Math 1540 Test1 Study Guide

1. Calculate the Average Rate of Change (ARC) of a function $y = f(x)$ over an interval.

   ARC = $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{y_2-y_1}{x_2-x_1}$, For example, $y = f(x) = \frac{10}{x^2+1}\ over\ (1,3) = \frac{\frac{10}{3^2+1}-\frac{10}{1^2+1}}{3-2} = \frac{1-5}{2} = -2$

2. Calculate limits. $\lim_{x\to x_o} f(x) = L$ using various methods.

   a. Plug-In. Example: $\lim_{x\to 2} x^3 - 2x^2 + 3 = 2^3 - 2(2^2) + 3 = 8 - 8 + 3 = 3$

   b. A function with a "hole" Example: $\lim_{x\to -5} \frac{x^2+13x+40}{x+5} = \lim_{x\to -5} \frac{(x+8)\cancel{(x+5)}}{\cancel{x+5}} = \lim_{x\to -5} x + 8 = 3$

   c. A function with a radical. Example: $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x\to 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{(x-1)}}$

   d. A function with $\sin nx. Remember \lim_{anything\to 0} \sin(anything) = anything$

$$\lim_{x\to 0} \frac{6x + \sin 10x}{x} = \lim_{x\to 0} \frac{6x}{x} + \frac{\sin 10x}{x} = \lim_{x\to 0} 6 + \frac{10x}{x} = 16$$

$$\lim_{x\to 0} \frac{6x^2 + \sin 5x}{x} = \lim_{x\to 0} \frac{6x^2}{x} + \frac{\sin 5x}{x} = \lim_{x\to 0} 6x + \frac{5x}{x} = 0 + 5 = 5$$

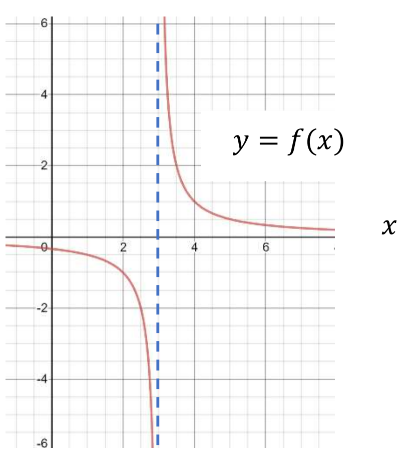
   e. Limits as x→ ∞

$$\lim_{x\to\infty} \frac{3x^3 + 4x^2 - 2x + 1}{2x^3 - 6x^2 + x - 2} = \lim_{x\to\infty} \frac{3x^3}{2x^3} = \frac{3}{2}$$

$$\lim_{x\to\infty} \frac{10x^2 - 4x + 3}{5x^3 + 2x + 12} = \lim_{x\to\infty} \frac{10x^2}{5x^3} = \lim_{x\to\infty} \frac{2}{x} = 0$$

$$\lim_{x\to\infty} \frac{10x^4 + 3x^3 - 2x + 1}{2x^3 - 5x^2 + 6x - 10} = \lim_{x\to\infty} \frac{10x^4}{2x^3} = \lim_{x\to\infty} 5x = \infty$$

   f. Infinite limits (see graph)




$$\lim_{x\to 3^-} f(x) = -\infty$$

$$\lim_{x\to 3^+} f(x) = +\infty$$

3. Calculate the slope M and the equation of the tangent line of $y = f(x)$ at the point $(x_o, y_o)$. You may calculate M for any x by the formula: (You may also calculate for a specific value of $x$ by substituting a specific value for $x$.)

$$M = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Example: $y = f(x) = 3x^2 - 2x + 5 \; at \; (2, 13)$

$$M = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2 - 2(x+h) + 5 - (3x^2 - 2x + 5)}{h}$$

$$M = \lim_{h\to 0} \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 5 - 3x^2 + 2x - 5}{h}$$

$$M = \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 5 - 3x^2 + 2x - 5}{h} = \lim_{h\to 0} \frac{6xh + 3h^2 - 2h}{h}$$

