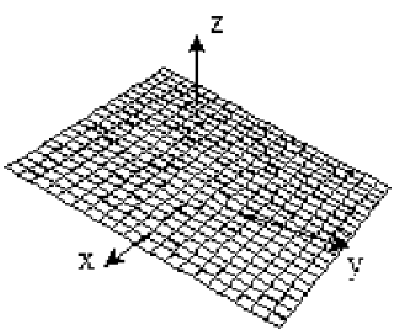
A)



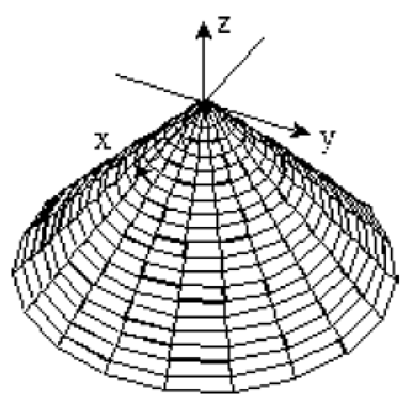
B)



C)



D)



3) Find an equation for the level curve of the function $f(x, y) = 9 - x^2 - y^2$ that passes through the point $(\sqrt{5}, \sqrt{7})$.

A) $x^2 + y^2 = -12$

B) $x^2 + y^2 = 21$

C) $x^2 - y^2 = 12$

D) $x^2 + y^2 = 12$

$f(\sqrt{5}, \sqrt{7}) = 9 - 5 - 7 = -3$

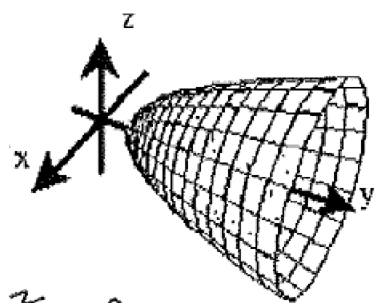
$9 - x^2 - y^2 = -3$

$x^2 + y^2 = 12$

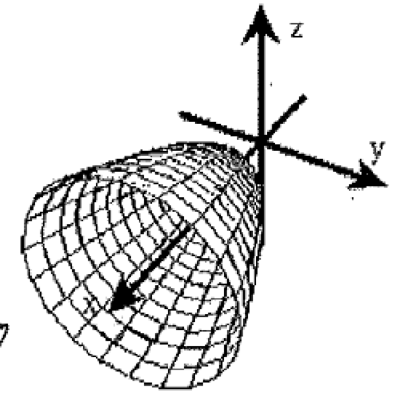
Sketch a typical level surface for the function.

4) $f(x, y, z) = x - y^2 - z^2$

A)



B)



$f(x,y,z) = x - y^2 - z^2 = c$

$x = c + y^2 + z^3$

say $c = 2$

$\cancel{x} = 2 + y^2 + z^2$

$z = 0$

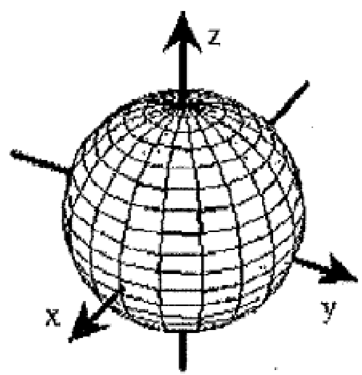
$x = 2 + z^2$ parabola

$y = 0$

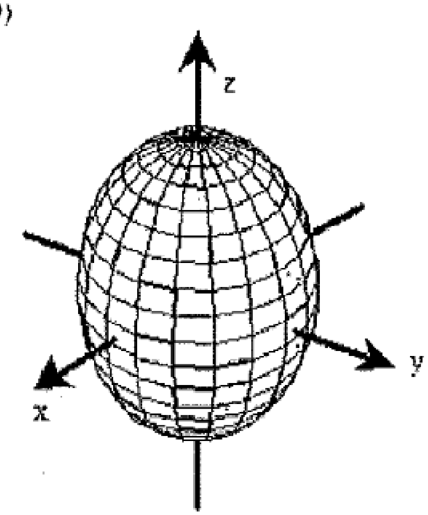
$x = 2 + z^2$ parabola

up toward x axis

C)



D)



Find the limit.

5)

$$\frac{7(1)(0) - 6(1)(-1)}{1^2 + (-1)^2} = \frac{6}{2} = 3$$

$$\lim_{P \to (1,-1,0)} \frac{7xz - 6xy}{x^2 + y^2 - z^2}$$

A) 7 B) -6 C) 3 D) -7

At what points is the given function continuous?

6)

$$f(x, y) = \frac{x - y}{2x^2 + x - 6}$$

everywhere denom $\neq 0$

$2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

$x \neq -2$

$x \neq 3/2$

A) All (x, y) such that $x \neq 0$

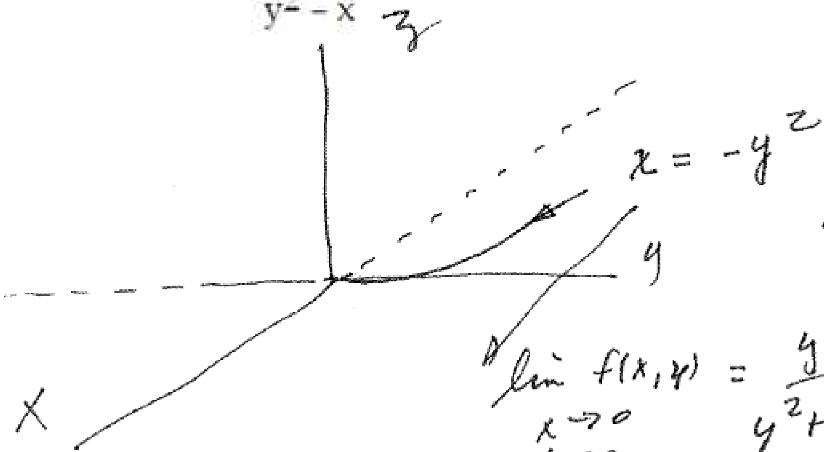
B) All (x, y) satisfying $x - y \neq 0$

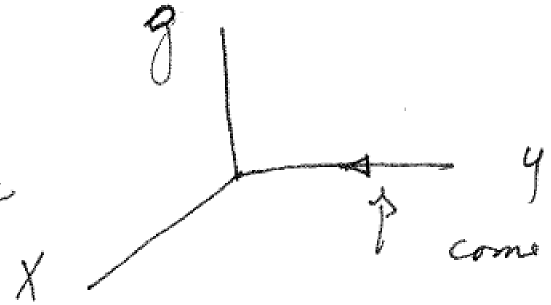
C) All (x, y)

D) All (x, y) such that $x \neq \frac{3}{2}$ and $x \neq -2$

Find two paths of approach from which one can conclude that the function has no limit as (x, y) approaches (0, 0).

7) $f(x, y) = \dfrac{y^2}{y^2 - x}$





come in on $x = 0$, $y = t$

means come in on y axis

$$\lim_{\substack{x\to 0\\ y\to 0}} f(x,y) = \frac{y^2}{y^2+y^2} = \frac{1}{2} \neq 1$$

$$\lim = \lim_{y\to 0} \frac{y^2}{y^2 - 0} = 1$$

18) $f(x, y) = \dfrac{x^2}{x^4 + y^2}$ try $y = x$

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{y^2}{y^4+y^2} = \lim_{y\to 0} \frac{\cancel{y^2}}{\cancel{y^2}\,y^2+1} = \frac{1}{1}$$

try $y = 10x$ $\quad \displaystyle\lim_{x\to 0} \frac{x^2}{x^4+100x^2} = \lim_{x\to 0} \frac{\cancel{x^2}(1)}{\cancel{x^2}(x^2+100)} = \frac{1}{100} \neq 1$

Find all the first order partial derivatives for the following function.

8) $f(x, y) = \dfrac{1}{\sqrt{x^2 + y^2}}$

A) $\dfrac{\partial f}{\partial x} = \left(\dfrac{x}{2(x^2 + y^2)^{3/2}}\right)$; $\dfrac{\partial f}{\partial y} = \left(\dfrac{y}{2(x^2 + y^2)^{3/2}}\right)$

B) $\dfrac{\partial f}{\partial x} = -\left(\dfrac{x}{2(x^2 + y^2)^{3/2}}\right)$; $\dfrac{\partial f}{\partial y} = -\left(\dfrac{y}{2(x^2 + y^2)^{3/2}}\right)$

C) $\dfrac{\partial f}{\partial x} = -\left(\dfrac{1}{2(x^2 + y^2)^{3/2}}\right)$; $\dfrac{\partial f}{\partial y} = -\left(\dfrac{1}{2(x^2 + y^2)^{3/2}}\right)$

D) $\dfrac{\partial f}{\partial x} = -\left(\dfrac{x}{(x^2 + y^2)^{3/2}}\right)$; $\dfrac{\partial f}{\partial y} = -\left(\dfrac{y}{(x^2 + y^2)^{3/2}}\right)$

$$f(x, y) = (x^2 + y^2)^{-1/2}$$

$$\frac{\partial f}{\partial x} = -\frac{1}{2}(x^2 + y^2)^{-3/2}\, 2x$$

$$= \frac{-x}{(x^2 + y^2)^{3/2}}$$

$$\frac{\partial f}{\partial y} = \frac{-y}{(x^2 + y^2)^{3/2}}$$

9) $f(x, y) = \dfrac{e^{-x}}{x^2 + y^2}$

A) $\dfrac{\partial f}{\partial x} = -\dfrac{e^{-x}(x^2 + y^2 + x)}{(x^2 + y^2)^2}$; $\dfrac{\partial f}{\partial y} = -\dfrac{ye^{-x}}{(x^2 + y^2)^2}$

B) $\dfrac{\partial f}{\partial x} = -\dfrac{2xe^{-x}}{(x^2 + y^2)^2}$; $\dfrac{\partial f}{\partial y} = -\dfrac{2ye^{-x}}{(x^2 + y^2)^2}$