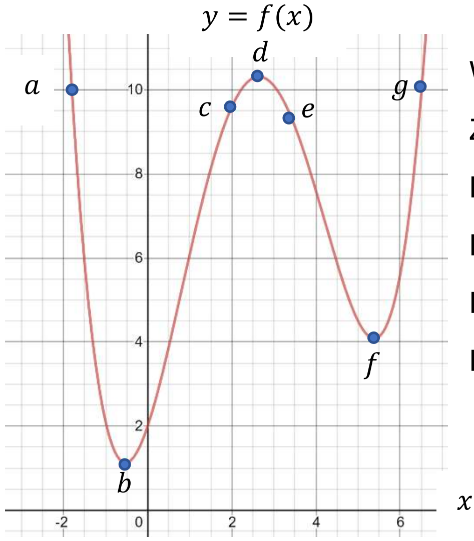
$$M = \lim_{h\to 0} \frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}} = 6x - 2, \qquad at\; x = 2, M = 6(2) - 2 = 10$$

$So, y = Mx + b = 10x + b, \quad must\; go\; through\; (2,13), \quad 13 = 10(2) + b, \quad b = -7$

$y = 10x - 7 \; is\; the\; equation\; of\; tangent\; line.$

4. Evaluate the relative values of slope (instantaneous rate of change) at a point on a graph.

$y = f(x)$




Which points are described as?

Zero Slope ____

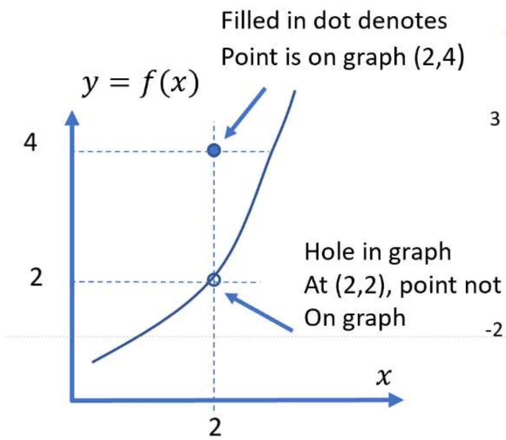
Large Negative Slope ___

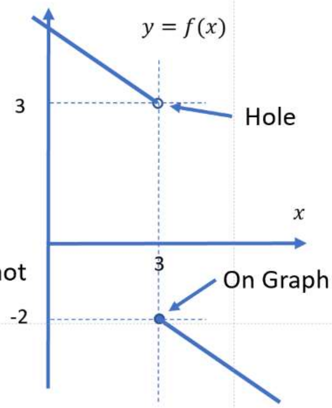
Large Positive Slope ___

Moderate Positive Slope ___

Moderate Negative Slope ___ Try first (See key at end)

5. Analyze graphs and interpret them to evaluate function values and limits.




$f(2) = 4$

$\lim_{x\to 2} f(x) = 2$

$f(3) = -2$

$\lim_{x\to 3^-} f(x) = 3$

$\lim_{x\to 3^+} f(x) = -2$

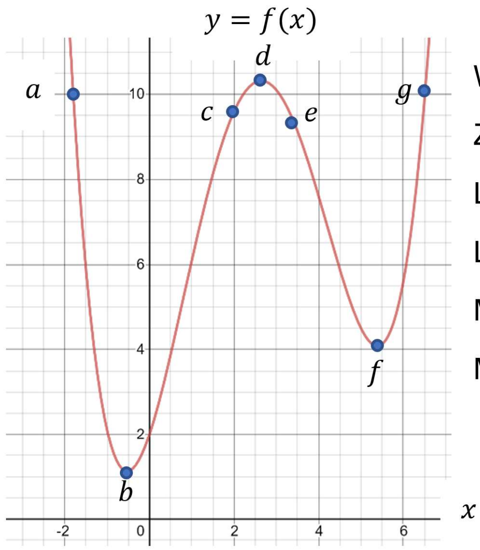
6. More Average and Instantaneous Rate of Change. On earth, an object falls a distance given by $s = 16t^2, s\ in\ feet\ and\ t\ in\ seconds$. What is the average rate of change (ARC) from 1 to 5 seconds? What is the instantaneous rate of change (IRC-velocity) at t = 5?

ARC = $\frac{16(5^2)-16(1^2)}{5-1} = \frac{400-16}{4} = 96\ ft/sec$

$$IRC = \lim_{h\to 0} \frac{s(t+h)-s(t)}{h} = \lim_{h\to 0} \frac{16(t+h)^2-16t^2}{h} = \lim_{h\to 0} \frac{16t^2+32th+16h^2-16t^2}{h}$$

$$= \lim_{h\to 0} \frac{h(32t+16h)}{h} = 32t, at\ t = 5, IRC = 160\ ft/sec$$

Key to slope question:




Which points are described as?

Zero Slope b, d, f

Large Negative Slope a

Large Positive Slope g

Moderate Positive Slope c

Moderate Negative Slope e Try first (See key at end)