C) $\dfrac{\partial f}{\partial x} = \dfrac{e^{-x}(x^2 + y^2 + 2x)}{(x^2 + y^2)^2}$; $\dfrac{\partial f}{\partial y} = \dfrac{2ye^{-x}}{(x^2 + y^2)^2}$

D) $\dfrac{\partial f}{\partial x} = -\dfrac{e^{-x}(x^2 + y^2 + 2x)}{(x^2 + y^2)^2}$; $\dfrac{\partial f}{\partial y} = -\dfrac{2ye^{-x}}{(x^2 + y^2)^2}$ ✓

Quotient Rule Check

$$\frac{\partial f}{\partial x} = \frac{(x^2+y^2)(-e^{-x}) - e^{-x}2x}{(x^2+y^2)^2} = -\frac{e^{-x}(x^2+y^2+2x)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{(x^2+y^2)0 - e^{-x}2y}{(x^2+y^2)^2} = \frac{-2ye^{-x}}{(x^2+y^2)^2}$$ ✓

10) $f(x, y, z) = \frac{\cos y}{xz^2}$

A) $\frac{\partial f}{\partial x} = -\frac{\cos y}{z^2}$; $\frac{\partial f}{\partial y} = -\frac{\sin y}{xz^2}$; $\frac{\partial f}{\partial z} = -\frac{2\cos y}{xz}$

B) $\frac{\partial f}{\partial x} = \frac{\cos y}{x^2z^2}$; $\frac{\partial f}{\partial y} = \frac{\sin y}{xz^2}$; $\frac{\partial f}{\partial z} = \frac{2\cos y}{xz^3}$

C) $\frac{\partial f}{\partial x} = -\frac{\cos y}{x^2z^2}$; $\frac{\partial f}{\partial y} = -\frac{\sin y}{xz^2}$; $\frac{\partial f}{\partial z} = -\frac{2\cos y}{xz^3}$

D) $\frac{\partial f}{\partial x} = \frac{\cos y}{z^2}$; $\frac{\partial f}{\partial y} = \frac{\sin y}{xz^2}$; $\frac{\partial f}{\partial z} = \frac{2\cos y}{xz}$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x}\left(\frac{\cos y}{z^2}\cancel{x}^{-1}\right) = -\frac{\cos y}{x^2 z^2}$$

$$\frac{\partial f}{\partial x} = -\frac{\sin y}{xz^2}$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\frac{\cos y(\cancel{0} z^{-2})}{x} = -\frac{2\cos y}{xz^3}$$

11) Evaluate $\frac{dw}{dt}$ at $t = \frac{3}{2}\pi$ for the function $w(x, y, z) = \frac{xy}{z}$; $x = \sin t$, $y = \cos t$, $z = t^2$.

A) $2\left(\frac{1}{\pi^2}\right)$ B) $-2\left(\frac{1}{\pi}\right)$ C) $-\frac{4}{9}\left(\frac{1}{\pi^2}\right)$ D) $-2\left(\frac{1}{\pi^2}\right)$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

chain rule

$$= \frac{y}{z}\cos t + \frac{x}{z}(-\sin t) - \frac{xy}{z^2}(2t) \quad \Big| \; t = \frac{3\pi}{2}$$

$$= \frac{\cos t}{t^2}\cos t + \frac{\sin t}{t^2}(-\sin t) - \frac{\sin t \cos t}{t^4}(3\pi)$$

$$= 0 - \frac{1}{(3\pi/2)^2} = -4/9 \left(\frac{1}{\pi}\right)^2$$

$$w(x,y,z) = e^{xyz^2}$$

12) Evaluate $\frac{\partial u}{\partial z}$ at (x, y, z) = (5, 4, 5) for the function $u(p, q, r) = p^2q^2 - r$; $p = y - z$, $q = x + z$, $r = x + y$.

A) –180 B) 440 C) 110 D) 220

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial z} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial z} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial z}$$

$$= 2pq^2(-1) + 2p^2q(1) - 1(0)$$

$$= 2(-1)(100)(-1) + 2(-1)^2(10)1$$

$$= 200 + 20 = 220$$

$p = y - z = 4 - 5 = -1$

$q = x + z = 10$

$r = x + y = 9$

Write a chain rule formula for the following derivative.

13) $\dfrac{\partial w}{\partial t}$ for $w = f(x, y, z)$; $x = g(r, s, t)$, $y = h(r, s, t)$, $z = k(r, s, t)$

A) $\dfrac{\partial w}{\partial t} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial r} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial s} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial t}$

B) $\dfrac{\partial w}{\partial t} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t} + \dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial t}$

C) $\dfrac{\partial w}{\partial t} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t}$

D) $\dfrac{\partial w}{\partial t} = \dfrac{\partial w}{\partial x} + \dfrac{\partial w}{\partial y} + \dfrac{\partial w}{\partial z}$

$$\frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial z}$$

Use implicit differentiation to find the specified derivative at the given point.

14) Find $\frac{dy}{dx}$ at the point (1, 1) for $5x^2 + 5y^3 + 5xy = 0$.

A) $-\frac{3}{4}$ B) $\frac{3}{4}$ C) -1 D) $-\frac{3}{2}$

$$\frac{d}{dx}(5x^2) + \frac{d}{dx}(5y^3) + \frac{d}{dx}(5xy) = 0$$

$$= 10x + 15y^2\frac{dy}{dx} + 5\left[x\frac{dy}{dx} + y\right] = 0$$

$$= 10x + 15y^2\frac{dy}{dx} + 5x\frac{dy}{dx} + 5y = 0$$

So $x=1$, $y=1$

$$= 10 + 15\frac{dy}{dx} + 5\frac{dy}{dx} + 5 = 0$$

$$20\frac{dy}{dx} = -15$$

$$dy/dx = -\frac{15}{20} = -3/4$$

15) Find $\frac{\partial z}{\partial y}$ at the point (8, 1, −1) for $\ln\left(\frac{yz}{x}\right) - e^{xy+z^2} = 0$.

A) $\frac{2e^9 - 1}{1 - 8e^9}$ B) $\frac{1 - 8e^9}{1 - 2e^9}$ C) $\frac{8e^9 - 1}{1 - 2e^9}$ D) $\frac{1 - 2e^9}{1 - 8e^9}$

let $F(x,y,z) = \ln\left(\frac{yz}{x}\right) - e^{xy+z^2} = 0$

$$\boxed{\frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}}$$

$$\frac{\partial F}{\partial y} = \frac{\cancel{x}}{y\cancel{z}}\left(\frac{\cancel{z}}{\cancel{x}}\right) - xe^{xy+z^2} = \frac{1}{y} - xe^{xy+z^2}$$

$$\frac{\partial F}{\partial z} = \frac{\cancel{x}}{\cancel{yz}}\left(\frac{\cancel{y}}{\cancel{x}}\right) - 2ze^{xy+z^2} = \frac{1}{z} - 2ze^{xy+z^2}$$

at (8, 1, −1)
$xy+z^2 = 9$

$$\text{So } \frac{\partial z}{\partial y} = -\frac{\left(\frac{1}{y} - xe^{xy+z^2}\right)}{\frac{1}{z} - 2ze^{xy+z^2}} = -($$

Compute the gradient of the function at the given point.

16) $f(x, y, z) = \ln(x^2 - 5y^2 + 8z^2)$, $(-5, -5, -5)$

A) $-\frac{1}{10}\mathbf{i} + \frac{1}{2}\mathbf{j} - \frac{4}{5}\mathbf{k}$

B) $-\frac{1}{10}\mathbf{i} + \frac{1}{2}\mathbf{j} - \frac{5}{16}\mathbf{k}$

C) $\frac{1}{16}\mathbf{i} + \frac{1}{2}\mathbf{j} - \frac{4}{5}\mathbf{k}$

D) $\frac{1}{16}\mathbf{i} + \frac{1}{2}\mathbf{j} - \frac{5}{16}\mathbf{k}$

same formula except at $\frac{\partial f}{\partial y}\, j$

$$x^2 - 5y^2 + 8z^2$$
$$= (-5)^2 - 5(-5)^2 + 8(-5)^2$$
$$= 25 - 5(25) + 8(25)$$
$$= 4(25) = 100$$

$$\nabla = \frac{2x}{(x^2-5y^2+8z^2)}\, i \quad \frac{-10y}{(x^2-5y^2+8z^2)}\, j \quad + \frac{16z}{(x^2-5y^2+8z^2)}\, j$$

$$= \frac{-10}{100}\, i + \frac{50}{100}\, j \quad -\frac{80}{100}\, k = -\frac{1}{10}\, i + \frac{1}{2}\, j - \frac{4}{5}\, k$$

Find the derivative of the function at the given point in the direction of A.

17) $f(x, y) = \ln(6x + 10y)$, $(-4, 3)$, $A = 6\mathbf{i} + 8\mathbf{j}$

A) $\frac{26}{15}$ B) $\frac{23}{15}$ C) $\frac{29}{15}$ D) $\frac{32}{15}$

$\nabla f = \frac{6}{6x+10y} i + \frac{10}{6x+10y} j$

$u = \frac{6}{\sqrt{36+64}} i + \frac{8}{\sqrt{36+64}} j$

$= \frac{6}{10} i + \frac{8}{10} \hat{j}$

$\nabla f = \frac{6}{-24+30} i + \frac{10}{-24+30} j = \left(i + \frac{10}{6} j\right)$

Derivative $= \nabla \cdot u = \left(i + \frac{10}{6} j\right) \cdot \left(\frac{6}{10} i + \frac{8}{10} j\right)$

$= \frac{6}{10} + \frac{8}{6} = \frac{3}{5} + \frac{4}{3} = \frac{29}{15}$

18) Find the derivative of the function $f(x, y) = \tan^{-1} \frac{y}{x}$ at the point $(-7, 7)$ in the direction in which the function decreases most rapidly.

A) $-\frac{\sqrt{2}}{21}$ B) $-\frac{\sqrt{2}}{14}$ C) $-\frac{\sqrt{3}}{21}$ D) $-\frac{\sqrt{3}}{14}$

19) Find the derivative of the function $f(x, y) = e^{xy}$ at the point $(0, 4)$ in the direction in which the function increases most rapidly.

A) 4 B) 8 C) 3 D) 12

20) Write an equation for the tangent line to the curve $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at the point $\left(\frac{4}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$.

A) $\frac{x}{3} + \frac{y}{4} = 1$ B) $\frac{x}{4} + \frac{y}{3} = 1$ C) $\frac{x}{3} + \frac{y}{4} = \sqrt{2}$ D) $\frac{x}{4} + \frac{y}{3} = \sqrt{2}$

for $f(x,y) = \frac{x^2}{16} + \frac{y^2}{9}$ the equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is level curve

$$\frac{2x}{16} + \frac{2y}{9}\frac{dy}{dx} = 0$$

at $\frac{4}{\sqrt{2}}, \frac{3}{\sqrt{2}}$

$$\frac{dy}{dx} = -\frac{\frac{\cancel{2}x}{16}}{\frac{\cancel{2}y}{9}} = \frac{-9x}{16y} = \frac{-9\,\frac{4}{\sqrt{2}}}{16\,\frac{3}{\sqrt{2}}} = \frac{-\cancel{9}^{3}(\cancel{4})}{\cancel{16}_{4}(\cancel{3})} = -3/4$$

$$y = -3/4\,x + b$$

$$\frac{3}{\sqrt{2}} = -\frac{3}{\cancel{4}}\left(\frac{\cancel{4}}{\sqrt{2}}\right) + b$$

$$b = \frac{6}{\sqrt{2}}$$

$$y = -3/4\,x + \frac{6}{\sqrt{2}}$$

$$\frac{y}{3} = -\frac{x}{4} + \frac{2}{\sqrt{2}}$$

$$\boxed{\frac{x}{4} + \frac{y}{3} = \sqrt{2}}$$

21) Find the equation for the tangent plane to the surface $z = -6x^2 - 2y^2$ at the point $(2, 1, -26)$.

A) $2x + y - 26z = -23$

B) $2x + y - 26z = 1$

C) $-24x - 4y - z = -26$

D) $-24x - 4y - z = -30$

$f(x, y, z) = \underline{-6x^2 - 2y^2 - z}$

$z = -6x^2 - 2y^2$ is surface

$(2, 1, -26)$

$\frac{\partial f}{\partial x} = -12x \Big|_{P_0} = -24$

$\frac{\partial f}{\partial y} = -4y \Big|_{P_0} = -4$

$\frac{\partial f}{\partial z} = -1 \Big|_{P_0} = -1$

$-24(x-2) + -4(y-1) - 1(z+26) = 0$

$-24x \underline{+48} - 4y + 4 - z - 26 = 0$

$-24x - 4y - z = -26$

$\frac{\begin{matrix} -2 \\ 52 \\ 26 \end{matrix}}{26}$

22) Find parametric equations for the normal line to the surface $x^2 + 7xyz + y^2 = 9z^2$ at the point (1, 1, 1).

A) $x = t - 9,\ y = t - 9,\ z = t + 11$

B) $x = 9t + 1,\ y = -9t + 1,\ z = -11t + 1$

C) $x = t + 9,\ y = t + 9,\ z = t - 11$

D) $x = 9t + 1,\ y = 9t + 1,\ z = -11t + 1$

$f(x, y, z) = x^2 + 7xyz + y^2 - 9z^2 = 0$

Normal line $x = x_0 + f_x(P_0)t$, $y = y_0 + f_y(P_0)t$, $z = z_0 + f_z(P_0)t$

$\left.\frac{\partial f}{\partial x}\right|_{P_0} = 2x + 7yz \Big|_{\substack{x=1\\y=1\\z=1}} = 9$, $\left.\frac{\partial f}{\partial y}\right|_{P_0} = 2xz + 2y \Big|_{\substack{x=1\\y=1\\z=1}} = 9$,

$\left.\frac{\partial f}{\partial z}\right|_{P_0} = (7xy - 18z)\Big|_{\substack{x=1\\y=1\\z=1}} = 7 - 18 = -11$

$x = 1 + 9t$, $y = 1 + 9t$, $z = 1 - 11t$

23) Write parametric equations for the tangent line to the curve of intersection of the surfaces $z = 2x^2 + 8y^2$ and $z = x + y + 8$ at the point $(1, 1, 10)$.

$f(x,y,z) = 2x^2 + 8y^2 - z$, $g(x,y,z) = x + y - z$

A) $x = -15t + 1,\ y = 3t + 1,\ z = -12t + 10$

B) $x = -3t + 1,\ y = 5t + 1,\ z = -12t + 10$

C) $x = -3t + 1,\ y = 3t + 1,\ z = -12t + 10$

D) $x = -15t + 1,\ y = 5t + 1,\ z = -12t + 10$

The tangent line is orthogonal to both the gradient of f and f

$$\nabla f\Big|_{(1,1,10)} = \left((4x)i + (16y)j - 1k\right)_{(1,1,10)} = 4i + 16j - k$$

$\nabla g\Big|_{1,1,10} = (i + j - k)$, The tangent line will then be parallel to

$$\nabla f \times \nabla g = \begin{vmatrix} i & j & k \\ 4 & 16 & -1 \\ 1 & 1 & -1 \end{vmatrix} = i(-16+1) - j(-4+1) + k(4-16) = -15i + 3j - 12k$$

Therefore the tangent line is parallel to $-15i + 3j - 12k$ at the point $(1, 1, 10)$

$$\boxed{x = 1 - 15t,\quad y = 1 + 3t,\quad z = 10 - 12t}$$

Find the linearization of the function at the given point.

24) $f(x, y, z) = -8x^2 - 3y^2 + 8z^2$ at $(1, -2, 3)$

A) $L(x, y, z) = -16x + 12y + 48z - 52$

B) $L(x, y, z) = -16x - 12y + 48z + 52$

C) $L(x, y, z) = -16x + 12y + 48z + 52$

D) $L(x, y, z) = -16x - 12y + 48z - 52$

144

$$L(x,y) = (-8x^2 - 3y^2 + 8z^2)\Big|_{\substack{x=1\\y=-2\\z=3}} - 16x\Big|_{x=1}(x-1) - 6y\Big|_{y=-2}(y+2) + 16z\Big|_{z=3}(z-3)$$

$$= (-8 - 12 + 72) - 16x + 16 + 12y + 24 + 48z - 144$$

$$= -16x + 12y + 48z - 52$$

Find all local extreme values of the given function and identify each as a local maximum, local minimum, or saddle point.

25) $f(x, y) = x^2 + 18x + y^2 + 2y - 2$ 25) ______

A) $f(9, 1) = 244$, local maximum
B) $f(-9, -1) = -84$, local minimum
C) $f(-9, 1) = -80$, saddle point
D) $f(9, -1) = 240$, saddle point

$$f(x, y) = x^2 + 18x + y^2 + 2y - 2$$

$$\frac{\partial f}{\partial x} = 2x + 18 = 0 \qquad \frac{\partial f}{\partial y} = 2y + 2 = 0$$

$$x = -9 \qquad y = -1$$

$(-9, -1)$ is a critical point

$$f(-9,-1) = 81 - 162 + 1 - 2 - 2 = -81 - 3 = -84$$

Use second derivative test

$$f_{xx} = 2 > 0 \quad f_{yy} = 2 \quad f_{xy} = 0, \quad f_{xy}^2 = 0$$

$$f_{xx} f_{yy} - f_{xy}^2 = 4 - 0 = 4 > 0 \Rightarrow \text{Local min}$$

if $f_{xx} > 0$ and $f_{xx} f_{yy} - f_{xy}^2 > 0$

26) $f(x, y) = (x^2 - 100)^2 + (y^2 - 64)^2$

A) $f(0, 0) = 14{,}096$, local maximum; $f(-10, -8) = 0$, local minimum

B) $f(0, 0) = 14{,}096$, local maximum; $f(0, 8) = 10{,}000$, saddle point; $f(0, -8) = 10{,}000$, saddle point; $f(10, 0) = 14{,}096$, saddle point; $f(10, 8) = 0$, local minimum; $f(10, -8) = 0$, local minimum; $f(-10, 0) = 4096$, saddle point; $f(-10, 8) = 0$, local minimum; $f(-10, -8) = 0$, local minimum

C) $f(0, 0) = 14{,}096$, local maximum; $f(10, 8) = 0$, local minimum; $f(10, -8) = 0$, local minimum; $f(-10, 8) = 0$, local minimum; $f(-10, -8) = 0$, local minimum

D) $f(0, 0) = 14{,}096$, local maximum; $f(0, 8) = 10{,}000$, saddle point; $f(10, 0) = 4096$, saddle point; $f(10, 8) = 0$, local minimum; $f(-10, -8) = 0$, local minimum

26) $f(x, y) = (x^2 - 100)^2 + (y^2 - 64)^2$

$\frac{\partial f}{\partial x} = 2(x^2 - 100)\, 2x$
$= 4x(x^2 - 100)$

$x = 0 \quad x = +10, x = -10,$

$\frac{\partial f}{\partial y} = 2(y^2 - 64)(2y) = 0$
$4y(y^2 - 64)$

$y = 0, \ y = -8, \ y = +8$

Check $(0,0)$ $(0,-8)$ $(0,+8)$ $(10,0)$ $(10,-8)$, $(10,8)$

$(-10,0)$ $(-10,8)$ $(-10,-8)$

$f_{xx} = \frac{\partial^2 f}{\partial x^2} = 4x(2x) + 4(x^2 - 100)$
$= 8x^2 + 4x^2 - 400$
$\frac{\partial^2 f}{\partial x^2} = \underline{12x^2 - 400}$

$\frac{\partial^2 f}{\partial y^2} = 4y(2y) + 4(y^2 - 64)$
$= f_{yy} = \boxed{12y^2 - 256}$

$\frac{\partial^2 f}{\partial x \partial y} = 0$ $f_{xy} = 0$ $\quad \frac{\partial^2 f}{\partial y \partial x} = 0$ $f_{yx} = 0$

Test $(0,0)$ $\frac{\partial^2 f}{\partial x^2} = -400 < 0$ $\quad \frac{\partial^2 f}{\partial y^2} = -256 < 0$ $\quad f_{xx} f_{yy} - f_{xy}^2 = (-400)(-256) - 0 > 0$

$\Rightarrow$ local MAX at $(0,0)$ $f(0,0) = 10^4 + 8^4$

$(0,-8)$. $f_{xx} = -400 < 0$ $\quad f_{yy}(0,-8) = 512$ $\quad f_{xx} f_{yy} - f_{xy}^2 \xrightarrow{} 0 = (-400)(512) < 0$ saddle point

$(0,8)$ $f_{xx} = -400 < 0$ $\quad f_{yy}(0,8) = 512$ $\quad f_{xx} f_{yy} - f_{xy}^2 \to 0 = (-400)(512) < 0$ saddle point

 Continued

Check $(0,0)$ $\quad f_{xx} = 12(100) - 400 = 800 > 0 \quad f_{yy} = -256 < 0 \quad f_{xx}f_{yy} - f_{xy}^2$

$f(10,0) = 14{,}096$

$= 800(-256) < 0$

$\Rightarrow$ saddle point

check $(-10,0)$ $\quad f_{xx} = 12(100) - 400 > 0 \quad f_{yy} = -256 < 0 \quad f_{xx}f_{yy} - f_{xy}^2$

$f(-10,0)$ 4096 $\quad = 800 > 0 \quad = 800(-256) < 0$

saddle point

check $(10,8)$ $\quad f_{xx} = 1200 - 400 = 800 > 0 \quad f_{yy} = 12(64) - 256 = 512 > 0 \quad f_{xx}f_{yy} - f_{xy}^2 > 0$

$f(10,8) = 0$ $\quad = 800(512) - 0 > 0$

Local MIN

check $(10,-8)$ $\quad f_{xx} = 800 > 0 \quad f_{yy} = 12(64) - 256 = 512 > 0 \quad f_{xx}f_{yy} - f_{xy}^2 > 0$

$f(10,-8) = 0$

$= 800(512) - 0 > 0$

Local MIN

Check $(-10,-8)$ $\quad f_{xx} = 1200 - 400 = 800 > 0 \quad f_{yy} = 12(64) - 256 = 512 > 0 \quad f_{xx}f_{yy} - f_{xy}^2 > 0$

$f(-10,-8) = 0$

$= 800(512) - 0 > 0$

Local MIN

check $(-10,8)$ $\quad f_{xx} = 800 > 0 \quad f_{yy} = 512 > 0 \quad f_{xx}f_{yy} - f_{xy}^2$

$= 800(512) - 0 > 0$

Local MIN

Find the absolute maxima and minima of the function on the given domain.

27) $f(x, y) = x^2 + xy + y^2$ on the square $-4 \le x, y \le 4$

A) Absolute maximum: 16 at $(4, -4)$ and $(-4, 4)$; absolute minimum: 12 at $(-2, 4)$, $(2, -4)$, $(4, -2)$, and $(-4, 2)$

B) Absolute maximum: 16 at $(4, -4)$ and $(-4, 4)$; absolute minimum: 0 at $(0, 0)$

C) Absolute maximum: 48 at $(4, 4)$ and $(-4, -4)$; absolute minimum: 16 at $(4, -4)$ and $(-4, 4)$

D) Absolute maximum: 48 at $(4, 4)$ and $(-4, -4)$; absolute minimum: 0 at $(0, 0)$

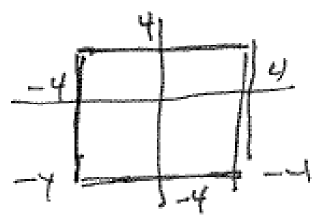
$f_x = 2x + y = 0$ $\quad f_y = x + 2y = 0$ $\quad$ only at $(0,0)$

$f_{xx} = 2$ $\quad f_{yy} = 2$ $\quad f_{xy} = 1$ $\quad f_{xx} f_{yy} - f_{xy}$

$= 4 - 1 = 3 > 0$

$\Rightarrow$ local MIN $(0,0)$

check boundary



if $x = 4$ on boundary $16 + 4y + y^2$ is max for $y = 4$ never goes to 0

if $x = -4$ on boundary $16 - 4y + y^2$ is max for $y = -4$ "

if $y = +4$ on boundary $x^2 + 4x + 16$ is max at $x = 4$ "

if $y = -4$ on boundary $x^2 - 4x + 16$ is max at $x = -4$ "

So $(4,4)$ $(-4,4)$ give absolute MAX

Find the extreme values of the function subject to the given constraint.

28) $f(x, y) = 8x^2 + 7y^2$, $x^2 + y^2 = 1$

A) Maximum: 7 at $(\pm 1, 0)$; minimum: 0 at $(0, 0)$

B) Maximum: 7 at $(0, \pm 1)$; minimum: 0 at $(0, 0)$

C) Maximum: 7 at $(\pm 1, 0)$; minimum: 8 at $(0, \pm 1)$

D) Maximum: 7 at $(0, \pm 1)$; minimum: 8 at $(\pm 1, 0)$

$f(x, y) = 8x^2 + 7y^2$, constraint $x^2 + y^2 = 1$

$y^2 = 1 - x^2$, $x^2 = 1 - y^2$

$f(x,y) = 8x^2 + 7(1 - x^2)$

$f(x,y) = 8x^2 + 7 - 7x^2 = \underline{x^2 + 7}$

MIN at $x = 0$ $y = \pm 1$

$x = 0$ $f(x,y) = \underline{7}$

$f(0, \pm 1) = 7$

$f(x,y)$ for $x^2 = 1 - y^2$

$f(x,y) = 8(1 - y^2) + 7y^2$

$= 8 - y^2$ has max at $\cancel{y} = 0$

at $\cancel{y} = 0$ $\cancel{x} = \pm 1$

$\cancel{(\pm 1, 0)}$ $f(\pm 1, 0) = 8$

MAX

29) $f(x, y, z) = x + 2y - 2z, \quad x^2 + y^2 + z^2 = 9$

A) Maximum: 9 at $(1, 2, -2)$; minimum: -9 at $(-1, -2, 2)$

B) Maximum: 1 at $(1, -2, -2)$; minimum: -1 at $(-1, 2, 2)$

C) Maximum: 8 at $(2, 1, -2)$; minimum: -8 at $(-2, -1, 2)$

D) Maximum: 1 at $(-1, -2, -3)$; minimum: -1 at $(1, 2, 3)$

$\nabla f = i + 2j - 2k \qquad \lambda \nabla g = \lambda 2x i + \lambda 2y j \lambda 2z k$

so $1 = 2\lambda x \qquad 2 = 2\lambda y \qquad -2 = 2\lambda z$

$\Rightarrow x = \frac{1}{2\lambda} \qquad y = \frac{1}{\lambda} \qquad z = -\frac{1}{\lambda}$

$y = 2x \qquad z = -y = -2x$

$x^2 + y^2 + z^2 = 9$

$x^2 + 4x^2 + 4x^2 = 9$

$9x^2 = 9$

$x = \pm 1$

$x = 1$, $y = 2$, $z = -2$

$x = -1$, $y = -2$, $z = 2$

$f(1, 2, -2) = 1 + 2(2) - 2(-2)$
$= 9$
MAX

$f(-1, -2, 2) = -1 + 2(-2) - 2(2$
$= -9$
MIN

Solve the problem.

30) Find the extreme values of $f(x, y, z) = x^2 + y^2 + z^2$ subject to $3x - y + z = 6$ and $x + 2y + 2z = 2$.

A) Maximum: none; minimum: $\frac{148}{45}$ at $\left(\frac{74}{45}, -\frac{20}{45}, \frac{28}{45}\right)$

B) Maximum: none; minimum: $\frac{148}{45}$ at $\left(-\frac{74}{45}, \frac{20}{45}, \frac{28}{45}\right)$

C) Maximum: none; minimum: $\frac{148}{45}$ at $\left(\frac{74}{45}, -\frac{20}{45}, -\frac{28}{45}\right)$

D) Maximum: none; minimum: $\frac{148}{45}$ at $\left(\frac{74}{45}, \frac{20}{45}, -\frac{28}{45}\right)$

$f(x,y,z) = x^2+y^2+z^2$, constraints, $g_1 = 3x-y+z-6=0$, $g_2 = x+2y+2z-2=0$

$\nabla f = 2xi+2yj+2zk$, $\nabla g_1 = 3i-j+k$, $\nabla g_2 = i+2j+2k$

$\nabla f = \lambda \nabla g_1 + \mu \nabla g_2$

$2xi + 2yj + 2zk = \lambda(3i-j+k) + \mu(i+2j+2k)$

① $2x = 3\lambda + \mu$ ② $2y = -\lambda + 2\mu$, ③ $2z = \lambda + 2\mu$

$2z = \lambda + 2\mu$

$4x = 6\lambda + 2\mu$

$2y = -\lambda + 2\mu$

$4x - 2y = 7\lambda$

$\boxed{\lambda = \frac{4}{7}x - \frac{2}{7}y}$

④ $2y + 2z = 4\mu$, $\mu = \frac{1}{2}y + \frac{1}{2}z$

$2z = \frac{4}{7}x - \frac{2}{7}y + y + z$

$2z = \frac{4}{7}x + \frac{5}{7}y + z$

$\boxed{z = \frac{4}{7}x + \frac{5}{7}y}$

$4x + 5y - 7z = 0$

Now have 3rd Equation in x,y,z!

$$\begin{pmatrix} 3x - y + z = 6 \\ x + 2y + 2z = 2 \\ 4x + 5y - 7z = 0 \end{pmatrix}$$

Now have three equations and three unknowns
Solve and get (74/45, -4/9, 28/45)

f (74/45, -4/9, 28/45) = 148/45 MIN   There is no max!